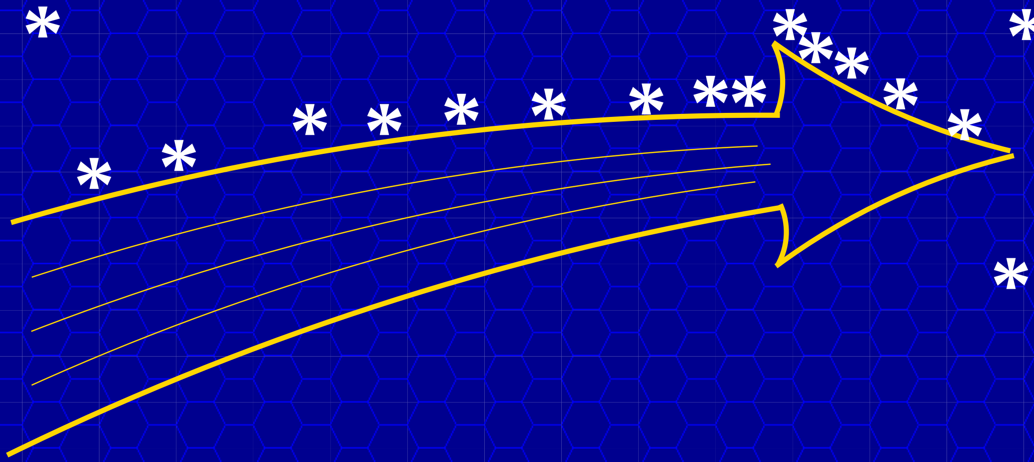
Ku optymalnej tomografii



IGF PAN (January 1998)

Optimal Tomography

- Planning stage
  - ...
  - ...
  - optimal source-receiver configuration
- Data acquisition
  - ...
  - ...
- Data inversion
  - ...
  - ...
  - choice of the optimal cell size

Optimization Criterion
(Resolution)

Proposition. *Spatial resolution analysis is the tool which can be conveniently use to solve posed questions.*

Definition. *The concept of resolution corresponds to the possibility of identifying individual points in the posterior model (image found) which, in the true model, are close together (*Tarantola, 1987*).*

[1]Tarantola, A. (1987). *Inverse Problem Theory: Methods for Data Fitting and Model Parameter Estimation*. Amsterdam: Elsevier.

Spatial Resolution

- size of cells or more generally way of a model discretization
- source-receiver configuration (rays coverage)
- data quality (outages, S/N ratio, etc.)

Assumptions

- linear tomography

$$\mathbf{d} = \mathbf{G} \cdot \mathbf{m}$$

- $l2$ - least square solution

$$\mathbf{m}^{ML} = \mathbf{m}_{apr} + \mathbf{H}^{-1} \cdot \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot (\mathbf{d}_{obs} - \mathbf{G} \cdot \mathbf{m}_{apr})$$

where

$$\mathbf{H} = \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot \mathbf{G} + \mathbf{C}_M^{-1}$$

- noise free data

$$\mathbf{d}_{obs} = \mathbf{G} \cdot \mathbf{m}_{true}$$

Resolution Operator (matrix)

$$\mathbf{m}^{ML} - \mathbf{m}_{apr} = \mathbf{R} \cdot (\mathbf{m}_{true} - \mathbf{m}_{apr})$$

$$\mathbf{R} = \left\{\mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot \mathbf{G} + \mathbf{C}_M^{-1}\right\}^{-1} \cdot \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot \mathbf{G}$$

$$\mathbf{C}_D = \sigma_D^2\, \mathbf{I} \qquad \mathbf{C}_D = \sigma_M^2\, \mathbf{I} \qquad g = \sigma_D / \sigma_M$$

$$\mathbf{R} = \left(\mathbf{G}^T \cdot \mathbf{G} + g^2\, \mathbf{I}\right)^{-1} \cdot \mathbf{G}^T \cdot \mathbf{G}$$

