Imaging seismic rupture process by Markov Chain Monte Carlo technique - the case of Rudna (Poland) copper mine events

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7 stycznia 2005

Plan of the talk

- Numerical background
  - Inversion techniques
  - Markov Chain Monte Carlo
  - Pseudo-spectral parametrization
- Relative Source Time Function
  - Information on rupture process
  - Empirical Green Function
- Mining Induced Events - case study
  - Local seismic network
  - Examples

Inverse problem - Indirect Measurements

$$\mathbf{d}^{obs} \implies \mathbf{m}$$

Solution

$$||\mathbf{d}^{obs} - \mathbf{d}^{th}(\mathbf{m})|| + \lambda||\mathbf{m}^{ml} - \mathbf{m}^{apr}|| = \min$$

Errors

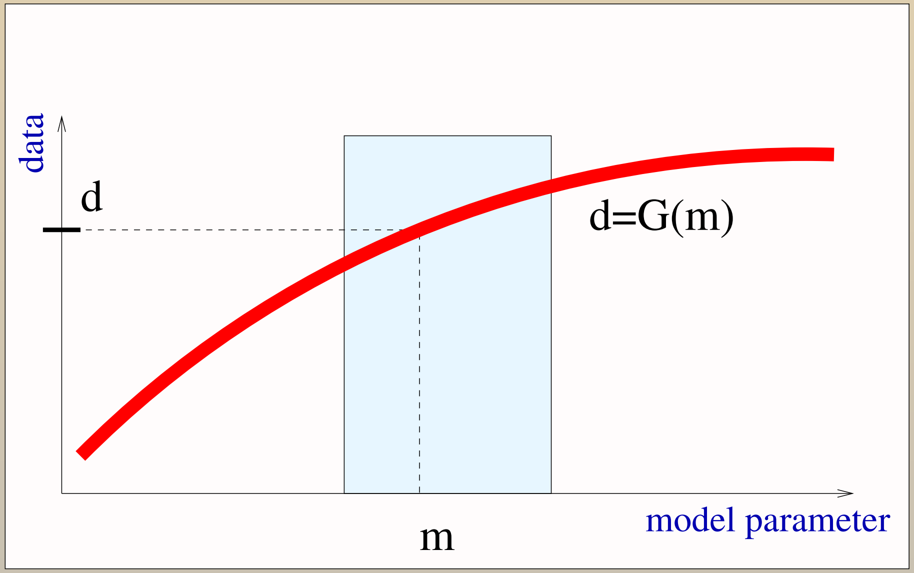
$$\mathbf{m}^{true} = \mathbf{m}^{ml} + \epsilon_{\mathbf{m}}$$

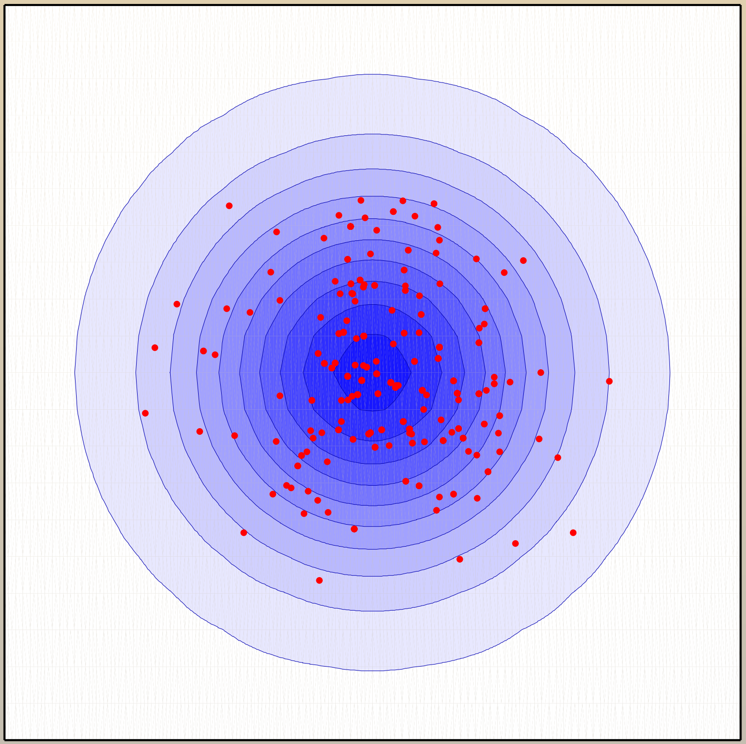
$$\epsilon_{\mathbf{m}} = ???$$

Inversion Algorithms

Method	Advantages	Limitations
Algebraic (LSQR)	- Simplicity	- Only linear problems
$\mathbf{m}^{ml} = (\mathbf{G}^T\mathbf{G} + \gamma\mathbf{I})^{-1}\mathbf{G}^T \cdot \mathbf{d}^{obs}$	- Large scale problems	- Lack of robustness
Optimization	- Simplicity	- Difficult error estimation
$\|\mathbf{G}(\mathbf{m}) - \mathbf{d}^{obs}\| + \lambda\|\mathbf{m}) - \mathbf{m}^a\| \;=\; \min$	- Fully nonlinear	
Bayesian	- Fully nonlinear	- More complex theory
$\sigma(\mathbf{m}) = f(\mathbf{m})L(\mathbf{m}, \mathbf{d}^{obs})$	- Full error handling	- Requires efficient sampler

