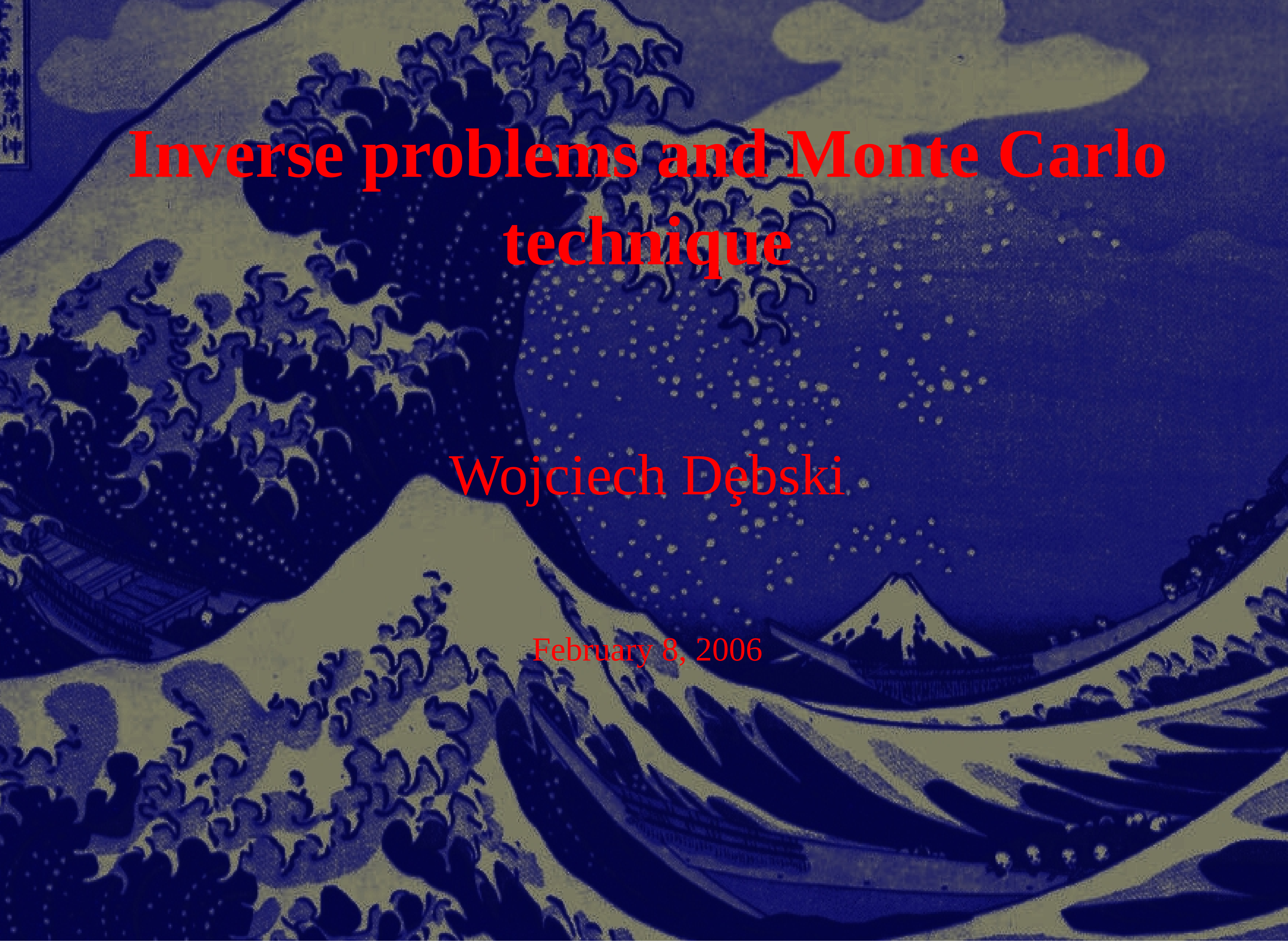

# Inverse problems and Monte Carlo technique

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February 8, 2006

# Plan of the talk

- Solutions to inverse problems
- Bayesian techniques - error estimation
- Examples
  - Location of seismic events
  - Imaging rupture process
  - seismic tomography

# Inverse problems

Inverse problems appears on the scen when:

1. we want to know parameters that cannot be measured directly (indirect measurements)

2. an inference from a given data (information) set is performed (non-parametric inversion)

3. given theory must be verified (!!!)

# Inverse problem - Indirect Measurements

$$\mathbf{d}^{obs} \implies \mathbf{m}$$

Solution

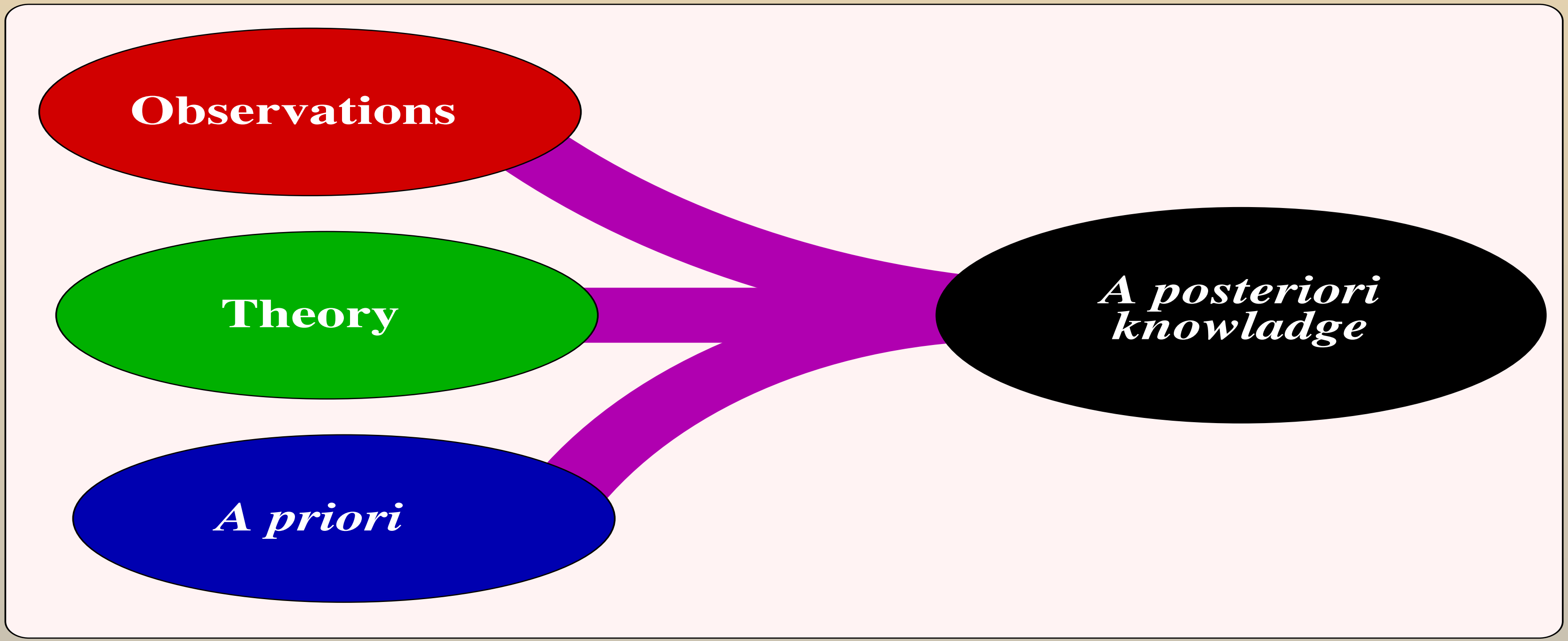
$$||\mathbf{d}^{obs} - \mathbf{d}^{th}(\mathbf{m})|| + \lambda||\mathbf{m} - \mathbf{m}^{apr}|| = \min$$