Properties

- Ideal case: $\mathbf{R} \approx \mathbf{I}$
  never achieved due to a limited number of observations $d^i_{obs} \quad i = 1 \ldots N_{obs}$

- Normalized diagonal elements:

$$0 \leq \mathbf{R}_{ii} \leq 1$$

- When medium is divided into homogeneous blocks $\mathbf{R}$ becomes a 2-D matrix of the size $\mathrm{N}_{cell} \times \mathrm{N}_{cell}$

- $\mathbf{R}_{ii} = 0$ only for cells probed by no ray

- Average (minimal) values of $\mathbf{R}_{ii} \quad ii = 1 \ldots \mathrm{N}_{cell}$ strongly depends on:
  - source-receiver configuration
  - cell size

Example - 1

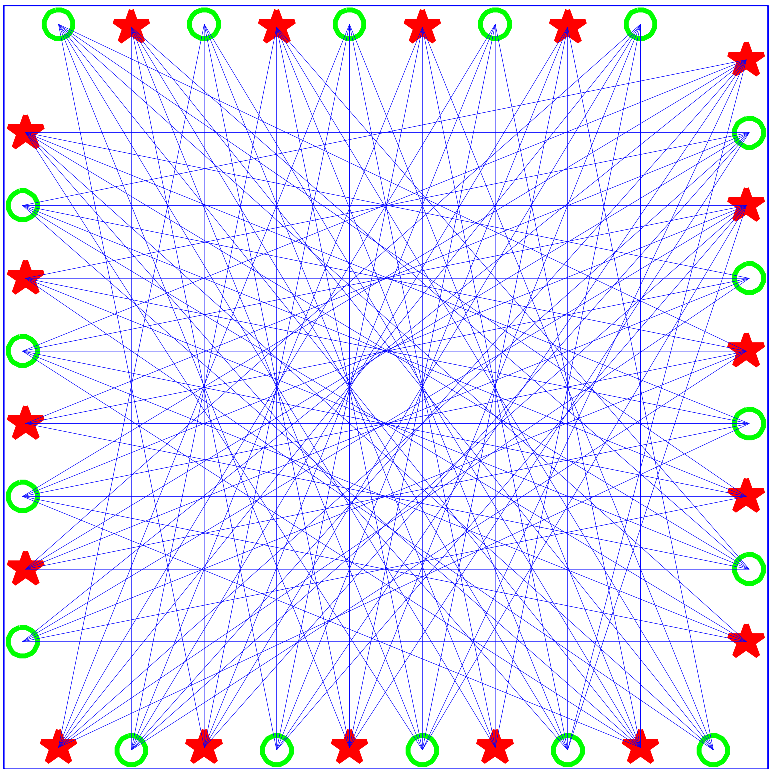


Figure 1: An example of the source-receiver configuration used in a pillar or rock sample tomography imaging.

Example - 1(a)

