Seismic Tomography

ideal (almost) imaging tool

Wojciech Dębski*

ggc00@cc.keele.ac.uk

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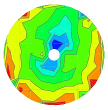
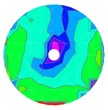
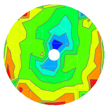
*On leave of absence of IGF PAS, Warsaw, Poland



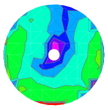
Plan of the seminar



- Part I – Principles
  - illumination and back-projection
  - data/theoretical uncerteinities
  - non uniqueness
  - Bayesian inversion
- Part II – Hot topics
  - Optimal Grid Choice
  - Enhanced Velocity Tomomography
  - Attenuation Tomography
- Part III – Conclusions
  - Iterative vs. Global Optimization



Tomography ...



The term tomography comes from the Greek word *tomos*, that is, slice, and means forming an image of an object from measurements made from slice (rays) trough it.

The waves passing through the object accumulate information on the spatial distribution of its physical properties and, recorded outside of the object's body, are used to infer this distribution.

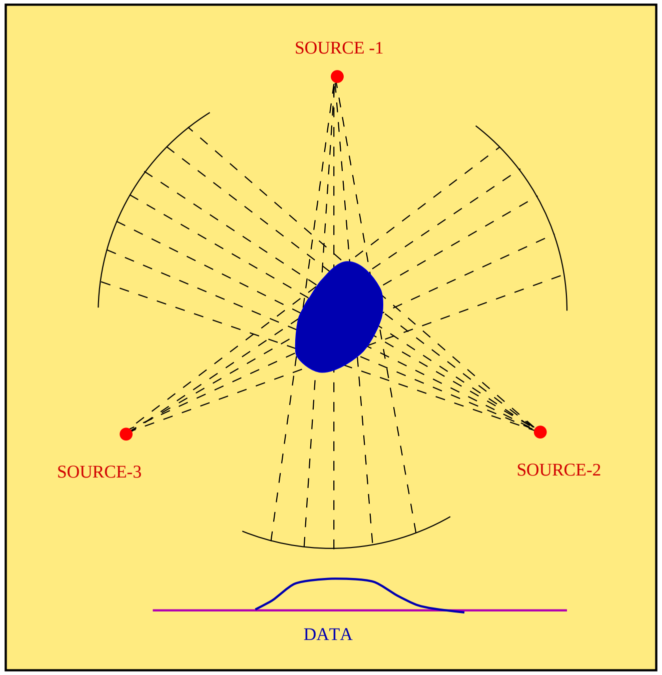
Basic Assumption: the wavelengths of the used (recorded) waves are smaller than the size of heterogeneities of the studied parameters; the waves propagate along lines and sample the medium only in points through which the rays pass.

Example (velocity tomography)

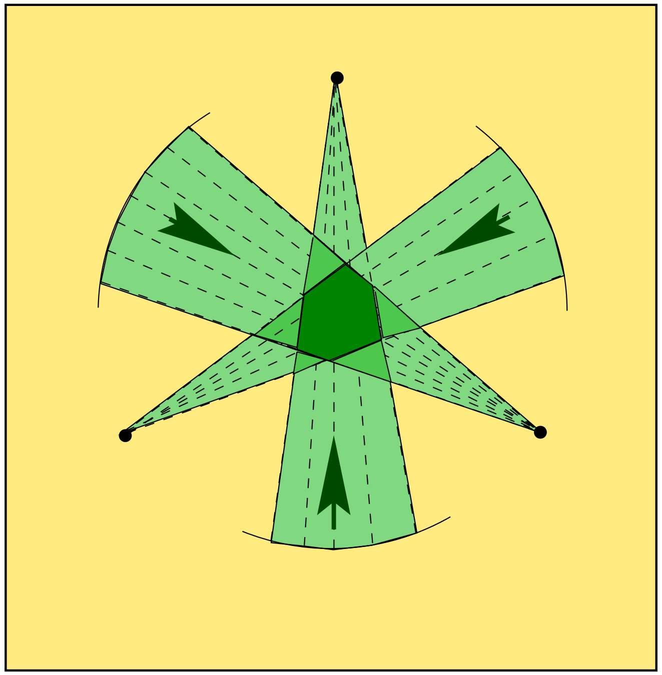
$$t = \int_{ray} \frac{ds}{v(s)}$$

"Back-projection"