Errors

$$\mathbf{m}^{true} = \mathbf{m}^{ml} + \epsilon_{\mathbf{m}}$$

$$\epsilon_{\mathbf{m}} = ???$$

# Bayesian Inversion - Basic Ideas




$$\sigma_{post}(\mathbf{m}|\mathbf{d}^{obs}) \sim \sigma_{apr}(\mathbf{m}) \int_{\mathbf{D}} \sigma_{th}(\mathbf{d}|\mathbf{m}) \star \sigma_{obs}(\mathbf{d}|\mathbf{d}^{obs}|)$$

# Bayesian Inversion

**(parameter estimation)**

*A posteriori* pdf

$$\sigma(\mathbf{m}) = f(\mathbf{m})L(\mathbf{m},\mathbf{d})$$

- $f$ - A priori pdf (prob. dens. funct.)
- $L$ - Likelihood function

Errors independent of $\mathbf{m}$ and $\mathbf{d}$

$$\mathbf{d}^{th} = \mathbf{G}(\mathbf{m})$$

$$\sigma(\mathbf{m}) = f(\mathbf{m})\exp\left(-||\mathbf{d}^{obs} - \mathbf{G}(\mathbf{m})||\right)$$

# Bayesian approach

*A posteriori* pdf $\sigma(m, d)$:

- always exists
- is unique
- describes all information
- is the solution of an inverse problem

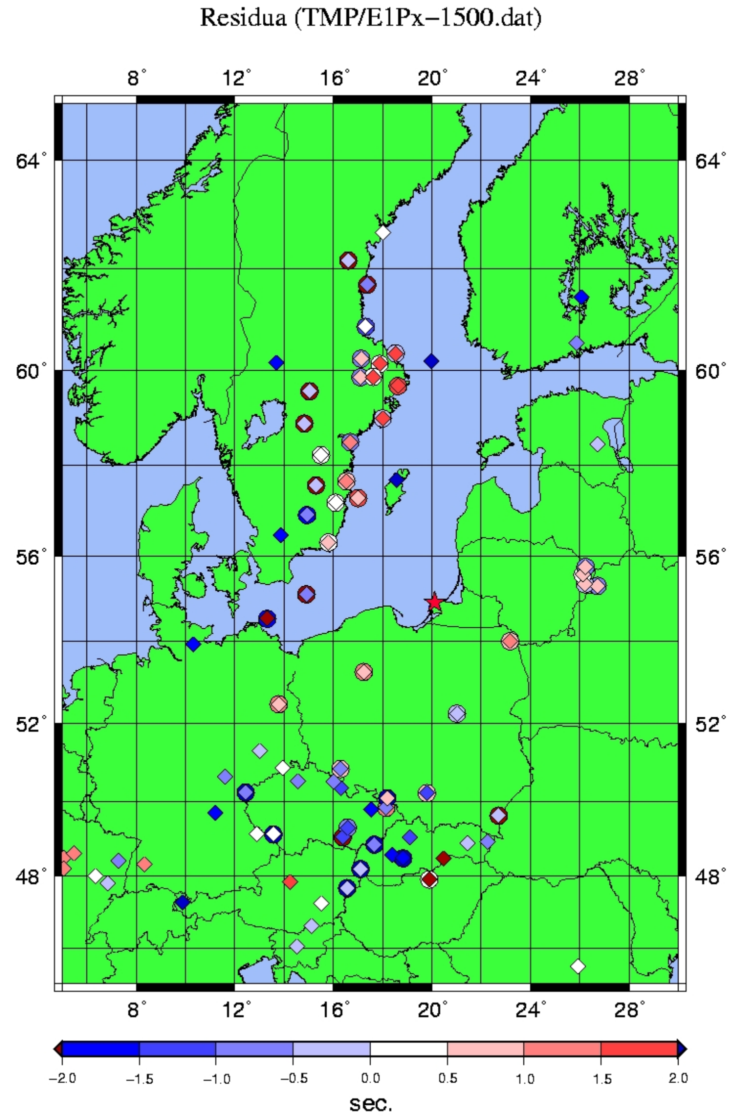
When and why we need to use this approach ???

ERROR ANALYSIS !!!