Inversion algorithms

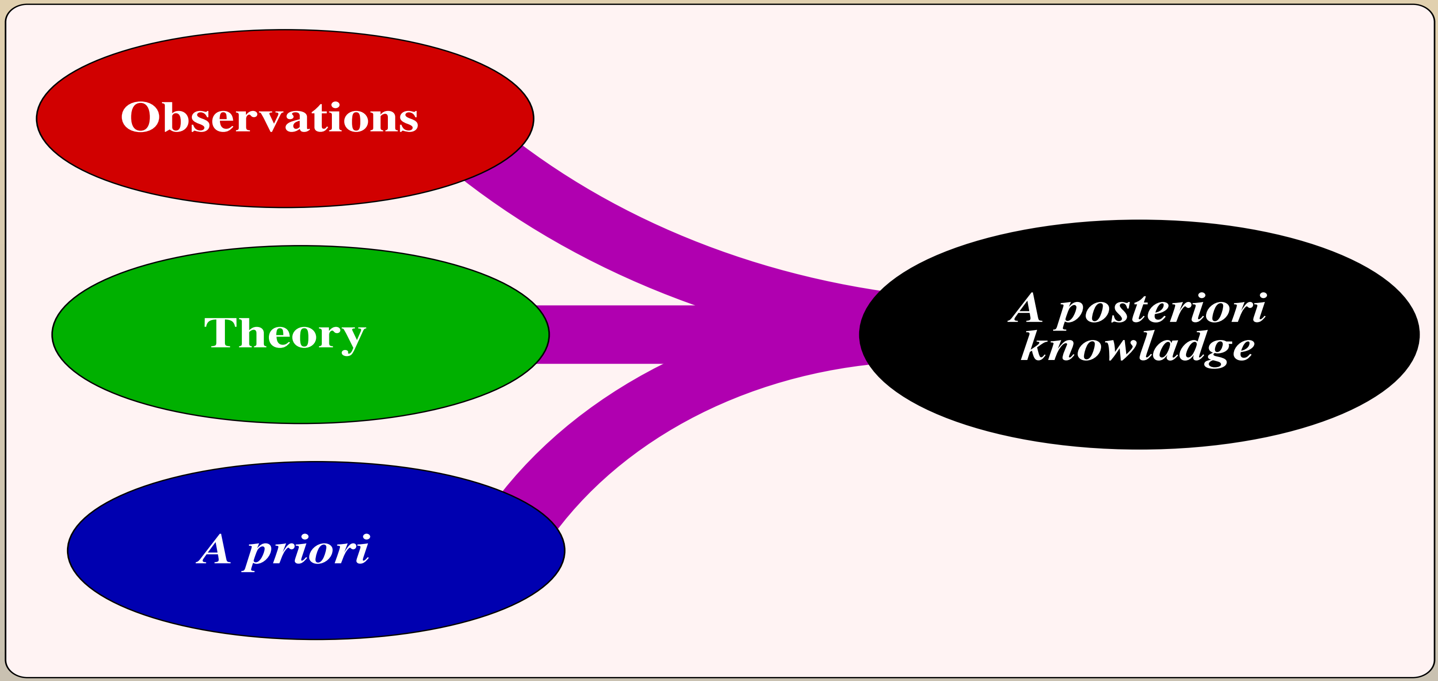




Back projection

Model space search

Bayesian Inversion - Basic Ideas



$$p_{post}(\mathbf{m}|\mathbf{d}) = \frac{p_{th}(\mathbf{d}|\mathbf{m})p_{apr}(\mathbf{m})}{p_{obs}(\mathbf{d})}$$

Bayesian Inversion

A posteriori pdf

$$\sigma(\mathbf{m}) = f(\mathbf{m})L(\mathbf{m}, \mathbf{d})$$

- f - A priori pdf (prob. dens. funct.)
- L - Likelihood function

Errors independent of $\mathbf{m}$ and $\mathbf{d}$

$$\mathbf{d}^{th} = \mathbf{G}(\mathbf{m})$$

$$\sigma(\mathbf{m}) = f(\mathbf{m}) \exp\left(-||\mathbf{d}^{obs} - \mathbf{G}(\mathbf{m})||\right)$$

Probabilistic approach

A posteriori pdf $\sigma(m, d)$:

- always exists
- is unique
- describes all information
- is the solution of an inverse problem

When and why we need to use this approach ???

ERROR ANALYSIS !!!