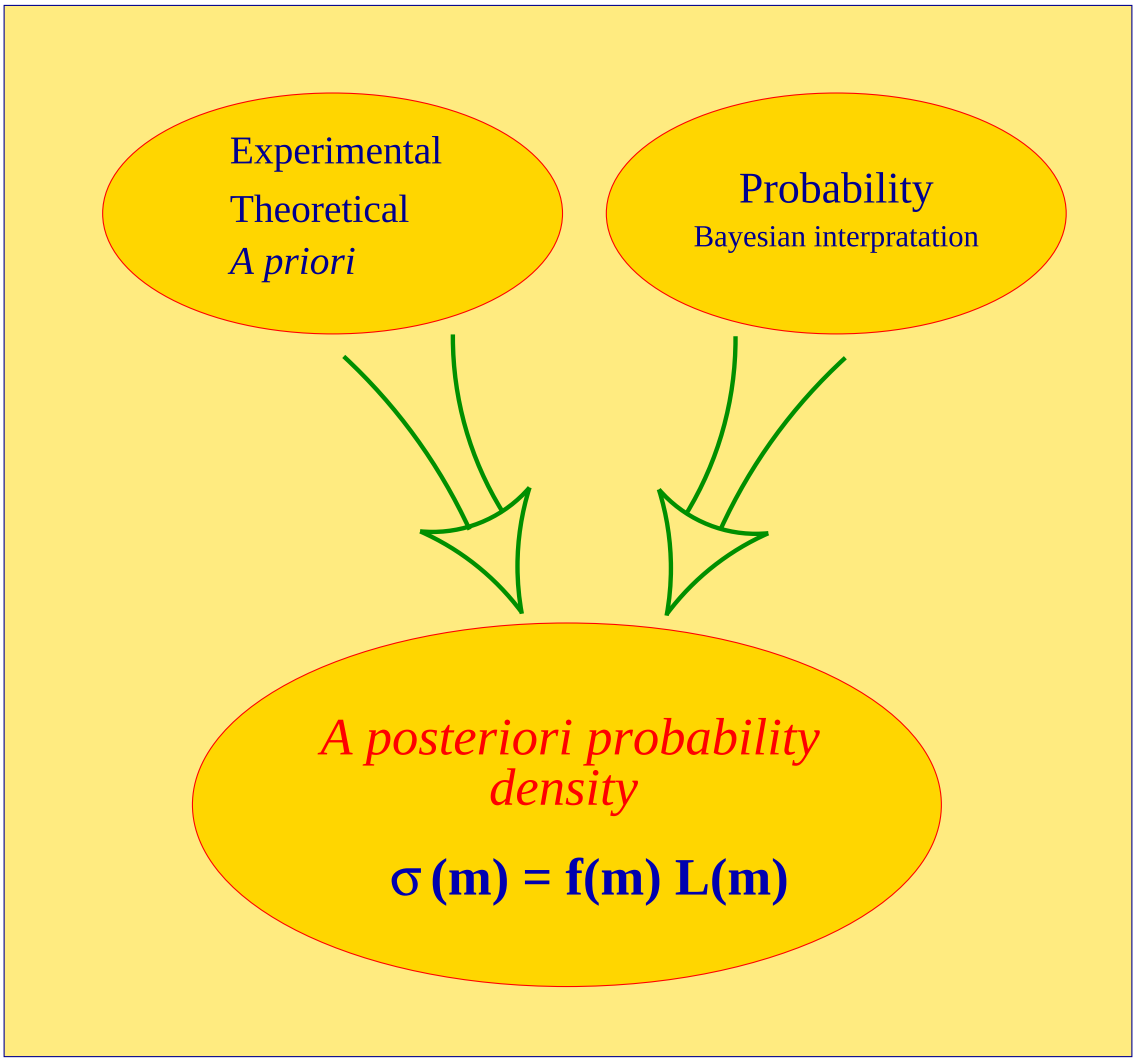
Projection

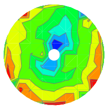


"BackProjection"

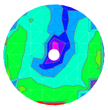


Solving Tomography Problem





Optimal grid



- linear tomography

$$\mathbf{d} = \mathbf{G} \cdot \mathbf{m}$$

- $l2$ - least square solution

$$\mathbf{m}^{ML} = \mathbf{m}_{apr} + \mathbf{H}^{-1} \cdot \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot (\mathbf{d}_{obs} - \mathbf{G} \cdot \mathbf{m}_{apr})$$

- resolution matrix

$$\boxed{\mathbf{m}^{ML} - \mathbf{m}_{apr} = \mathbf{R} \cdot (\mathbf{m}_{true} - \mathbf{m}_{apr})}$$

$$\mathbf{R} = \left(\mathbf{G}^T \cdot \mathbf{G} + g^2\, \mathbf{I}\right)^{-1} \cdot \mathbf{G}^T \cdot \mathbf{G}$$

Resolution matrix

- Ideal case: $\mathbf{R} \approx \mathbf{I}$

  never achieved due to a limited number of observations $d^i_{obs} \quad i = 1 \ldots N_{obs}$

- Normalized diagonal elements:

$$0 \le \mathbf{R}_{ii} \le 1$$

- $\mathbf{R}_{ii} = 0$ only for cells probed by no rays

- Average (minimal) values of $\mathbf{R}_{ii} \quad ii = 1 \ldots \mathrm{N}_{cell}$ strongly depends on:
  - source-receiver configuration
  - cell size