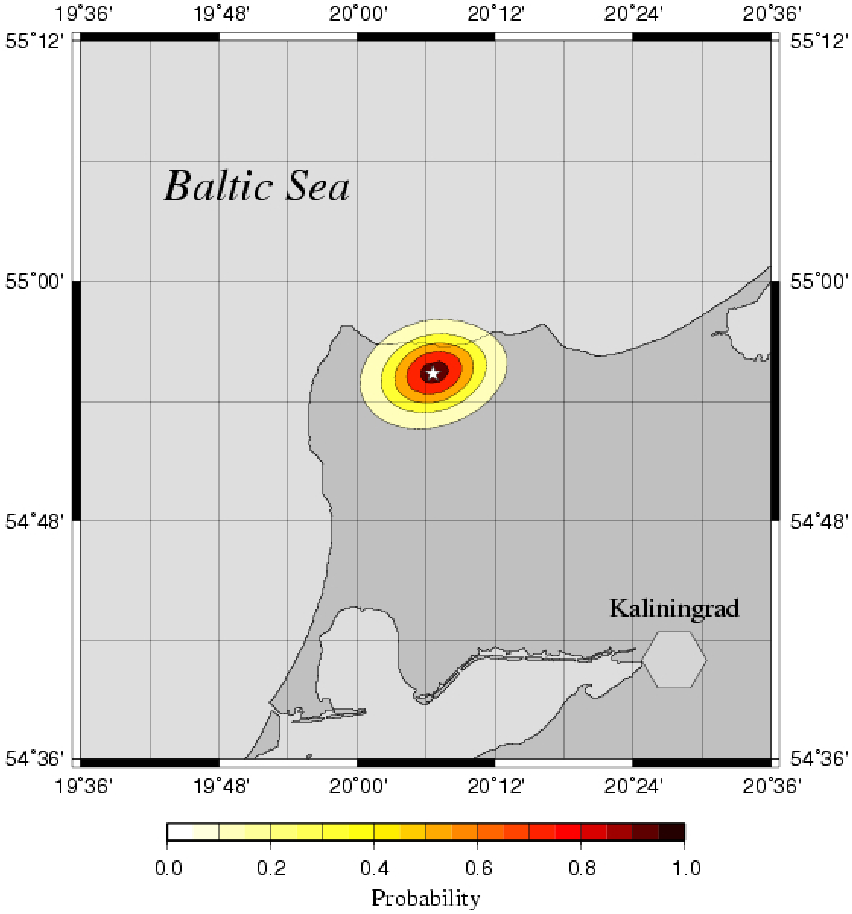
# Examination of $\sigma(\mathbf{m})$

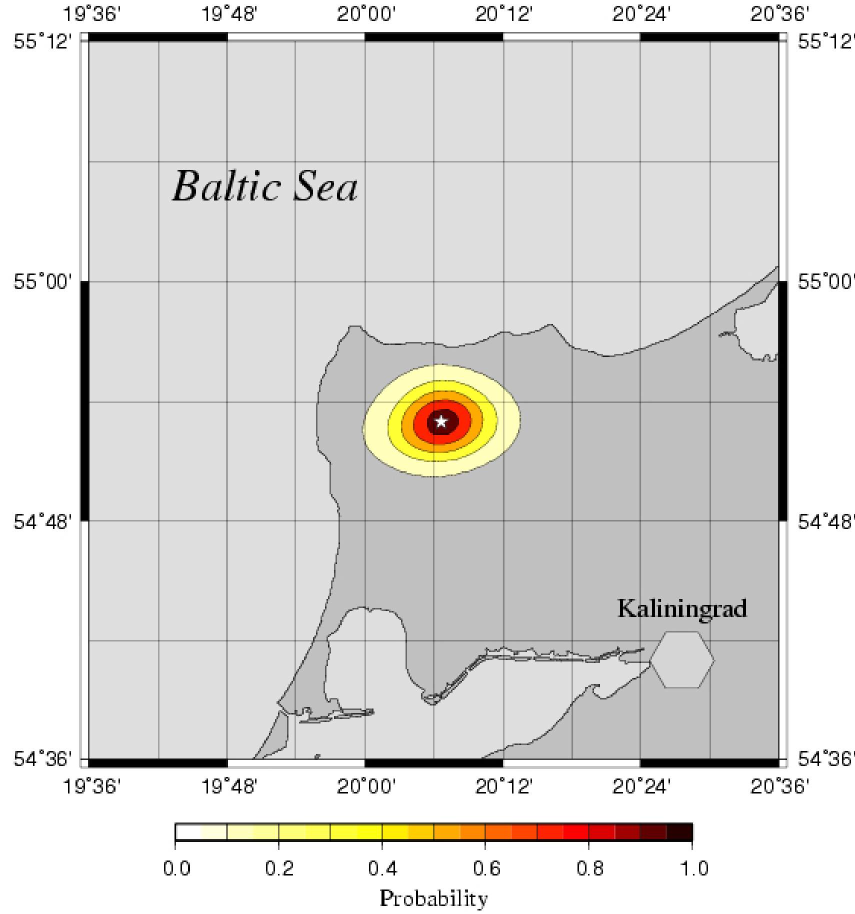
- Searching *Maximum Likelihood* solution
  - Gradient methods
  - Deterministic methods (e.g.  simplex)
  - Monte Carlo - Global Optimization
    - Simulated Annealing
    - Genetic Algorithm
- Sampling over regular grid
- **Random (Monte Carlo) Sampling**