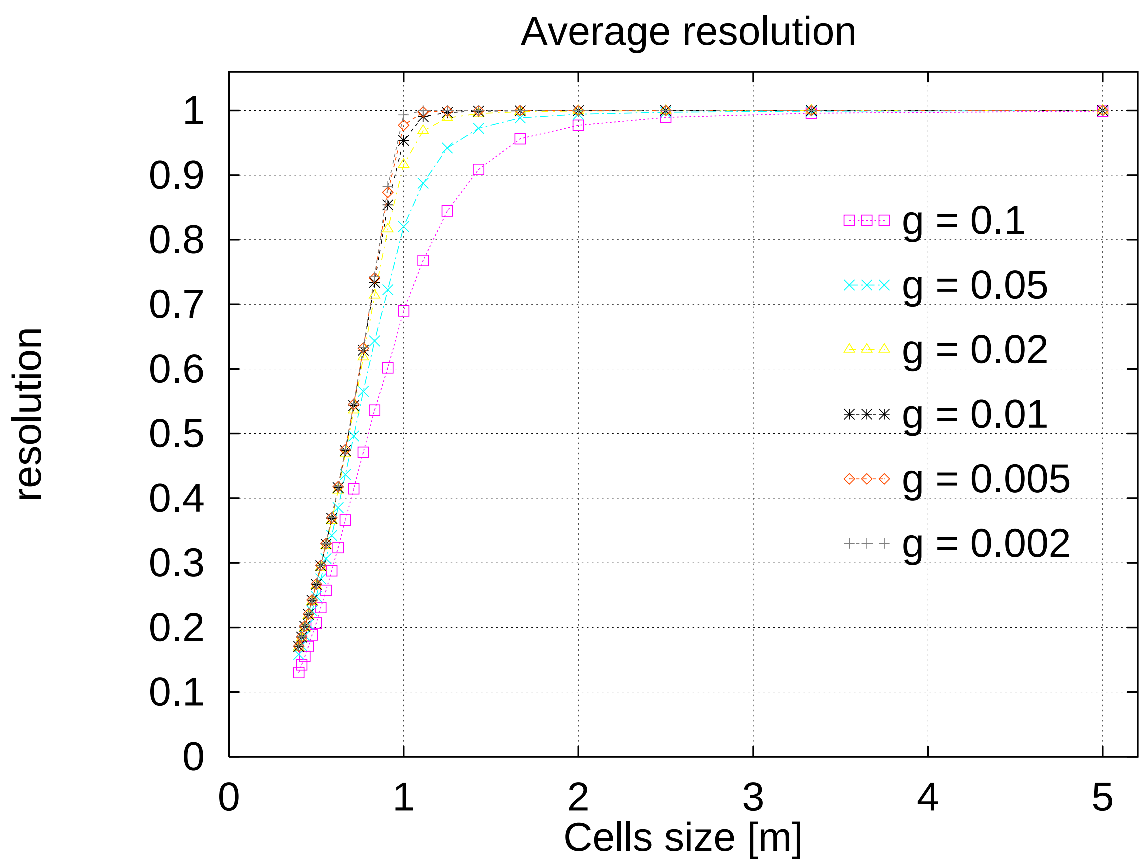


Figure 2: Average resolution as a function of cells size. The square cells were considered only