Examination of $\sigma(\mathbf{m})$

- Searching *Maximum Likelihood* solution
  - Gradient methods
  - Monte Carlo - Global Optimization
- Sampling over regular grid
- **Random (Monte Carlo) Sampling**

Monte Carlo sampling

Monte Carlo sampling

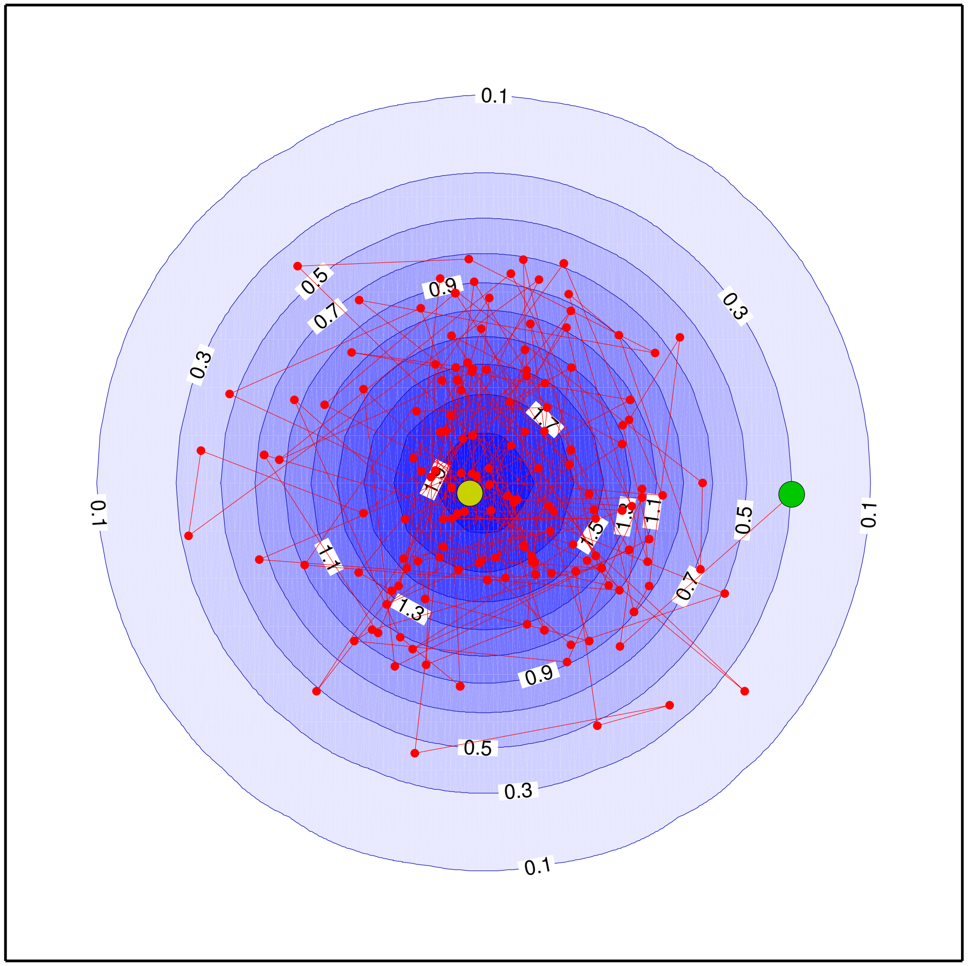
$$\{\mathbf{m}_1, \mathbf{m}_2 \cdots\cdots\cdots\mathbf{m}_N\}$$

$$N_{[\mathbf{m}-\Delta,\mathbf{m}+\Delta]} \sim \sigma(\mathbf{m})$$

Solution

- $\mathbf{m}^{ml} = \mathbf{m}_i : \sigma_M(\mathbf{m}_i) = \max$
- $\mathbf{m}^{avr} = \sum_{samples} \mathbf{m}_s$
- $\epsilon_{\mathbf{m}} = \sum_{samples} (\mathbf{m}^{avr} - \mathbf{m}_s)^2$

Markov Chain Monte Carlo technique



Elements of seismic tomography

Far field:

$$u_i^{obs}(t,\mathbf{r}) \;=\; \int\limits_{t'} \int\limits_{\Sigma} \mathcal{G}_i(t-t',\, \mathbf{r},\mathbf{r}')S(t',\mathbf{r}')\, d\mathbf{r}'\, dt'$$

$$u_i^{obs}(t) \;=\; \int\limits_{t'} \mathcal{G}_i(t-t')\bar{S}_i(t')\, dt'$$

Joseph Fourier Univ., 13.I.2005

Seismic tomography - two step inversion

Apparent STF

$$\bar{S}_i(t) \approx \int_{\Sigma} S(t + \delta_i(\mathbf{r}'),\, \mathbf{r}')\, d\mathbf{r}'$$