Rock Sample

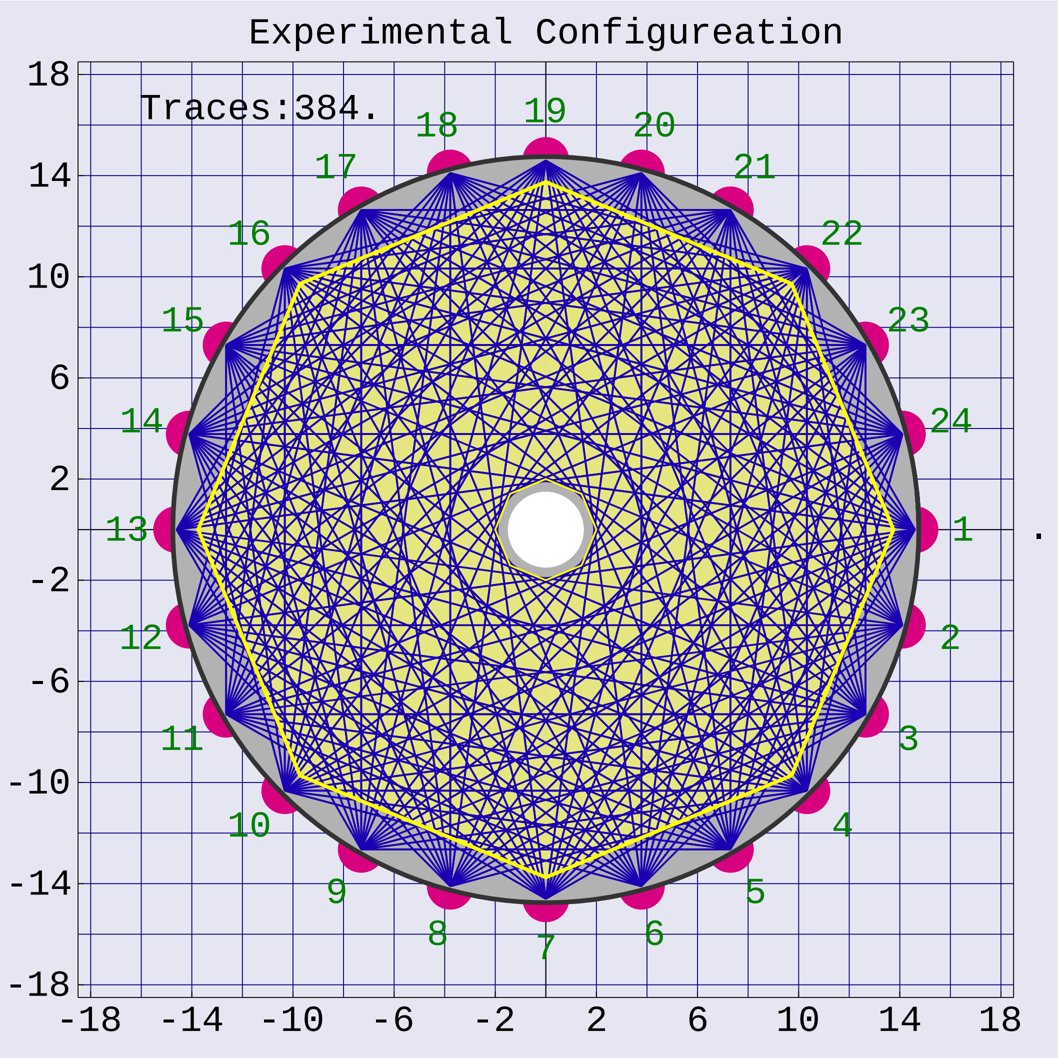


Figure 1: Layout of transducers and ray paths

Resolution Curve

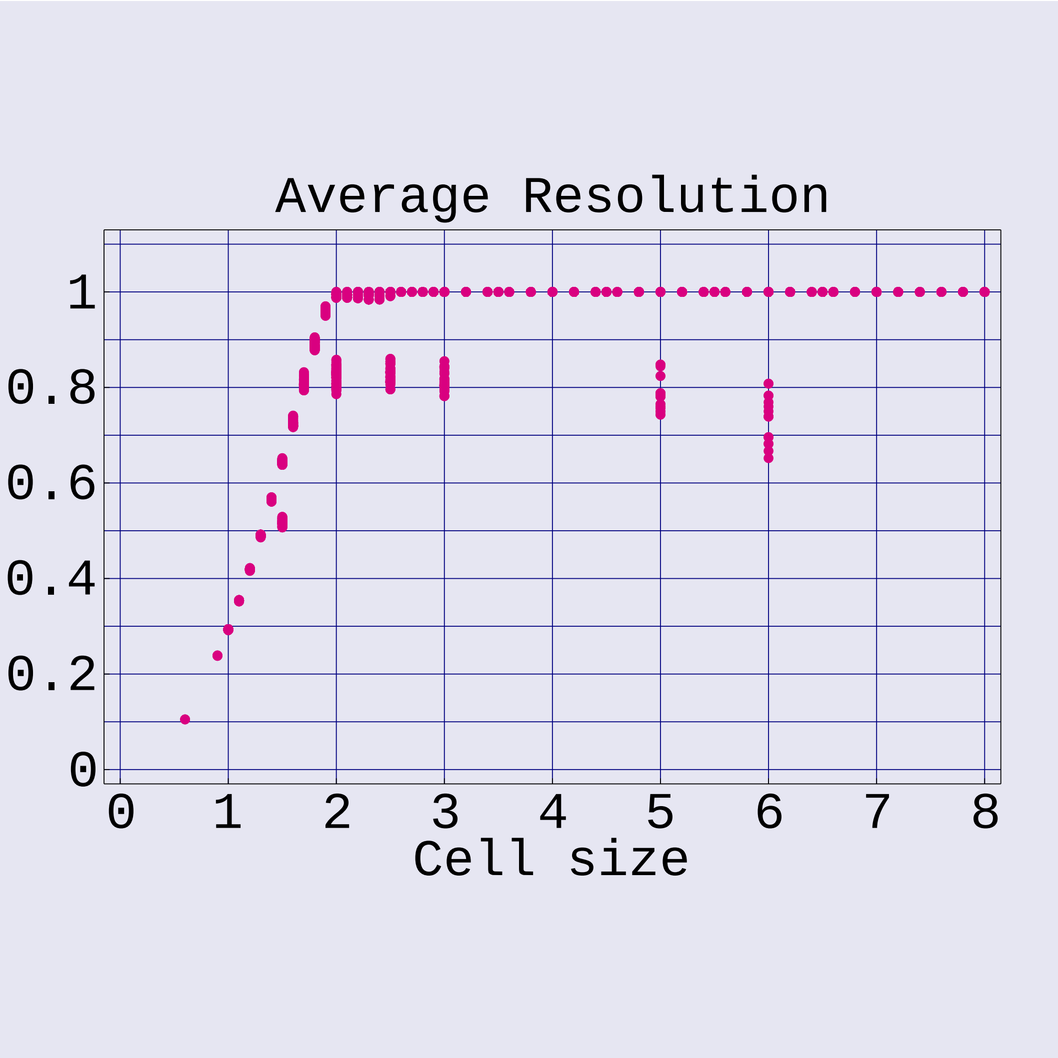
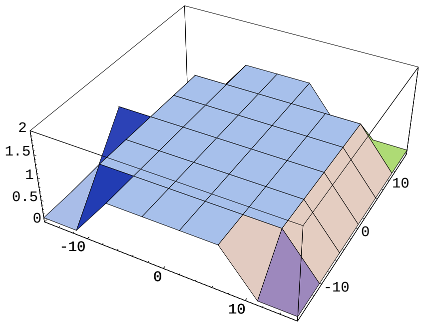


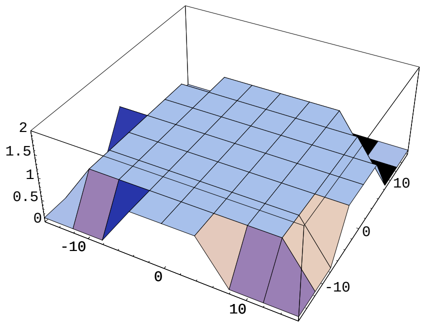
Figure 2: Monte Carlo simulation of the resolution curve