Example - 2

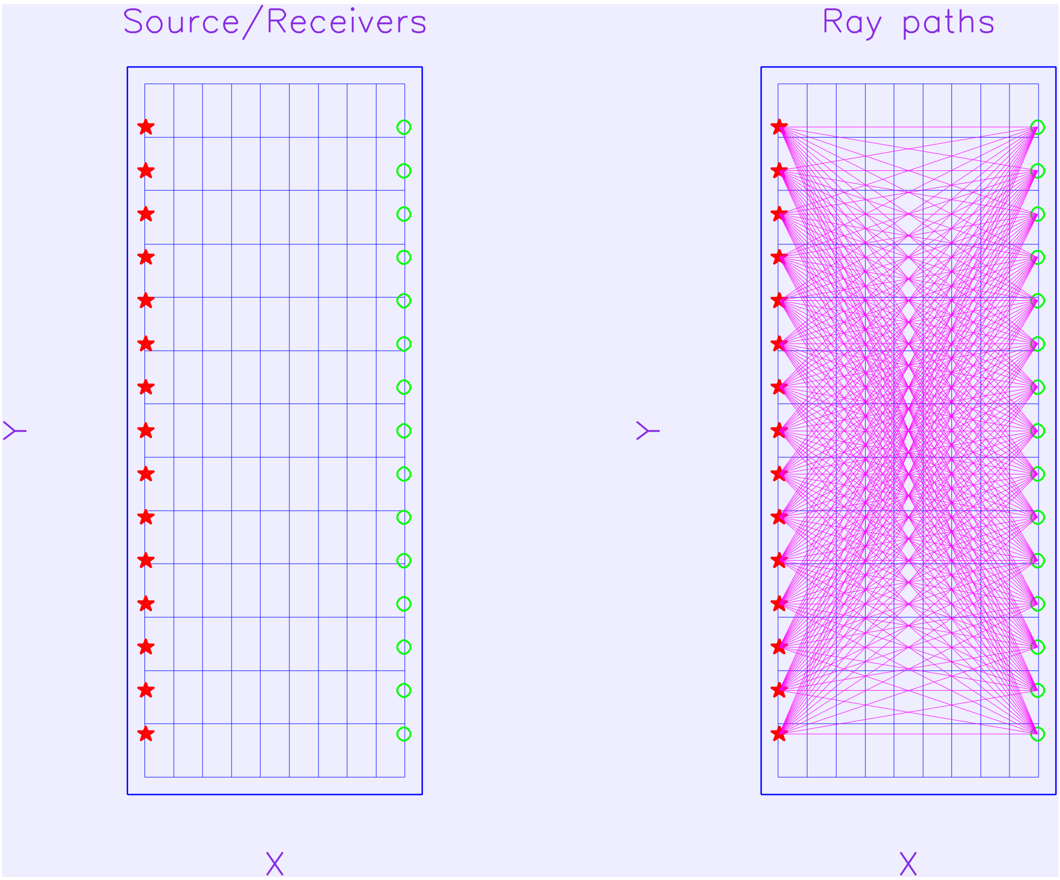


Figure 3: An example of the source-receiver configuration used in a cross-well tomography imaging. The distance between wells is 30m and their depth 80m.

Example - 2(a)