# 2004 earthquakes in Kalinigrad area ($M_L \approx 5/5.2$)

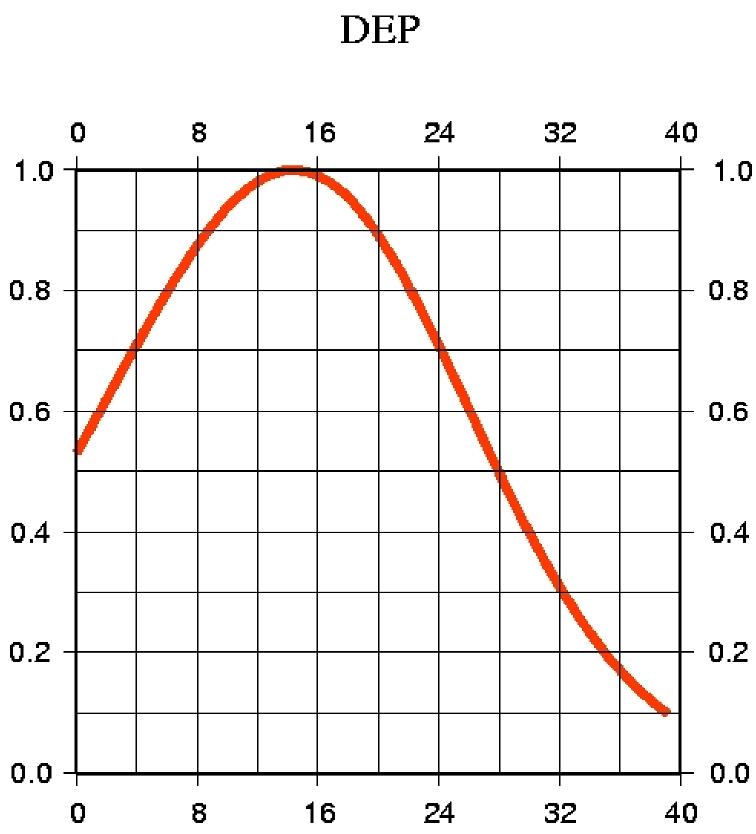


# Epicenter location



# Depth


DEP
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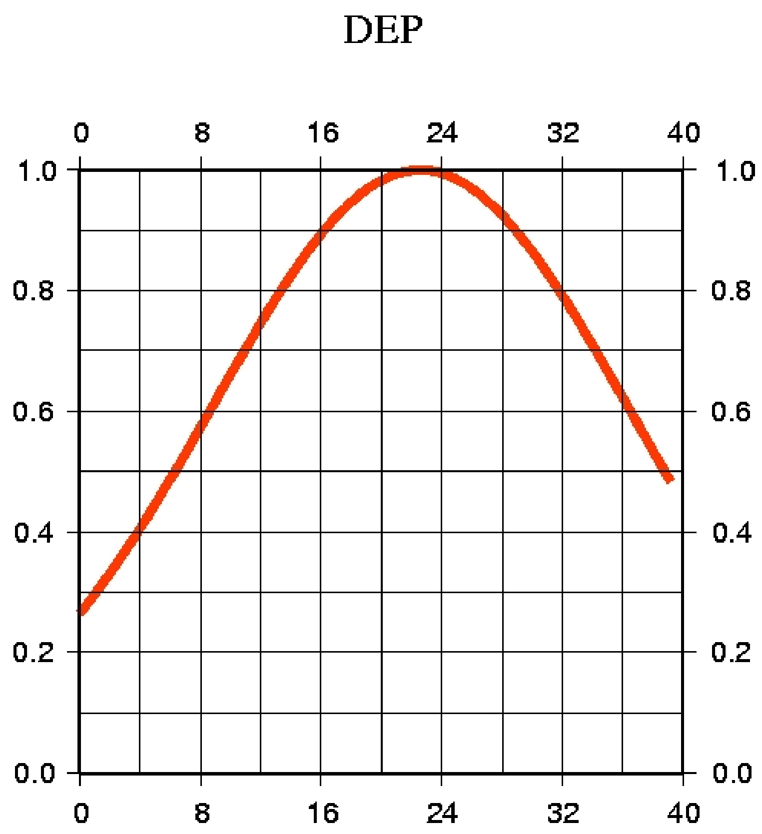
DEP
0
8
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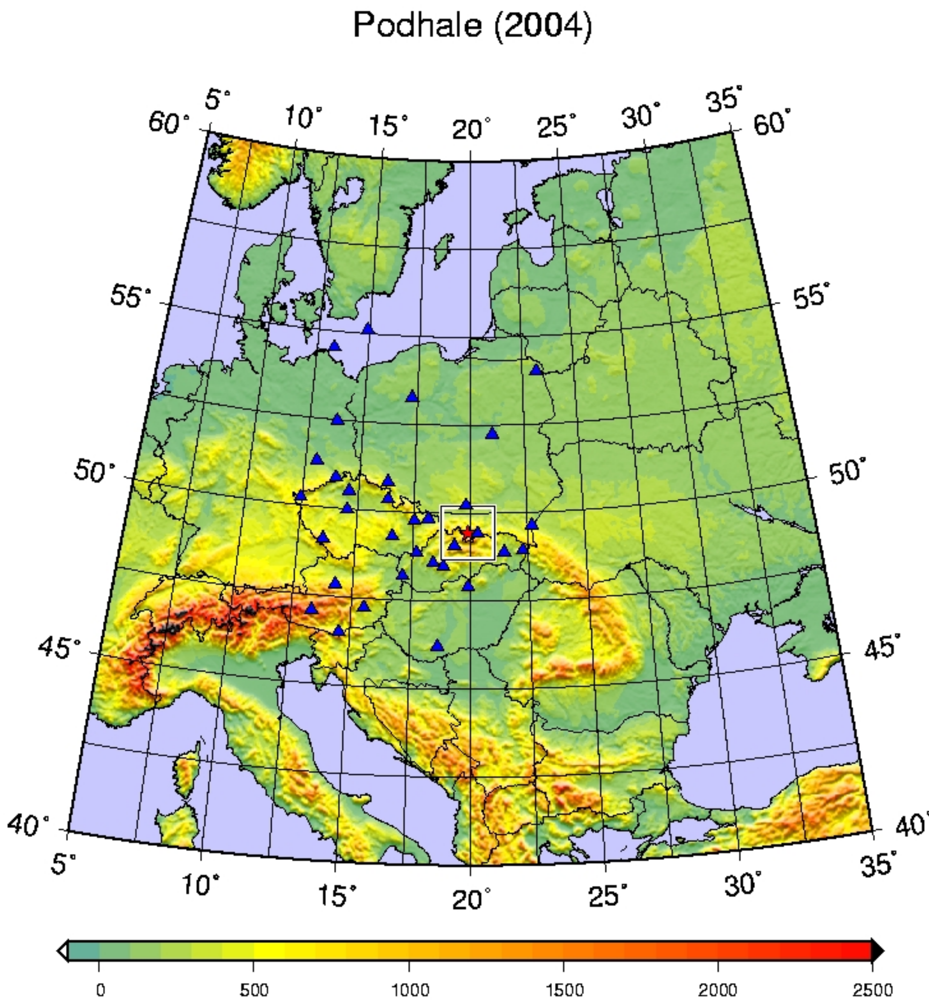
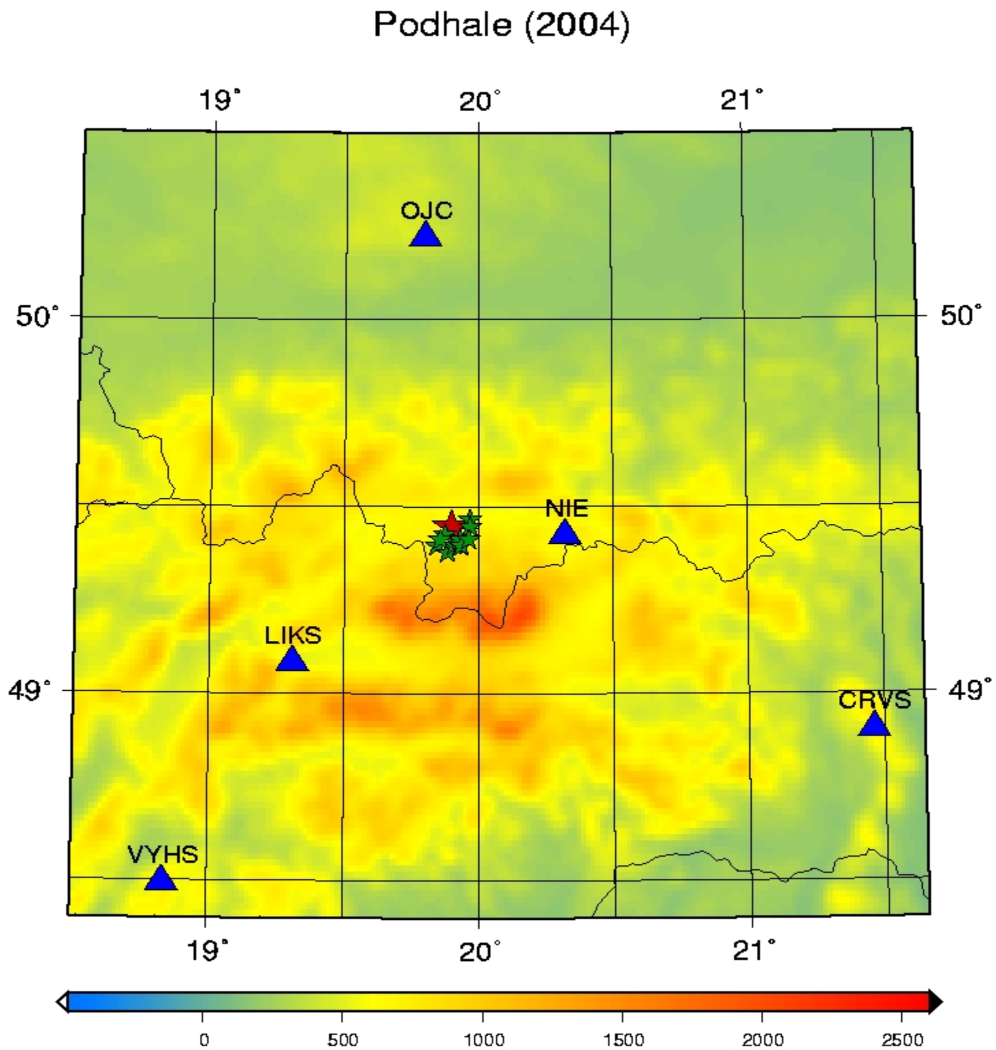
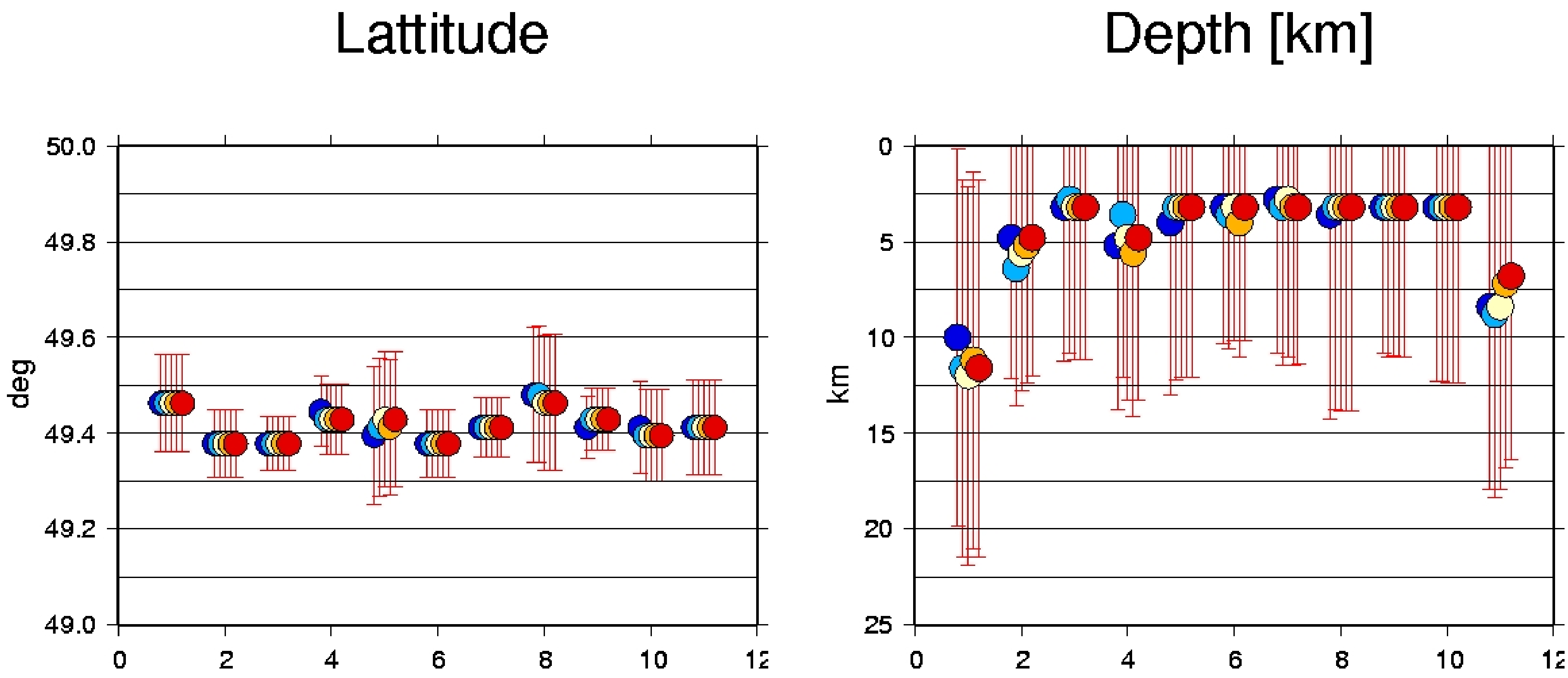
# Natural seismicity in Poland (A)