Synthetic images

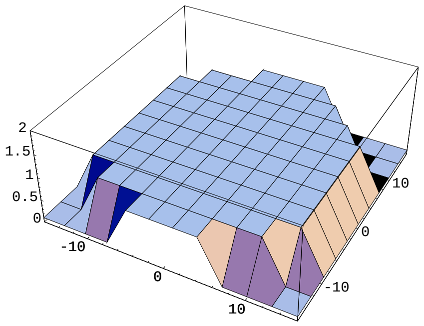
cs = 5cm



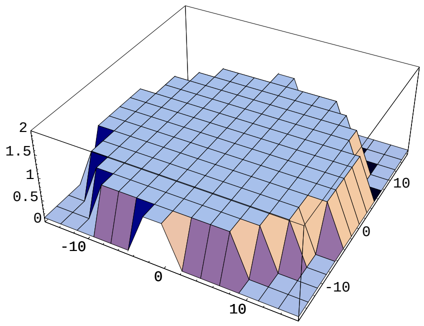
cs = 4cm



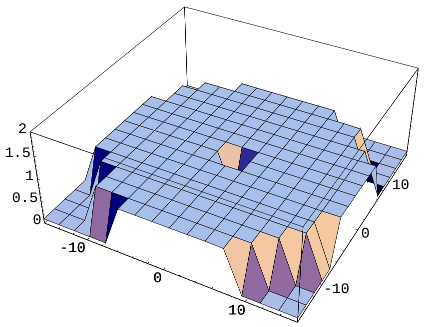
cs = 3cm



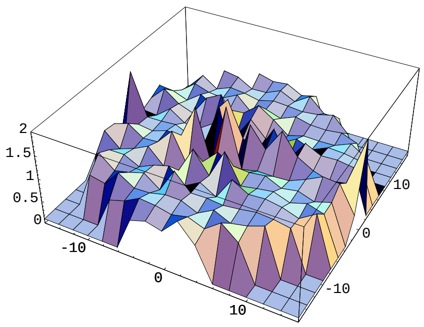
cs = 2.2cm



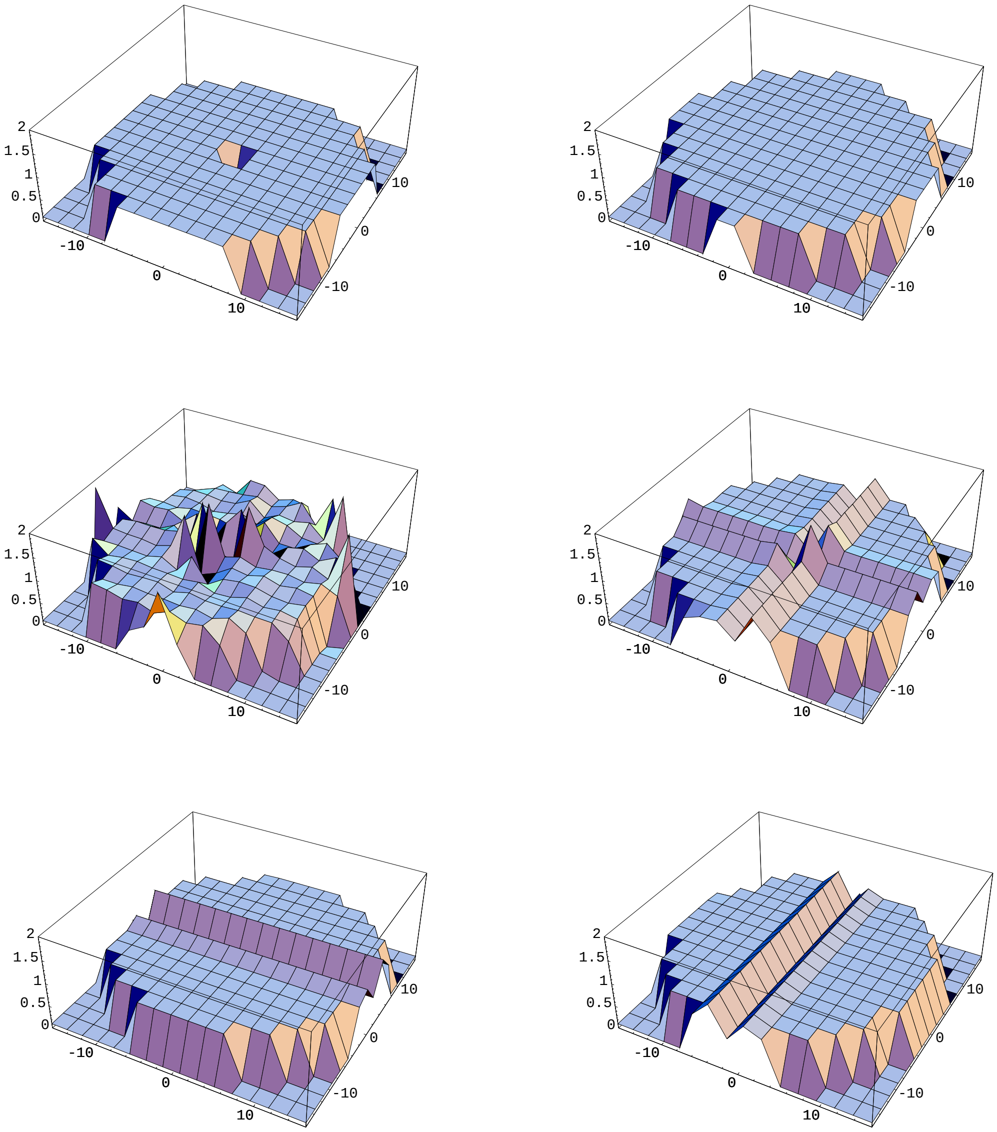