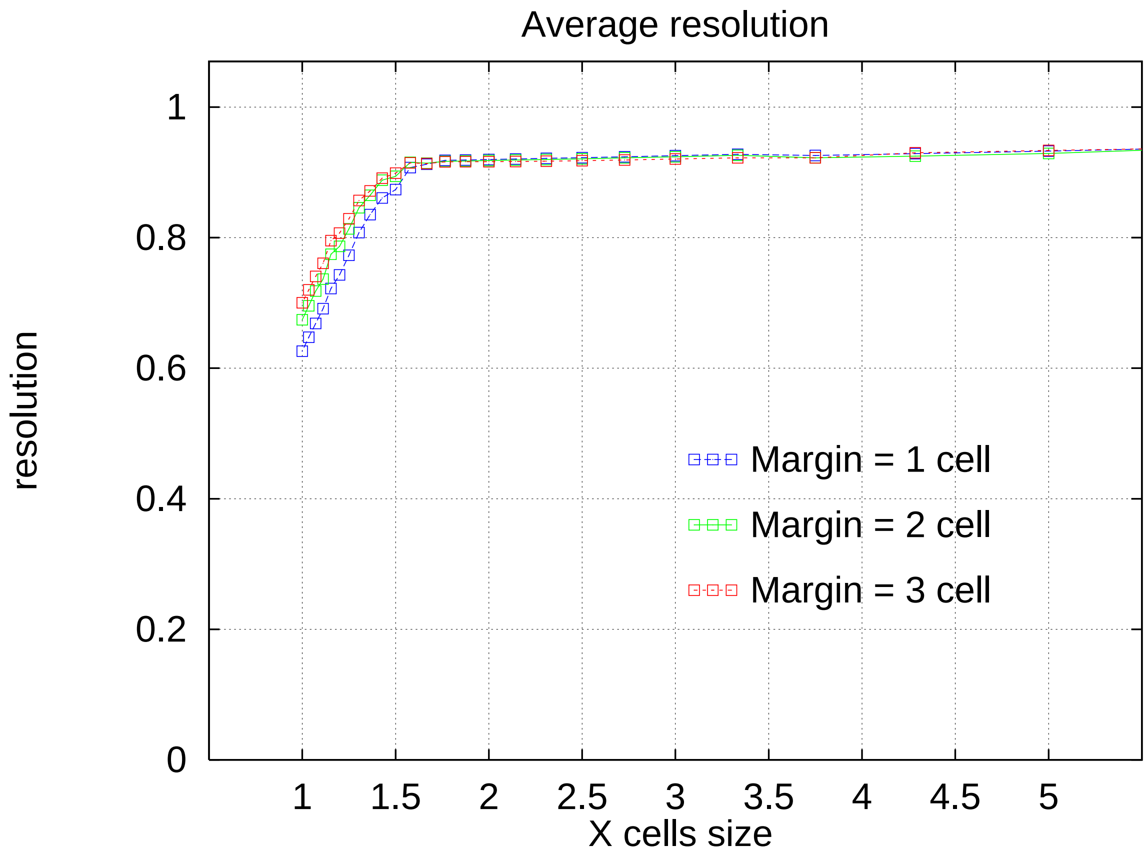


Figure 4: Average resolution as a function of cells size in meters. The vertical size of cell is equal around 6m

Example - 2(b)

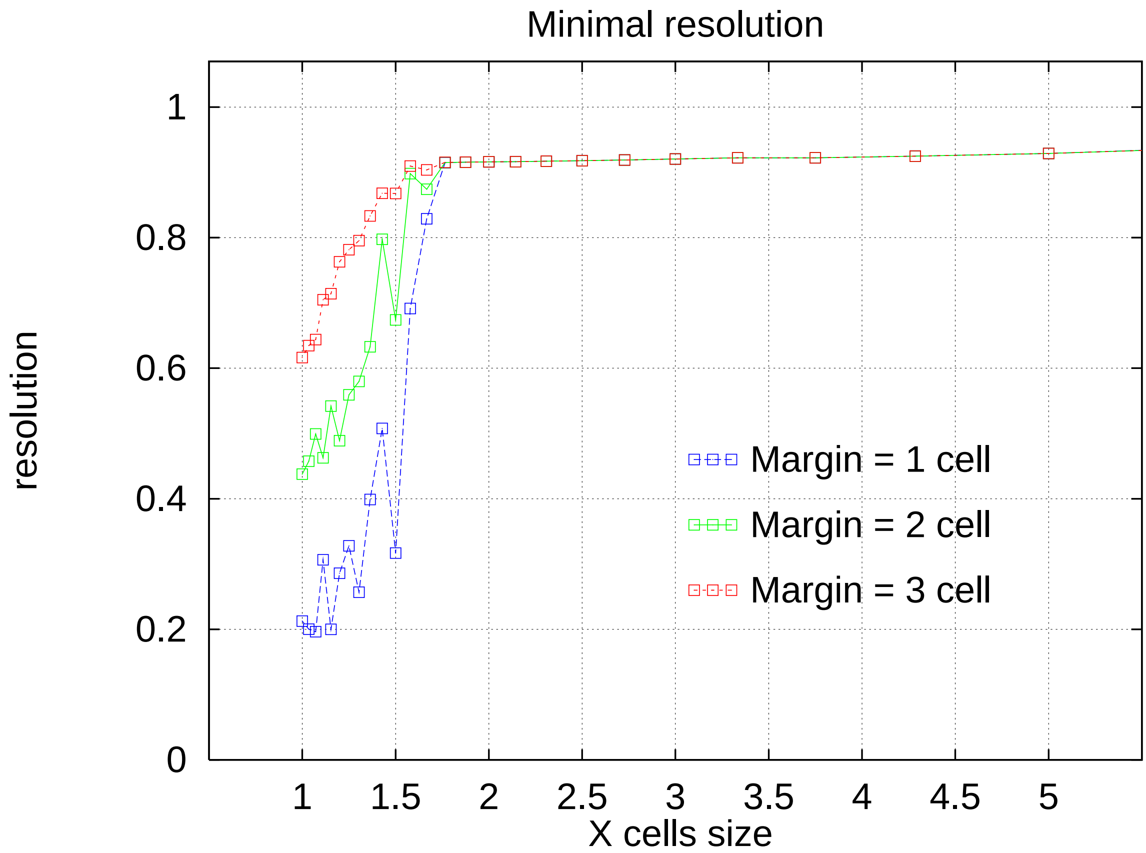


Figure 5: Minimal resolution as a function of cells size in meters. The vertical size of cell is equal around 6m

The Optimal Configuration

Problem: how to find the best source - receiver configuration?