Two steps inversion

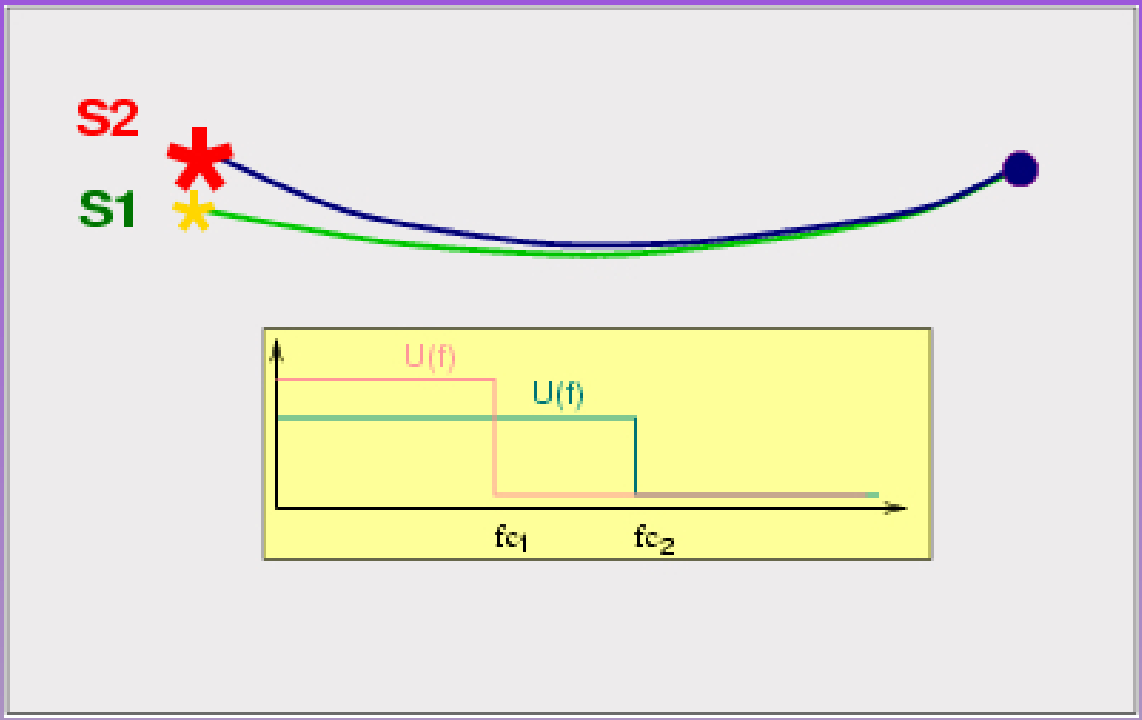
$$u_i^{obs}(t) \Longrightarrow \bar{S}_i(t') \Longrightarrow S(t, \mathbf{r})$$

Apparent STF

$$u_i^{obs}(t) \;=\; \int\limits_{t'} \mathcal{G}_i(t-t')\bar{S}_i(t')\,dt'$$

- Aproximation $\mathcal{G}$: (e.g. Empirical Green Function (EGF)
- Discretization $\bar{S}(t) \Longrightarrow \{m_1, m_2, \ldots\}$
- Constraints
  - causality
  - finit duration
  - positivity
  - limited frequency band