cs = 2cm

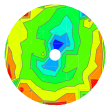


cs =1.9cm

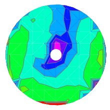


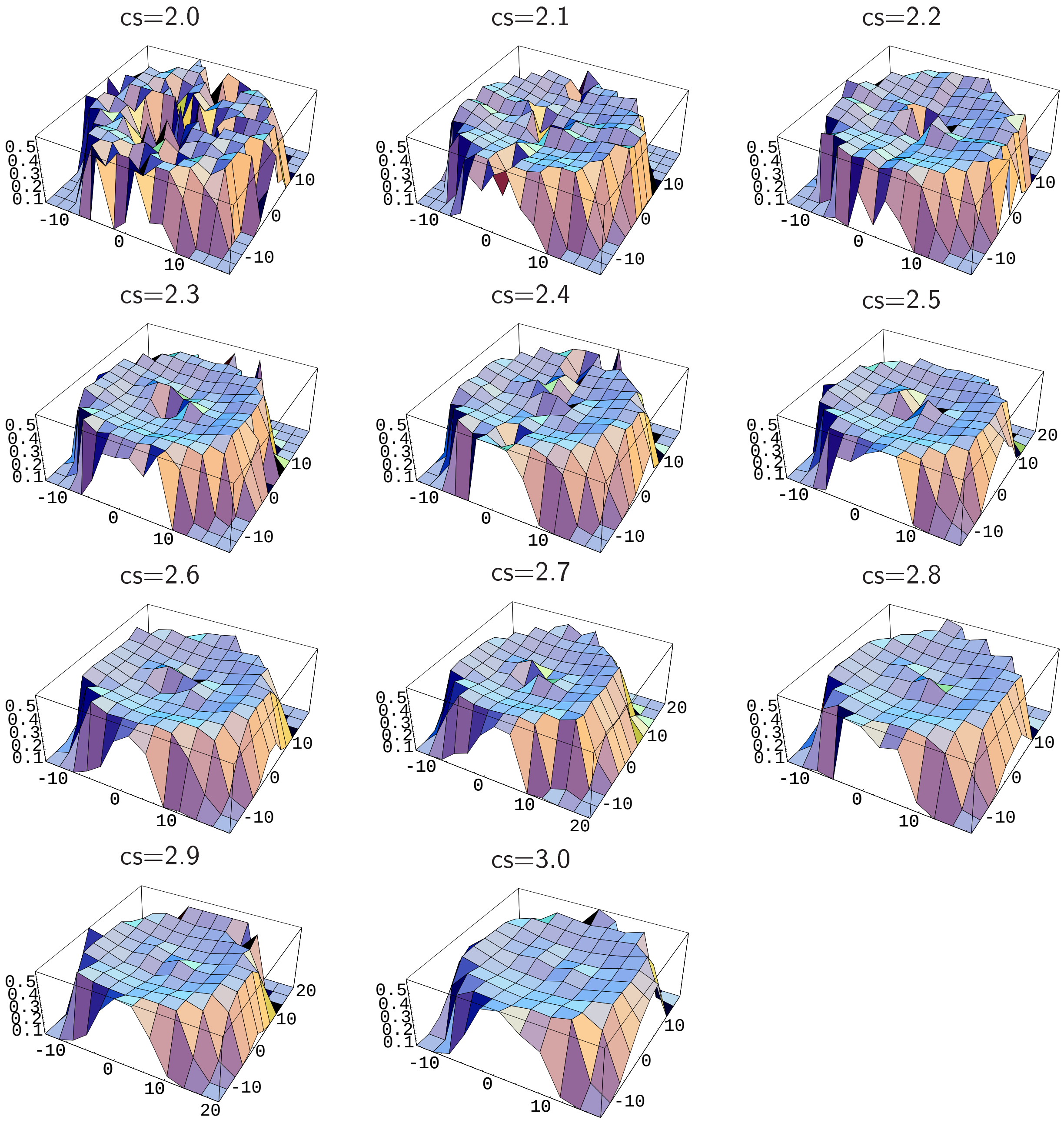
Synthetic images
(2cm cells)



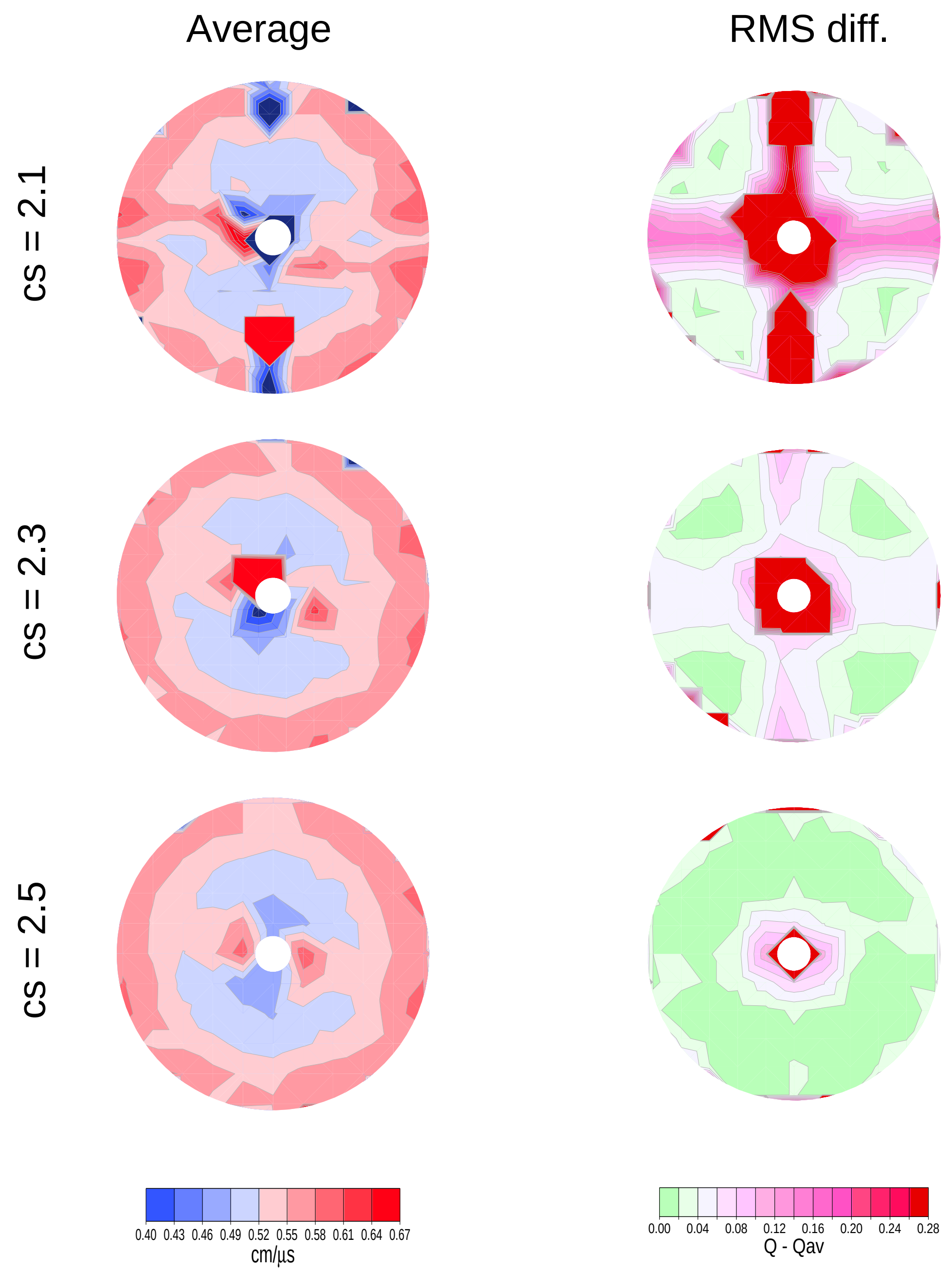


Velocity Distibution
(Lac du Bonnet granite sample)