# Natural seismicity in Poland (B)



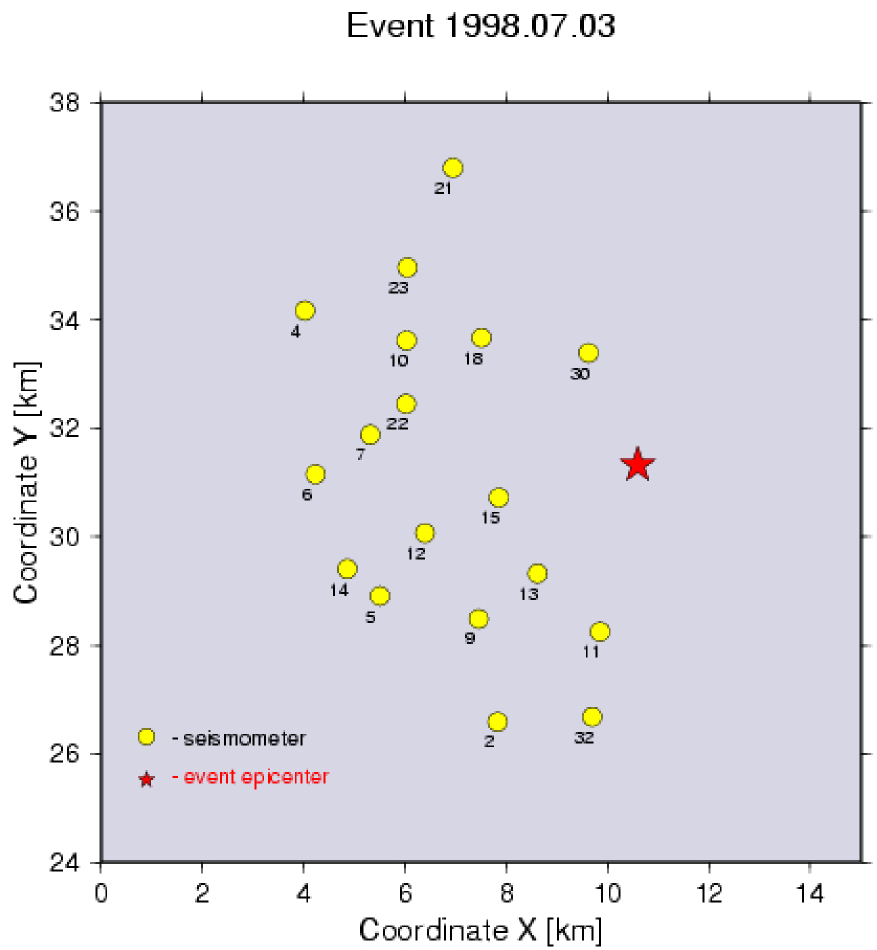
# 2004 Karpathy earthquakes (MC errors)