Empirical Green Function



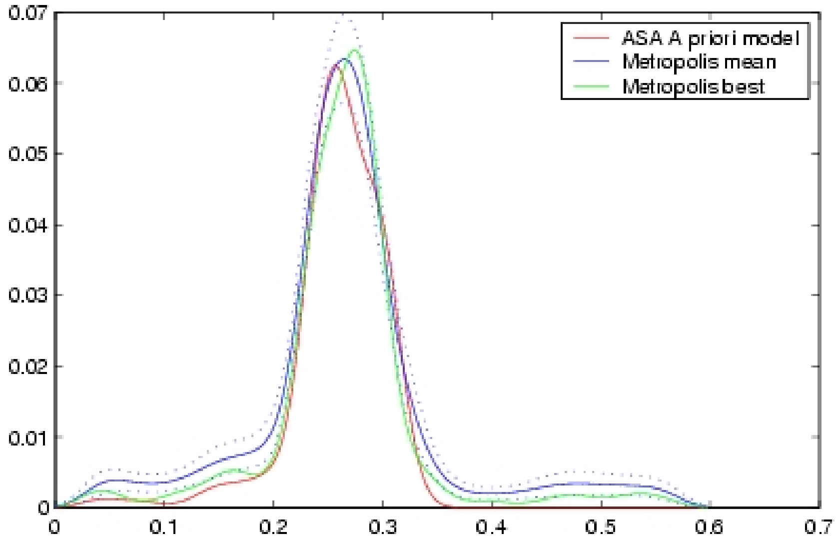
Discretization

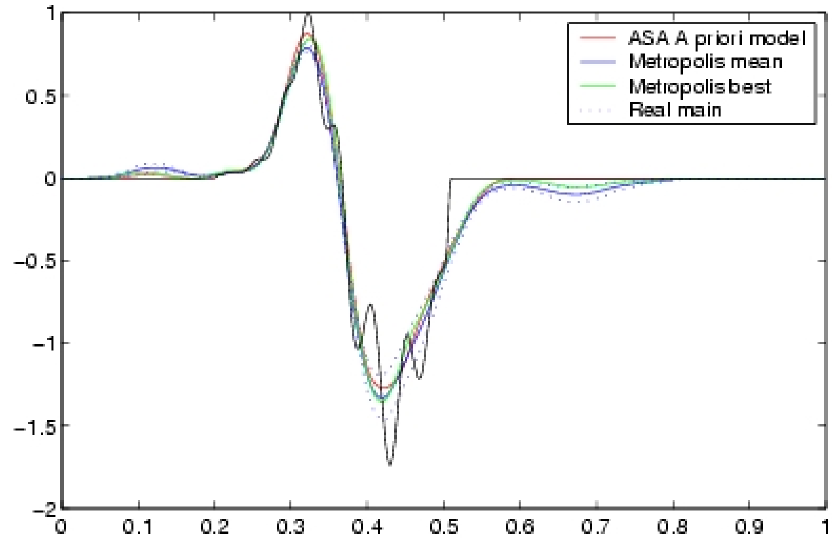
$$\bar{S}(t) \; = \; \sum_i a_i \phi_i(t)$$

$$\phi_i(t) = \begin{cases} \delta(t - t_i) \\ \exp\left(-\frac{1}{2}(t - t_i)^2\right) \\ \sin(\omega_i t) \end{cases}$$

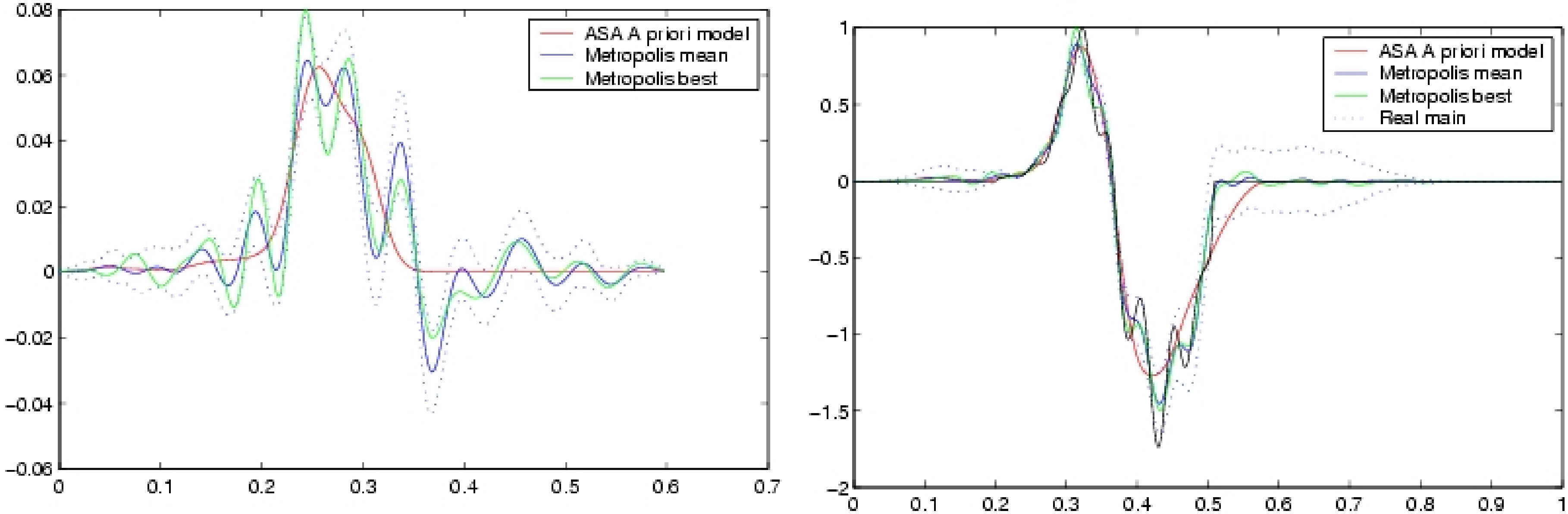
$$\bar{S}(t) \equiv \bar{S}(\mathbf{a}) \leftrightarrow \{a_i\}$$