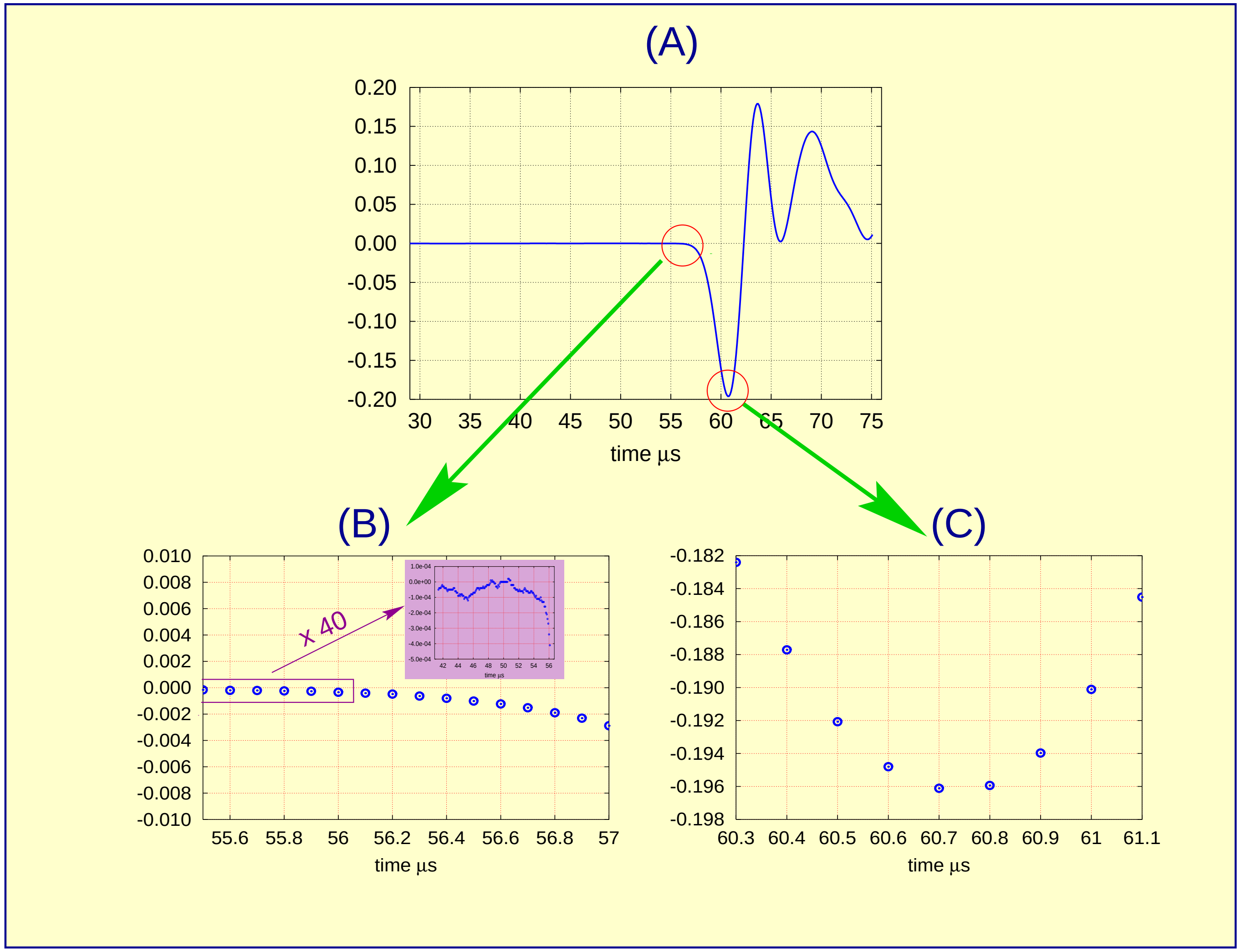


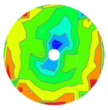


Stacking technique

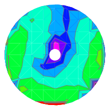


Data Quality





Enhanced velocity

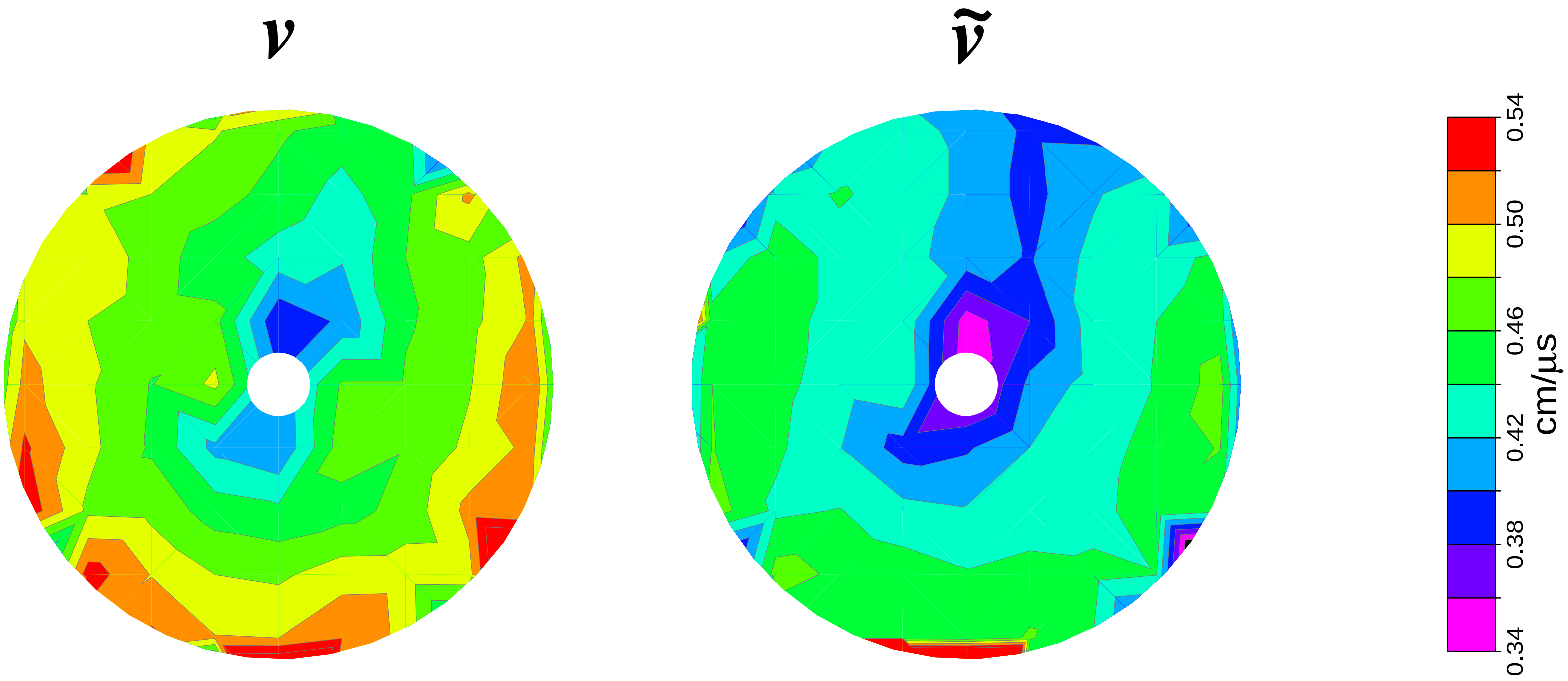


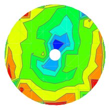
What parameters should we look for?

$$\tilde{V} = V \frac{Q}{Q+c} \qquad (c \approx 0.5)$$

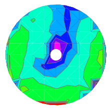
Parameter	V	Q	$\tilde{V}$
Data	travel time	pulse rise time	peak arrival
Data quality	moderate	low	high