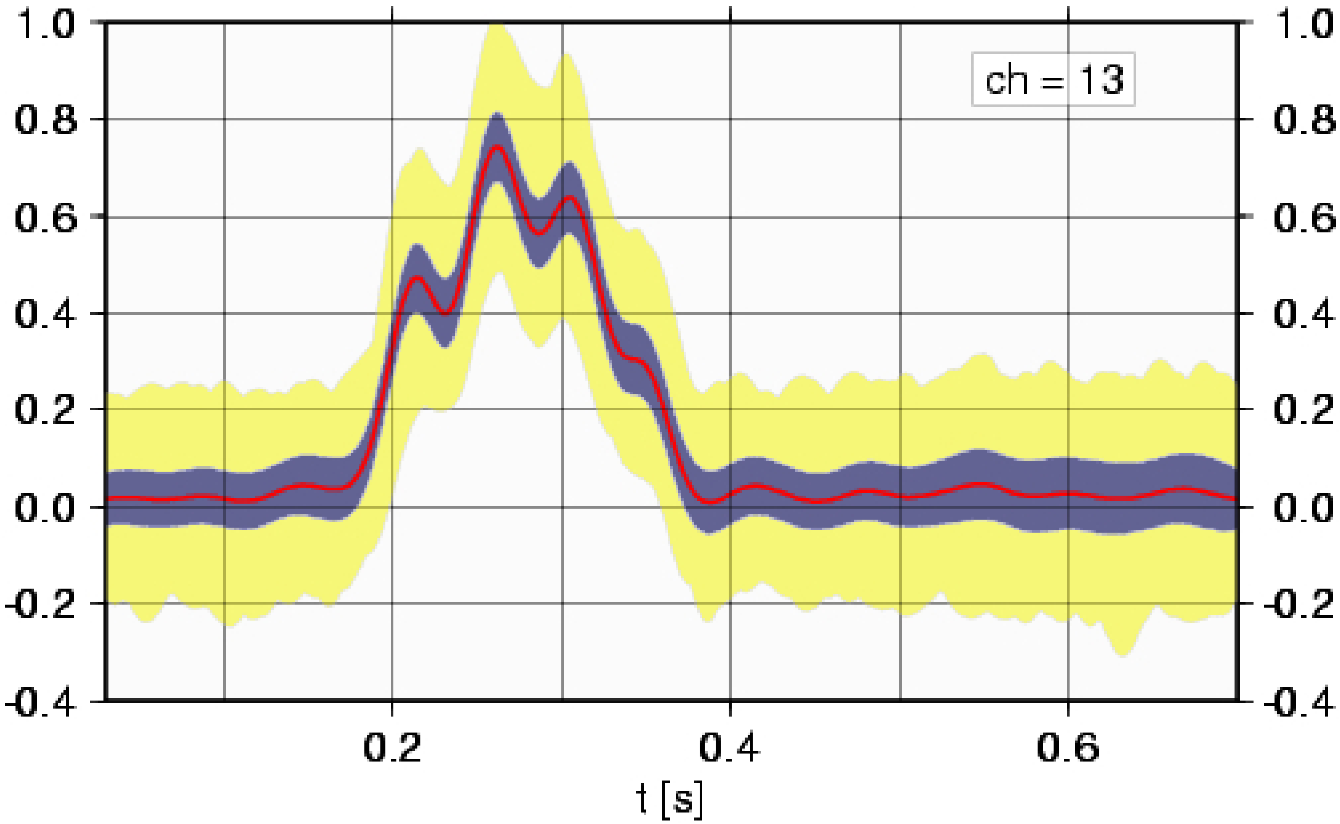
# Imaging rupture process

$$u(x,t) = \int_{t,x'} \dot{M}(t) G(t - t', x - x')$$

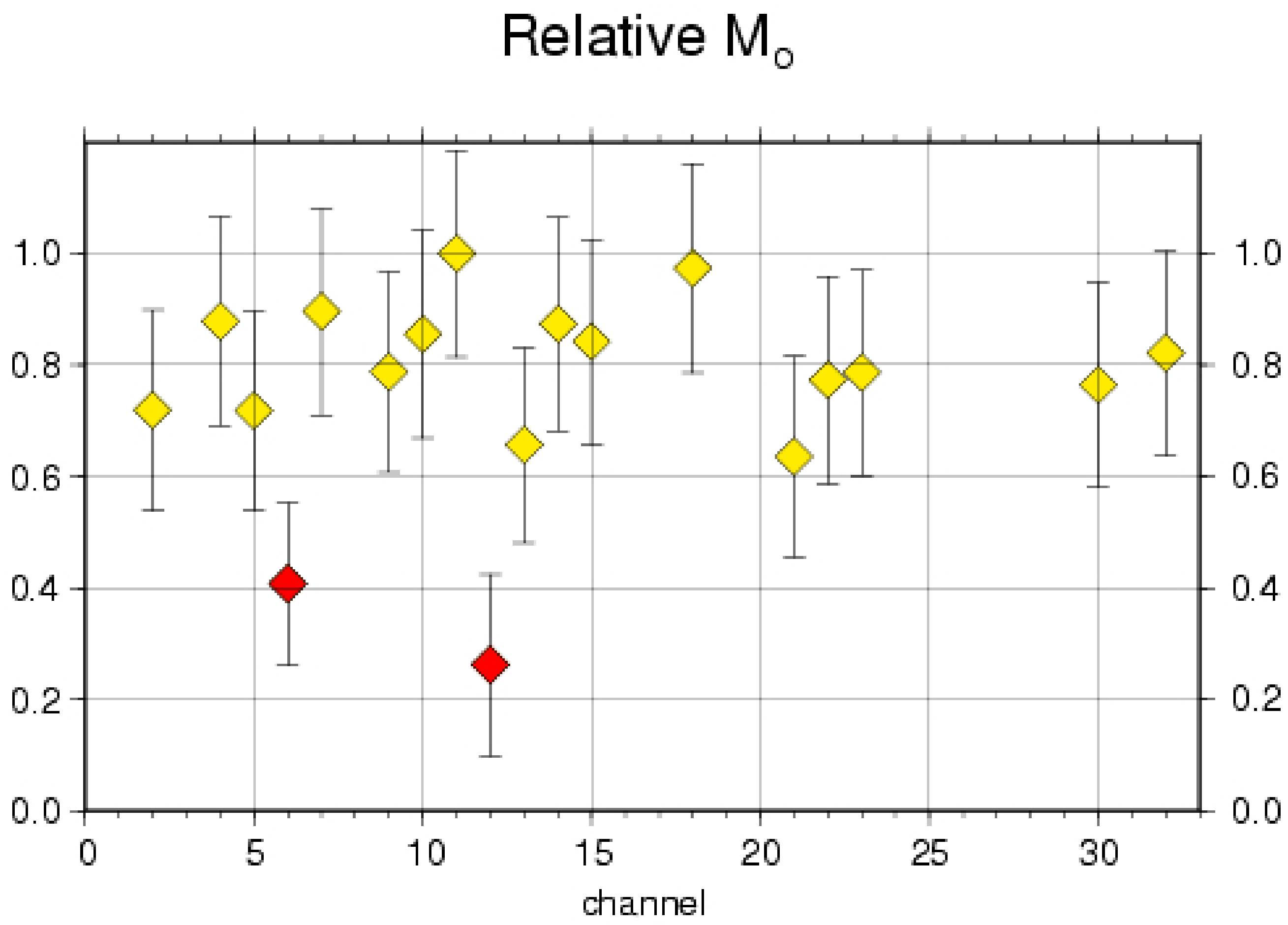
# Copper mines seismic network



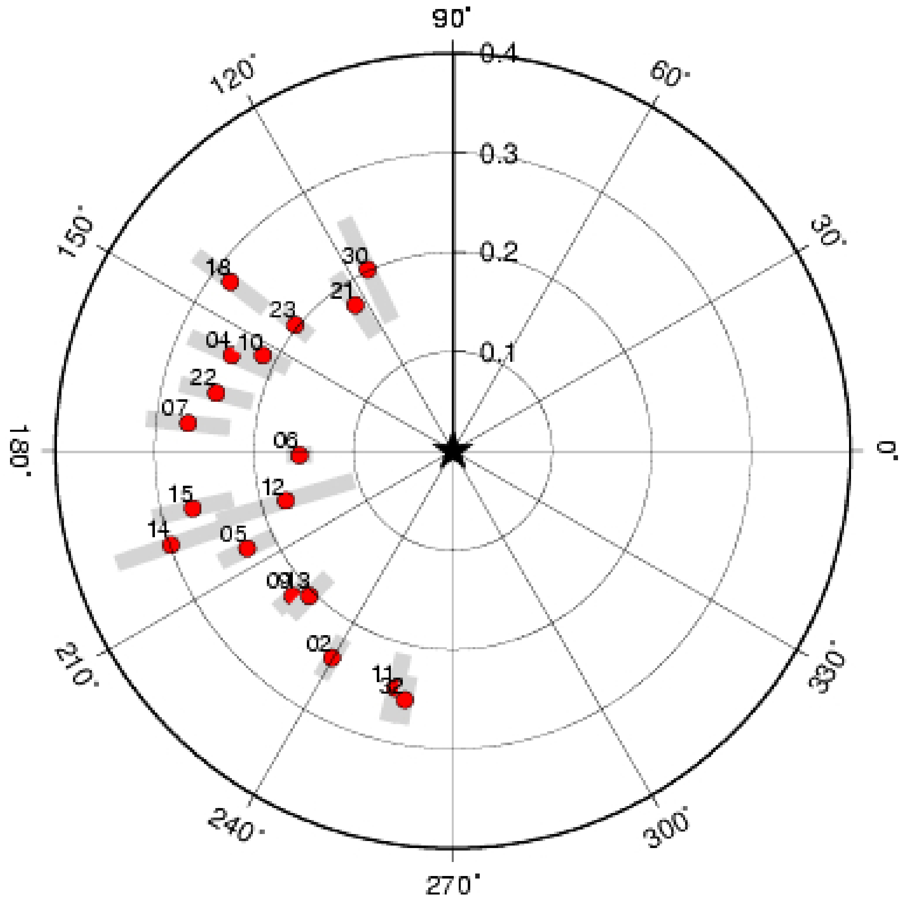
# STF - channel 13



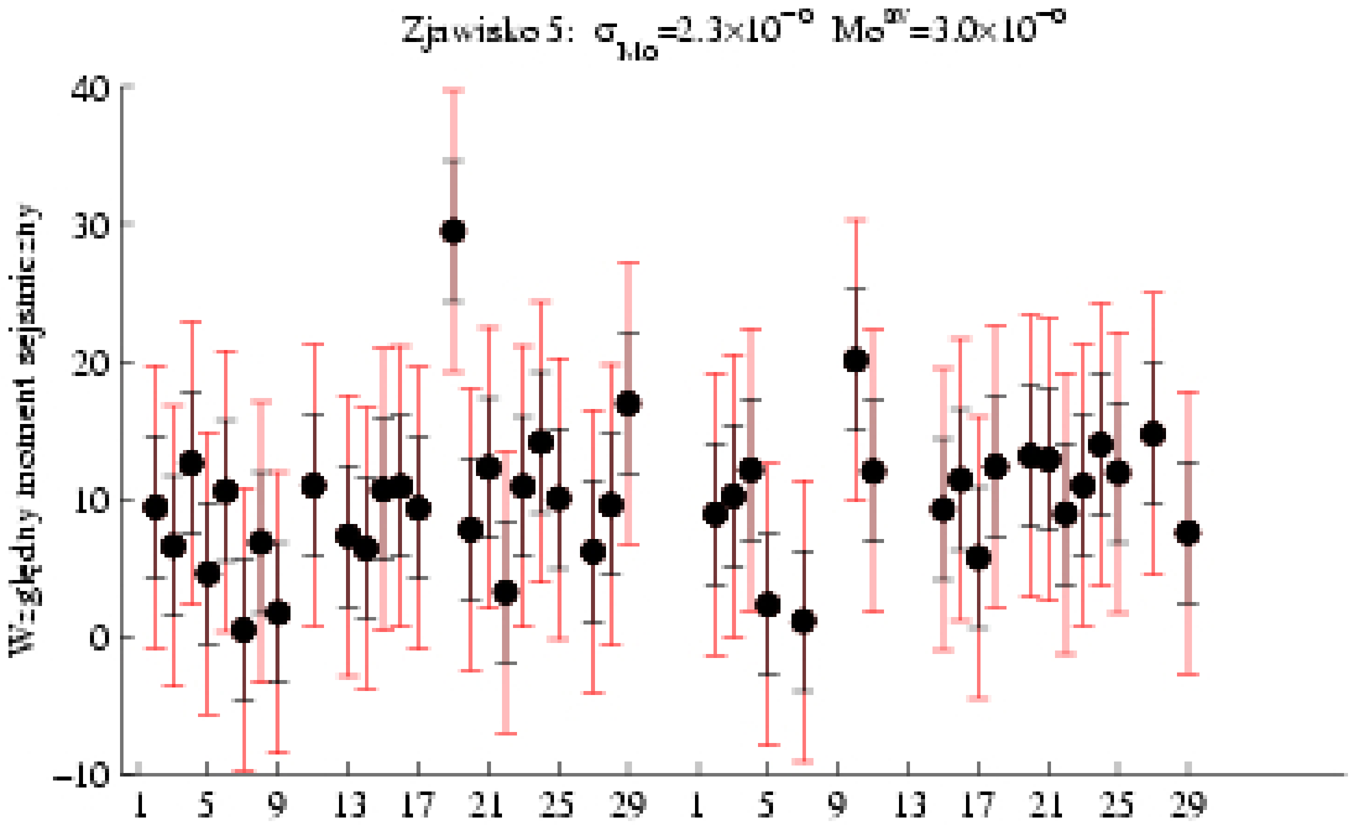