Positivity constraint

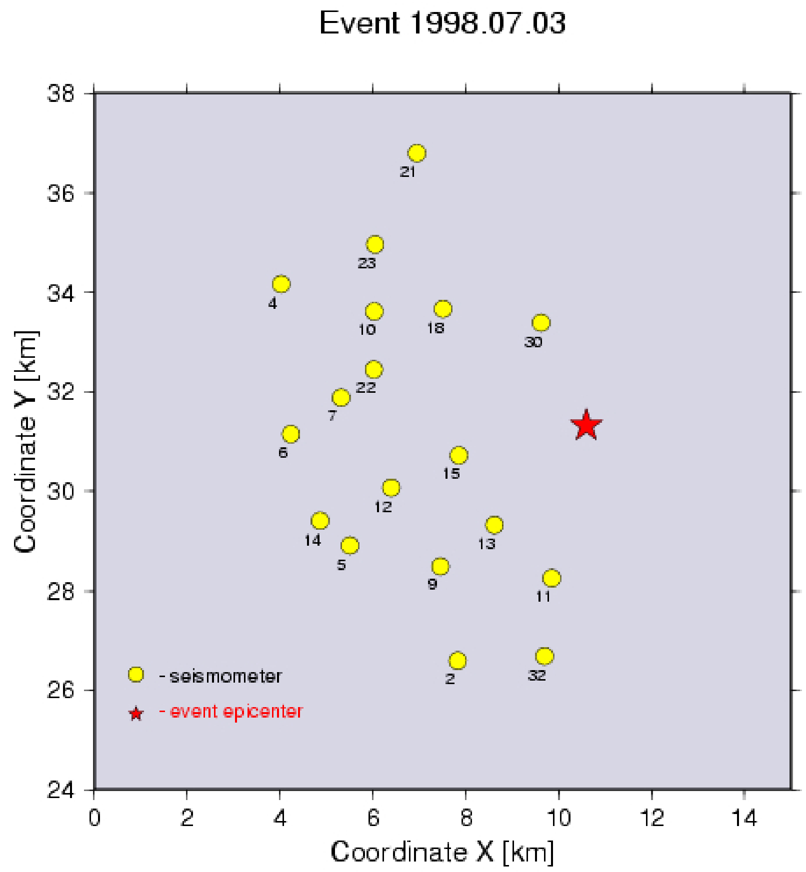




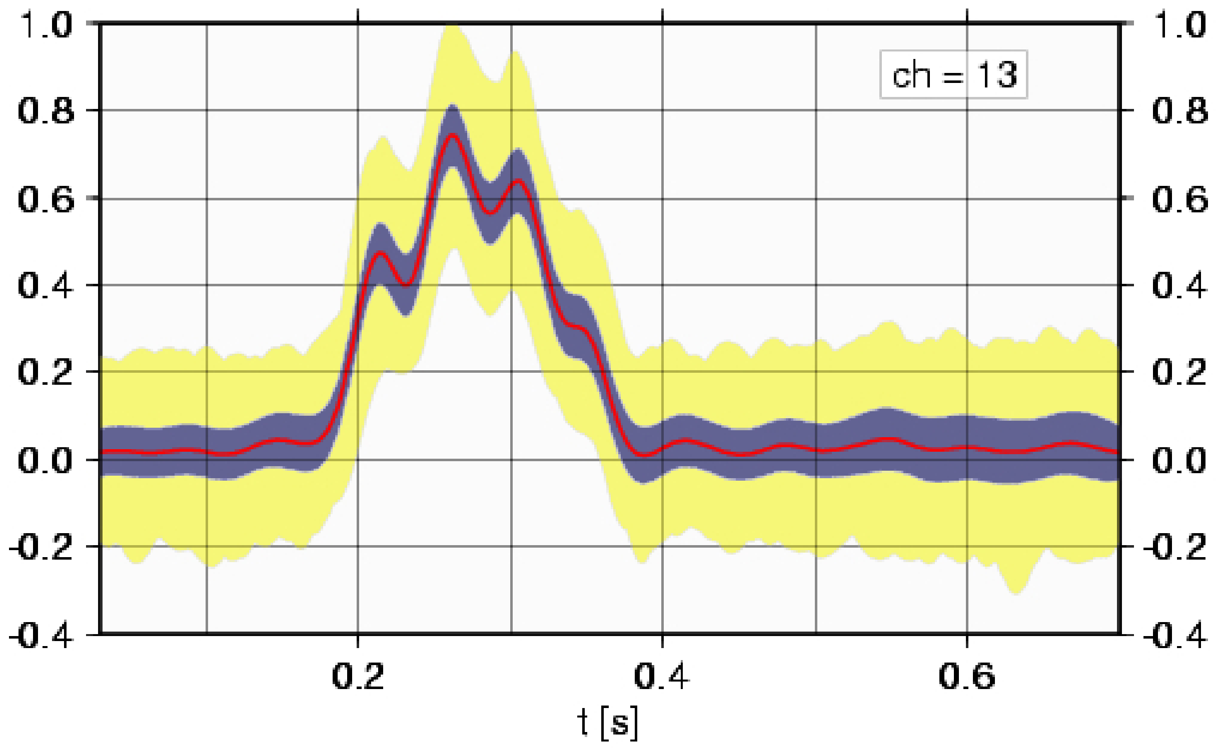