> **Definition.** *The source - receiver configuration which for a given number of sources and receivers provides the best spatial resolution in a selected region e.i. allows to use cells of the smallest size is called optimal.*

Solution: look for the configuration which gives the smallest critical cell size

Example (configuration)

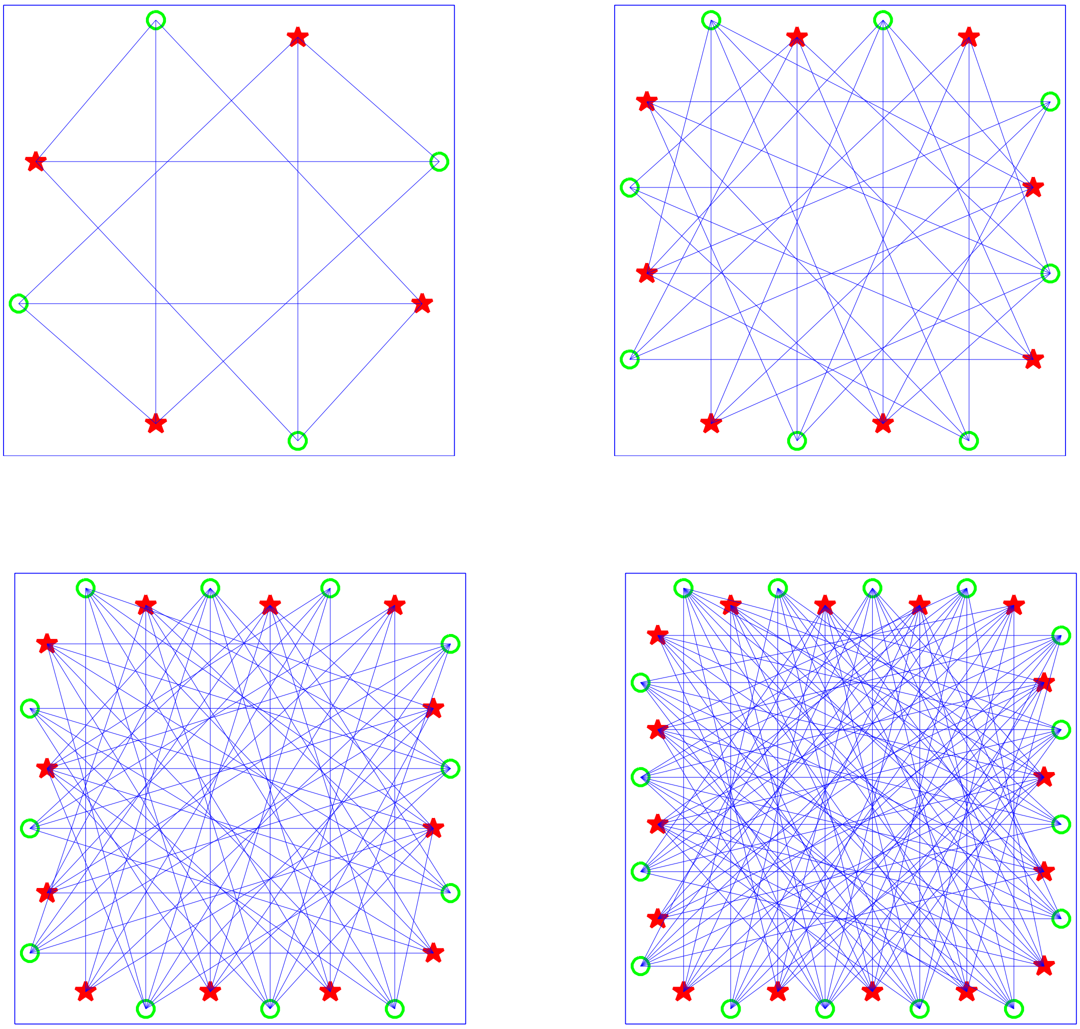


Figure 6: Source-receiver configurations used to calculate resolution curves shown in the next transparency