# Seismic moment $M_o$



# Radiation pattern



# Another example



# Tomography and expectations?

Tomography should provides:

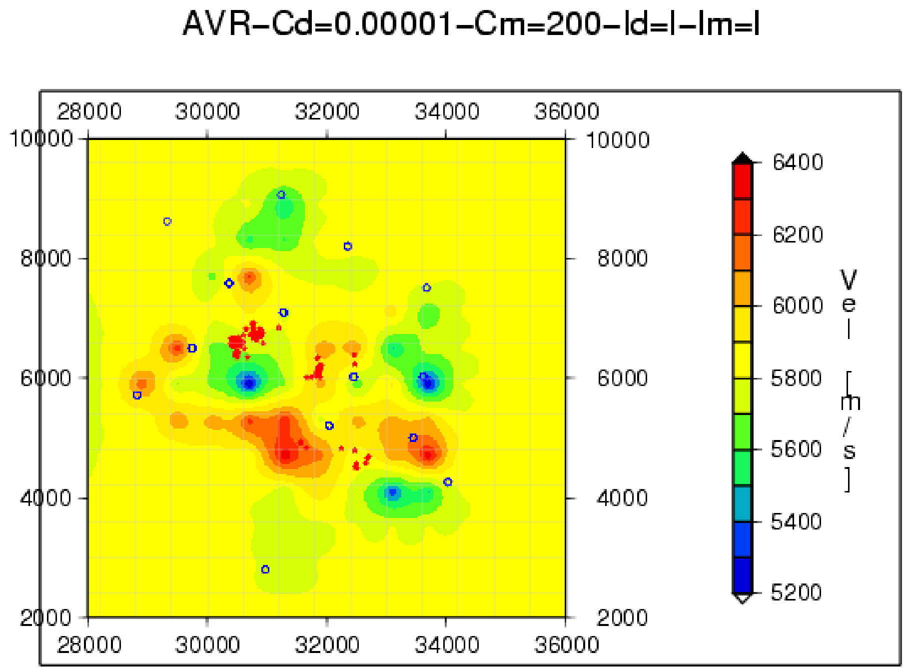
1. spatial heterogeneity distribution

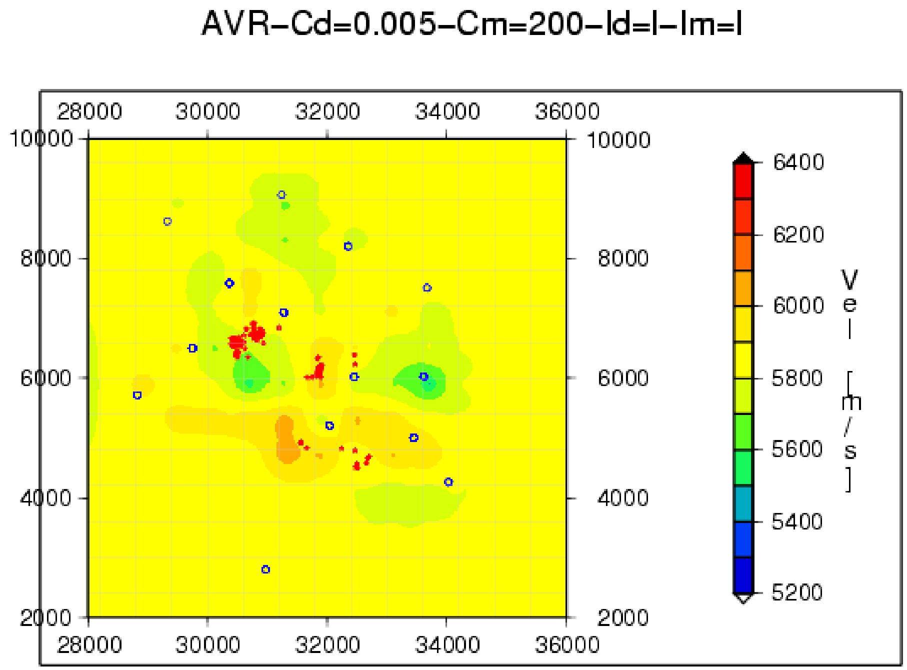
2. quantitfy "strenght" of heterogeneities