Modeling	$t_a = \int_{ray} \frac{ds}{v(s)}$	$t_r \sim \int_{ray} \frac{ds}{Q(s)V(s)}$	$t_a = \int_{ray} \frac{ds}{\tilde{V}(s)}$
Sensitivity	low	high	moderate (?)
Robustnes	high	low	moderate (?)

Lac du Bonnet Granite Sample





Attenuation Tomography



- amplitude decay

$$A_r = A_s(f)\exp\left(-\pi f \int_{ray} \frac{ds}{Q(s)v(s)}\right)$$

- spectrum decay

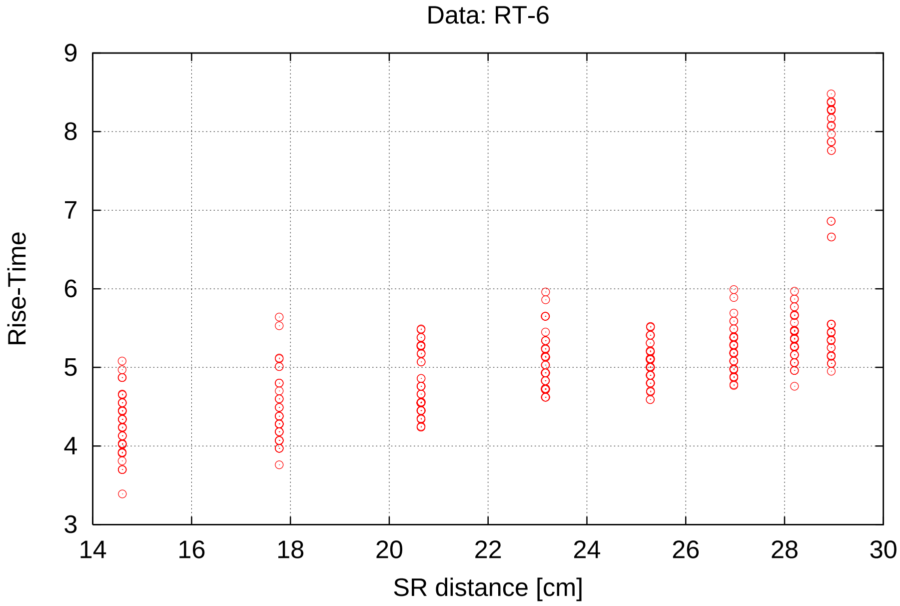
$$A_s \sim f^{-\gamma} \qquad (\gamma \approx 2)$$

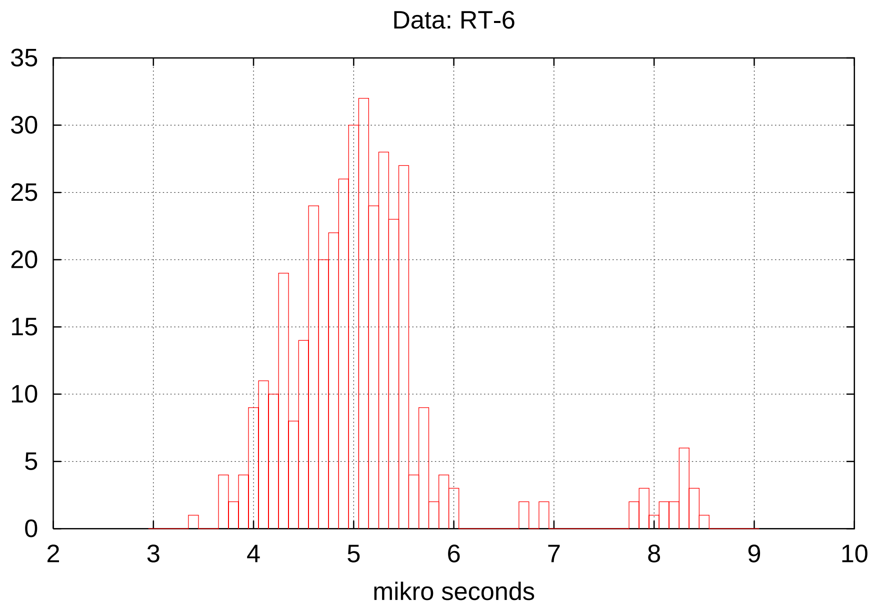
$$\ln A(f) const. - \gamma \ln(f) - a \cdot f$$