Example (resolution)

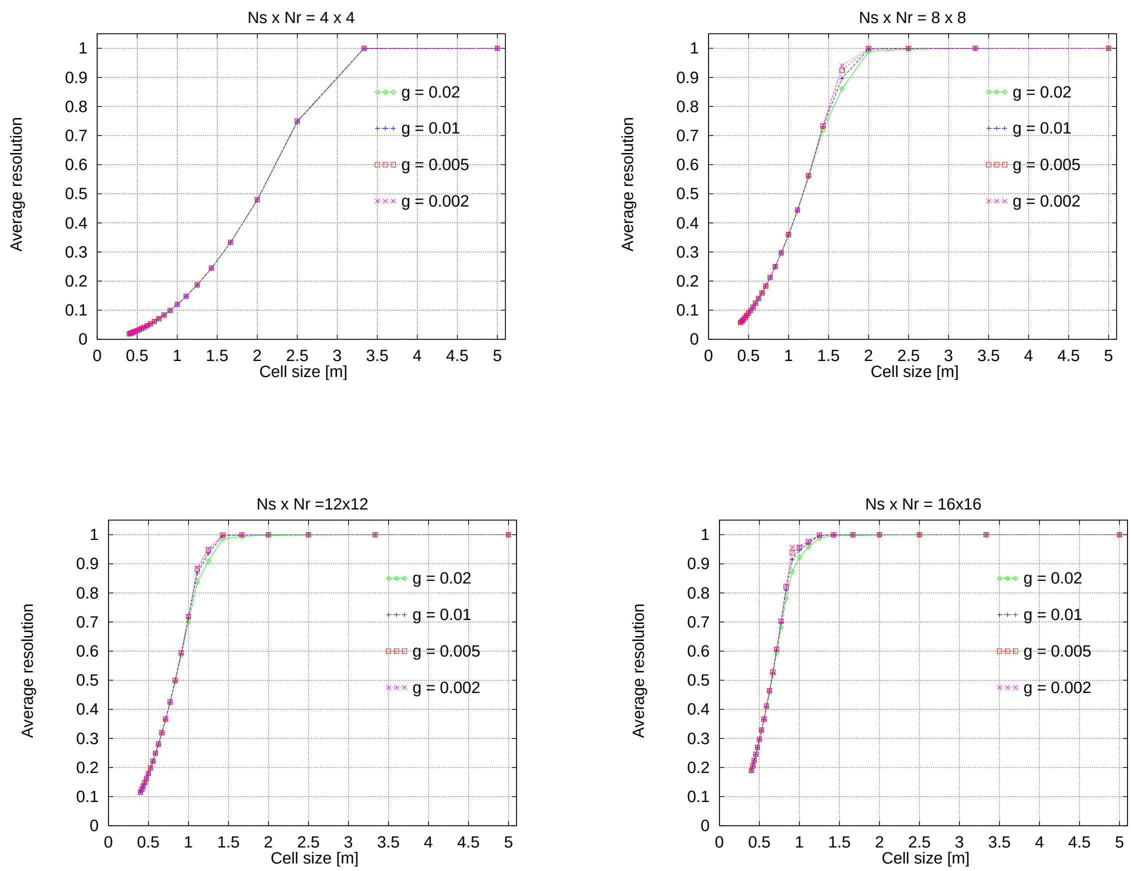


Figure 7: Examples of the resolution curves for four source-receiver configurations. The shift of the curve braking down point (optimal cell size) towards smaller values for increasing number of source and receivers is observed.