PROBLEM:

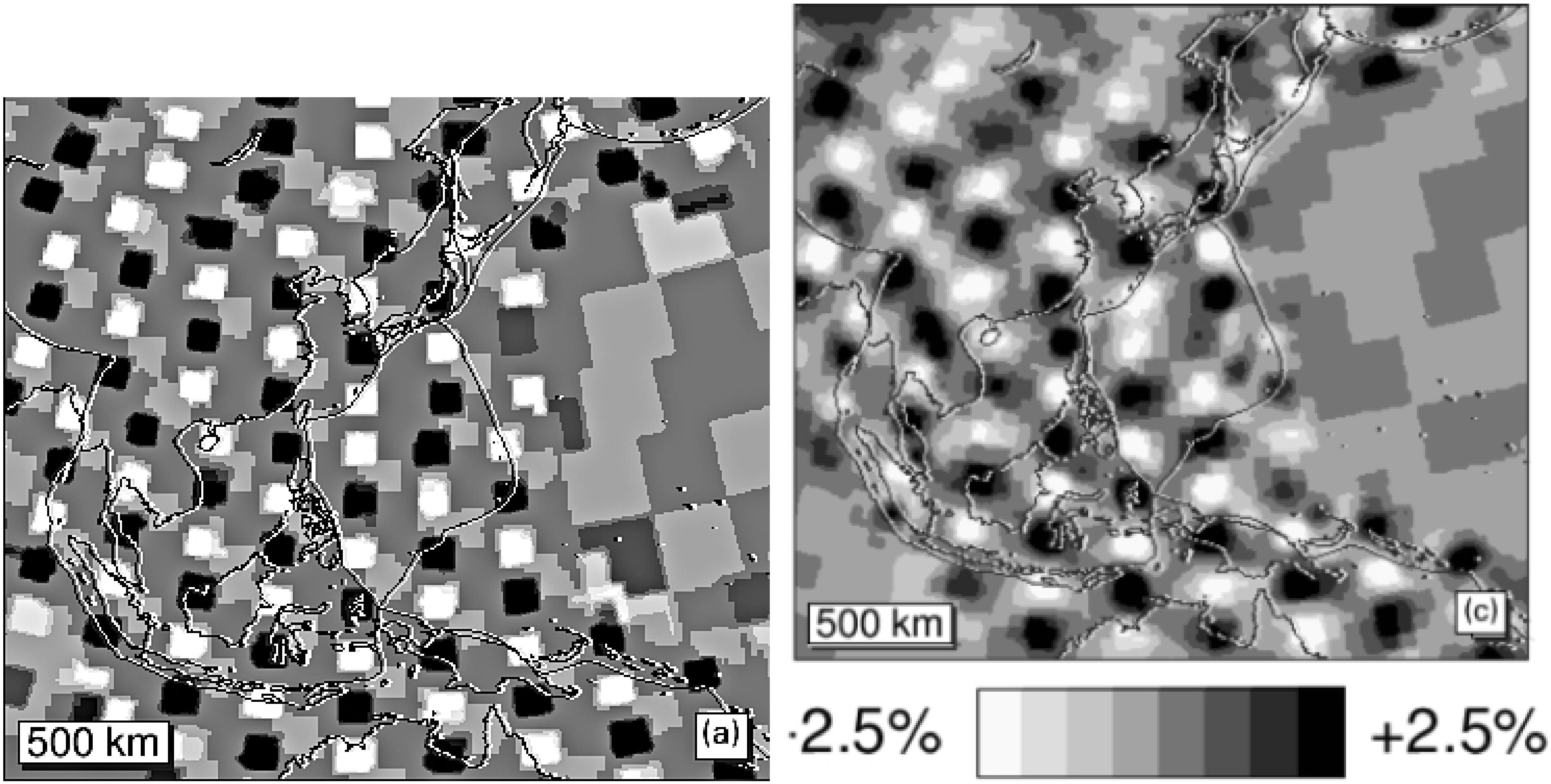
$$||\mathbf{d}^{obs} - \mathbf{d}^{th}(\mathbf{m})|| + \lambda||\mathbf{m} - \mathbf{m}^{apr}|| = \min$$

# Heterogeneities

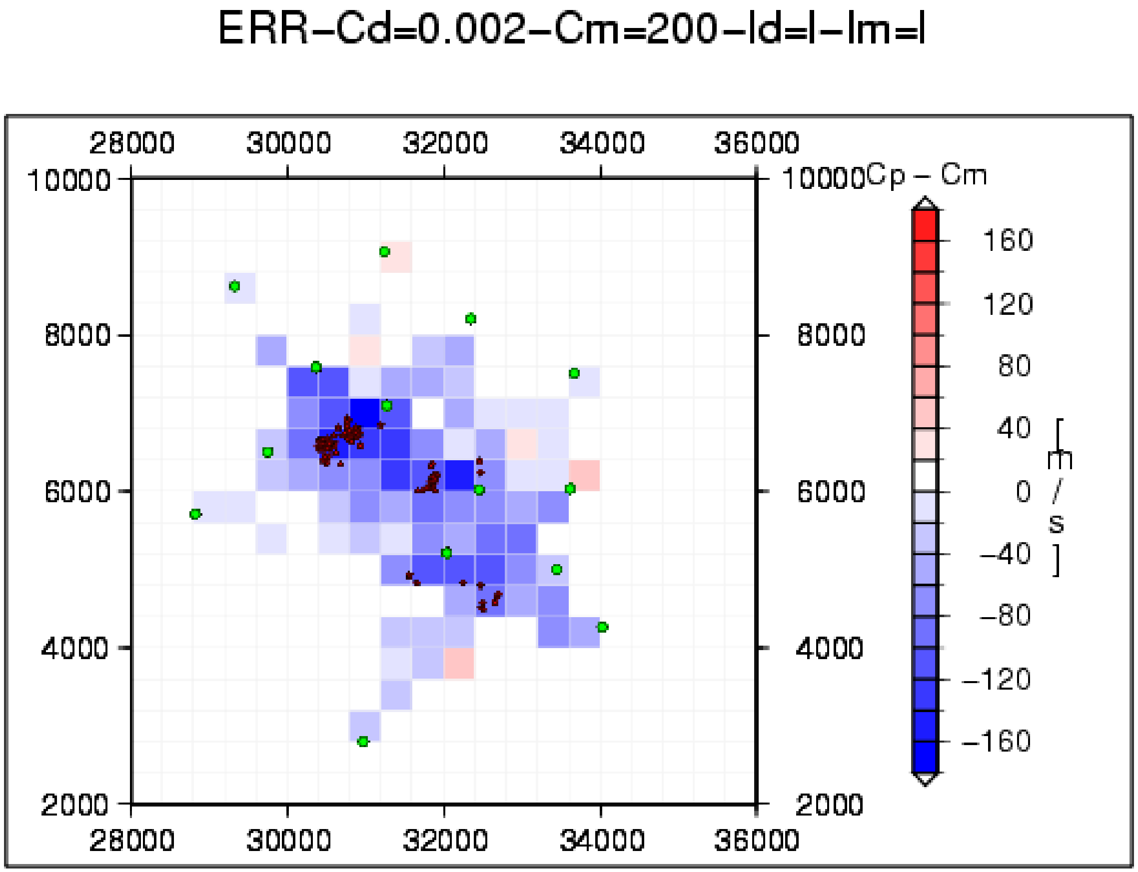


# Classical approach



# Example of an error estimation



## Sampling to tomography solution

SEE MOVIE