- pulse rise time

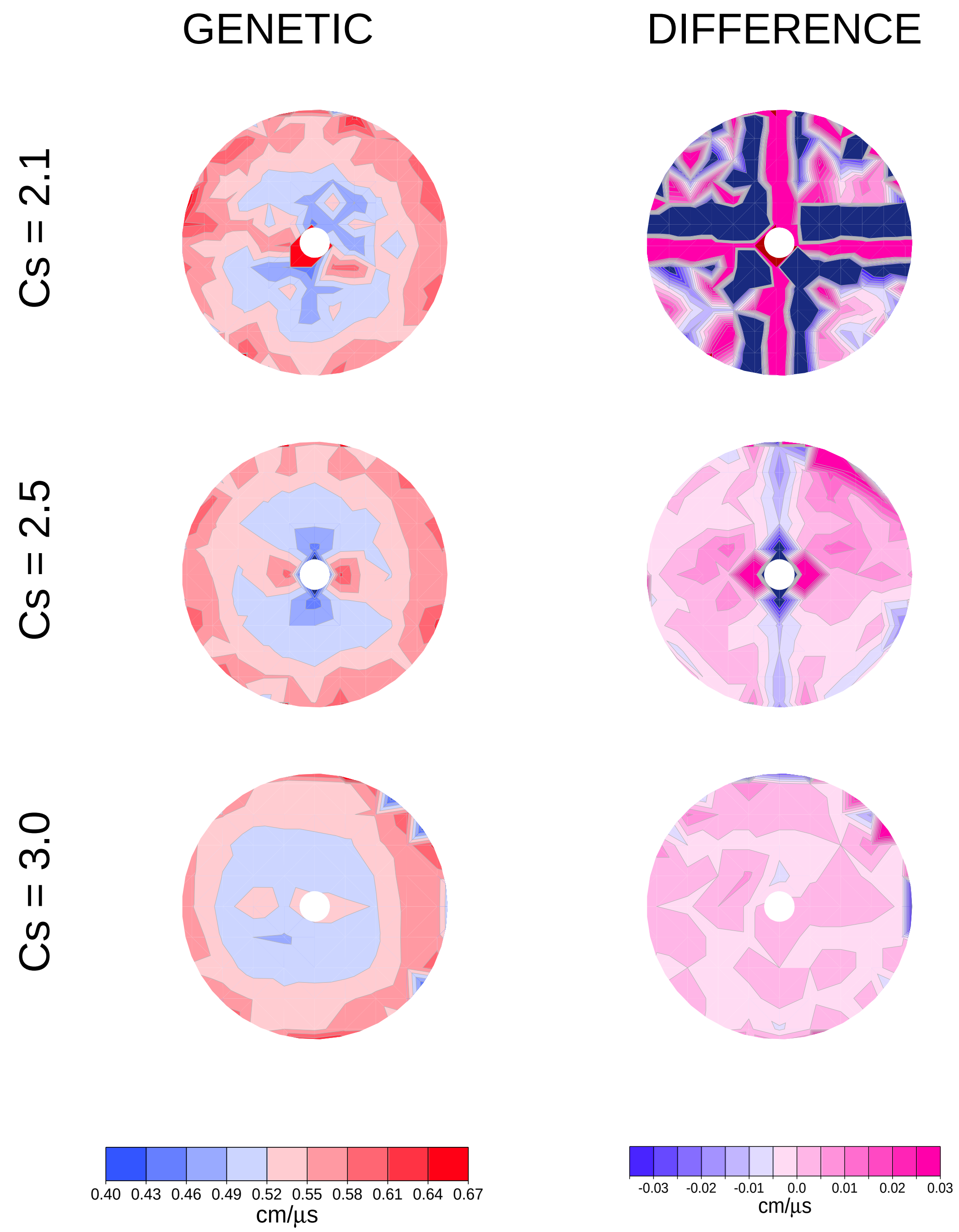
$$t_{rt} = t_o + c \cdot \int_{ray} \frac{ds}{Q(s)v(s)}$$

Rise Time Data





Genetic Algorithm



Local vs. Global Optimization

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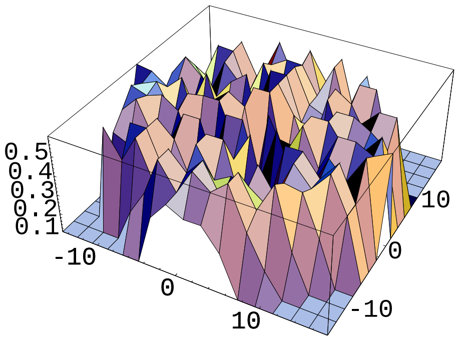
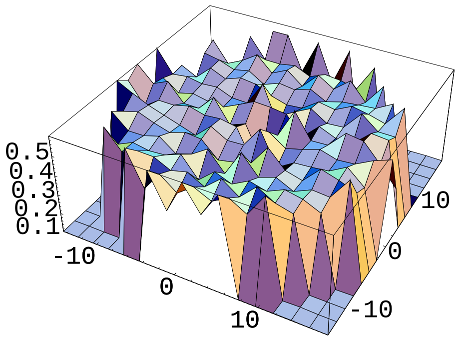
14 sec.</td><td>Genetic (l_C-norm)
37 hours</td></tr>
<tr><td>
</td><td>
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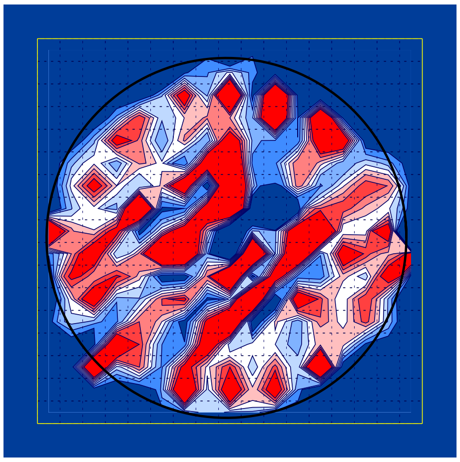
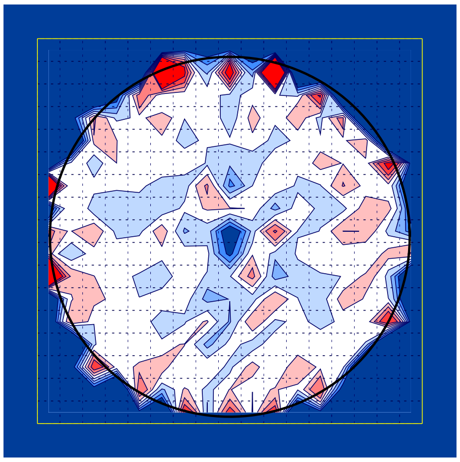
<tr><td></td><td></td></tr>
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