Example (dependence)

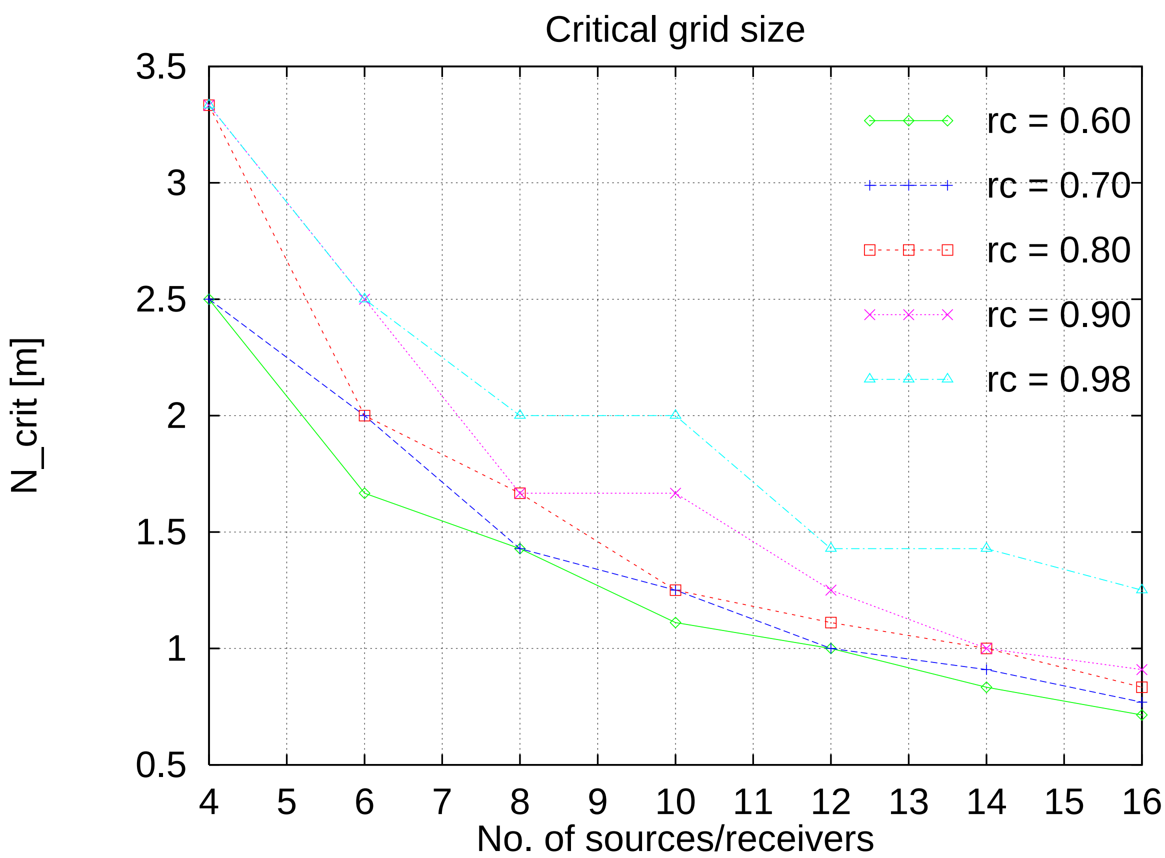


Figure 8: Dependence of the cell size for which average resolution exceeds *nc* level as the function of the source (receiver) number for a few *nc*.

Optimization

Optimization with respect to:

- grid type (coordinate system)
  - Cartesian coordinates (both 2D and 3D)
  - polar coordinates (2D)
  - spherical coordinate system (3D)
  - irregular
- resolution criterion
  - minimal resolution
  - average resolution
  - difference between minimal and maximal resolution
- local macro-structure

Conclusions

Resolution analysis approach

- Advantages:
  - convenient tool for analysis of the optimal tomography configuration
  - allows to analyze how to use experimental data in the optimal way
- Disadvantages:
  - does not account for a local medium macro-structure
  - does not take into account the real experimental data quality (signal to noise ratio, outages, etc.)