No constraint



Station distribution



STF - channel 13



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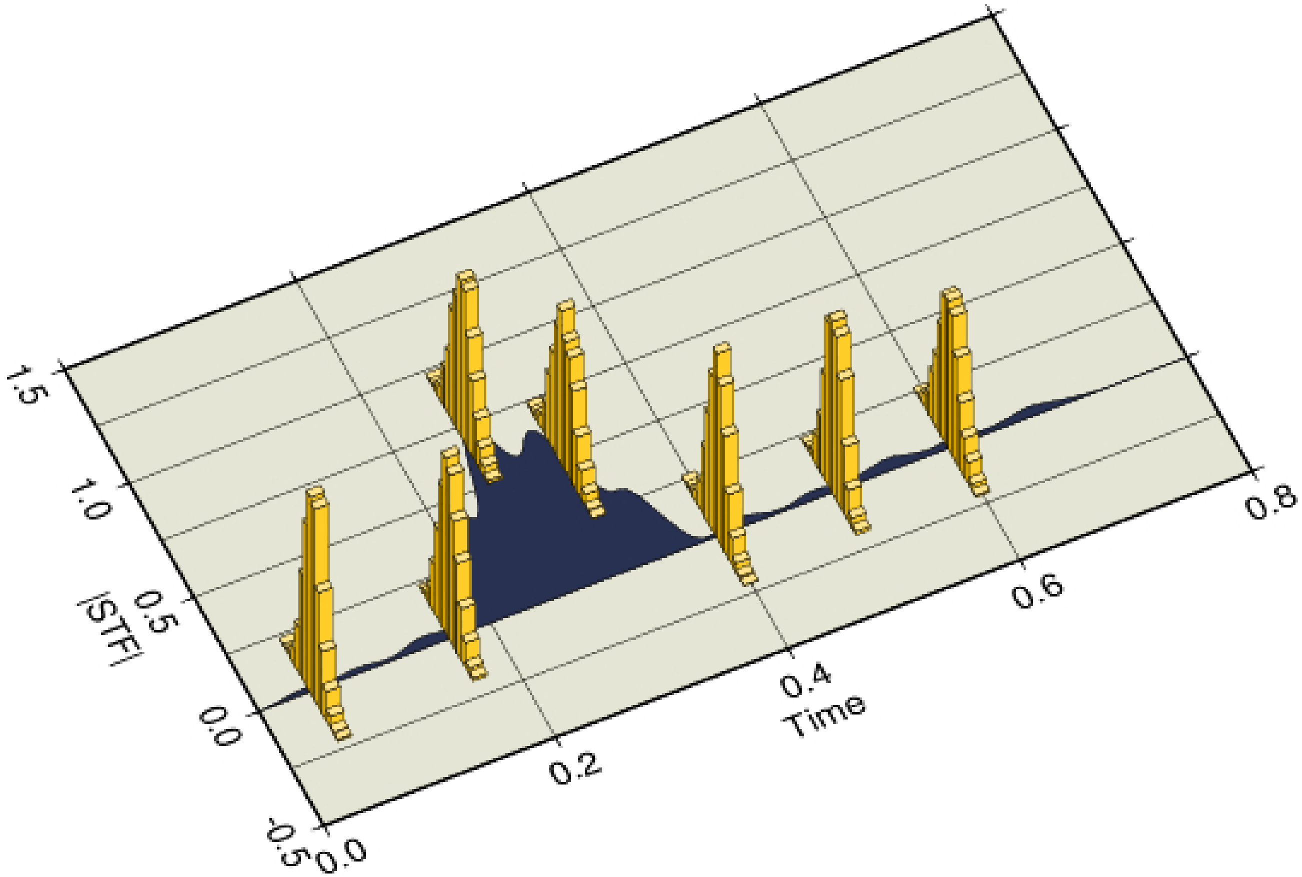
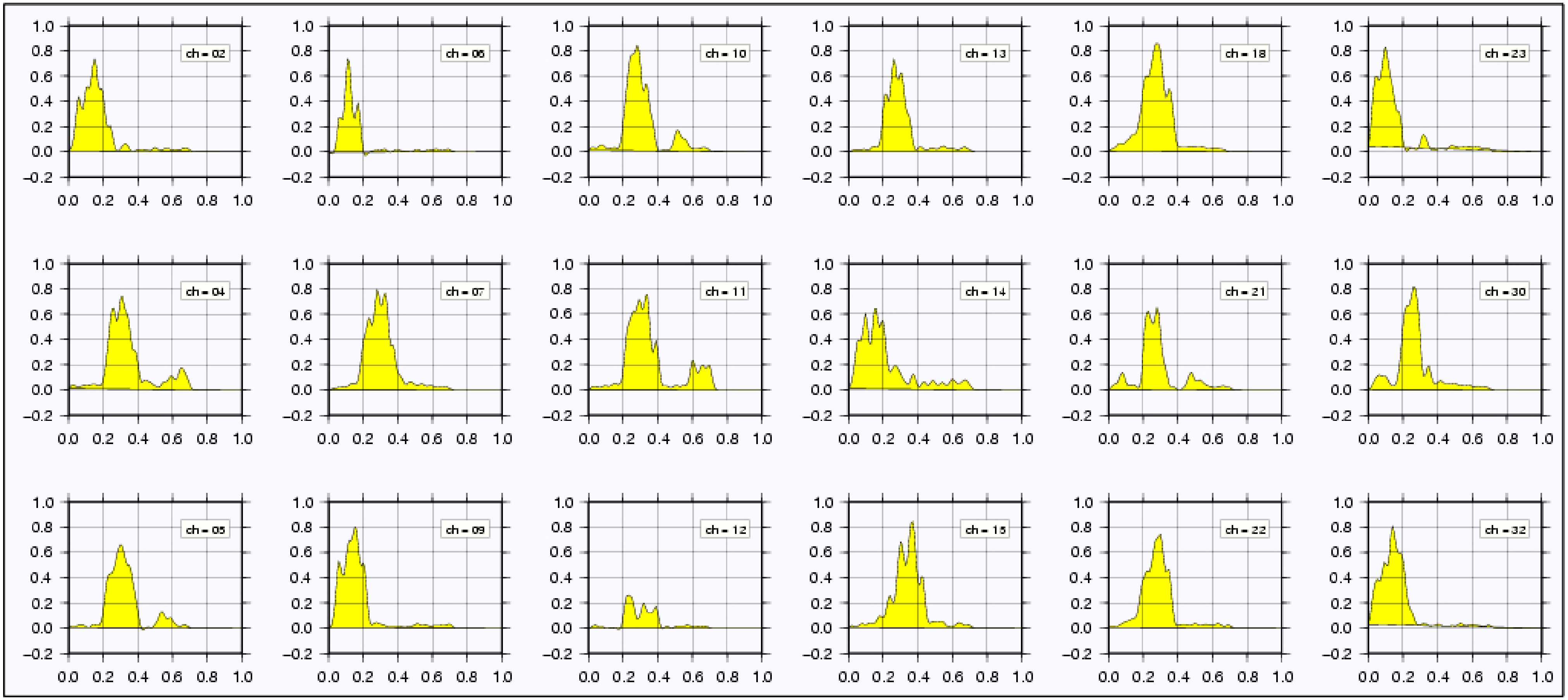
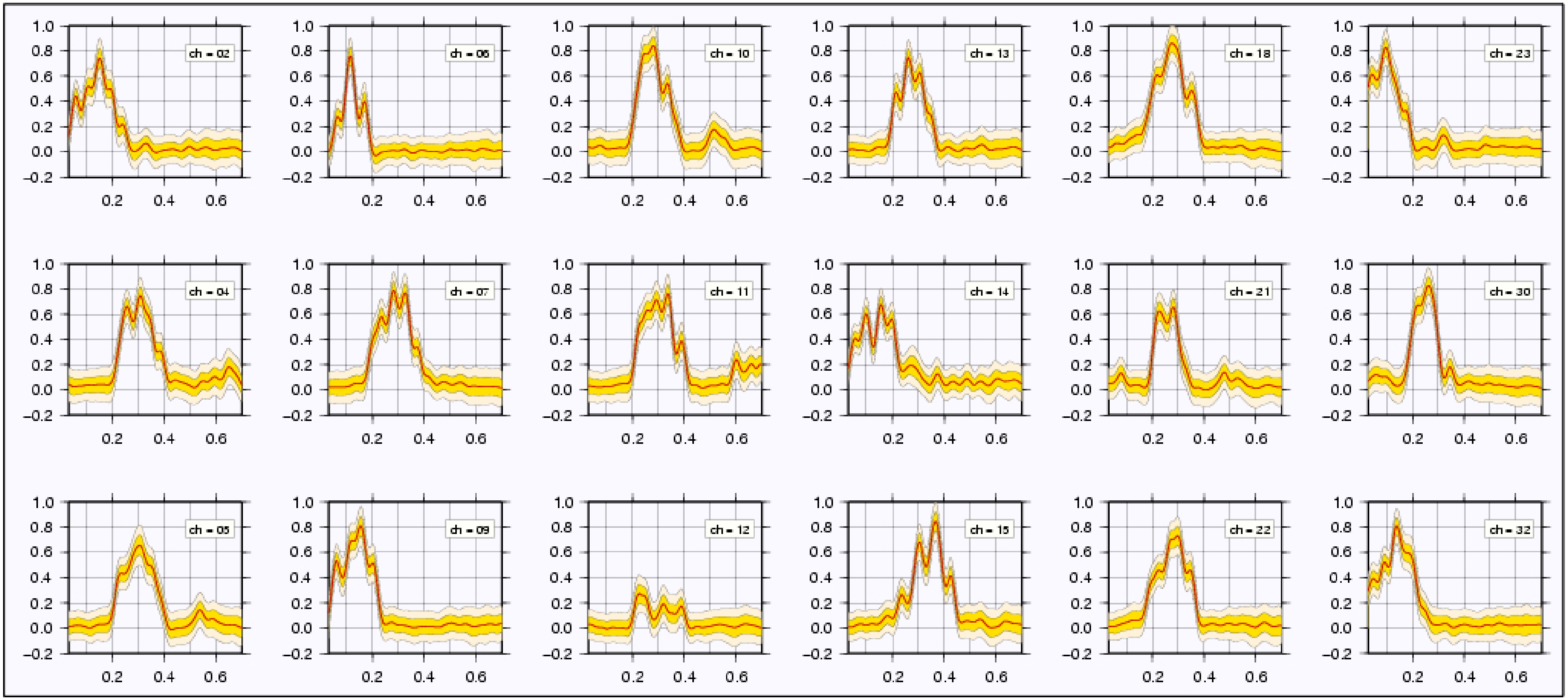


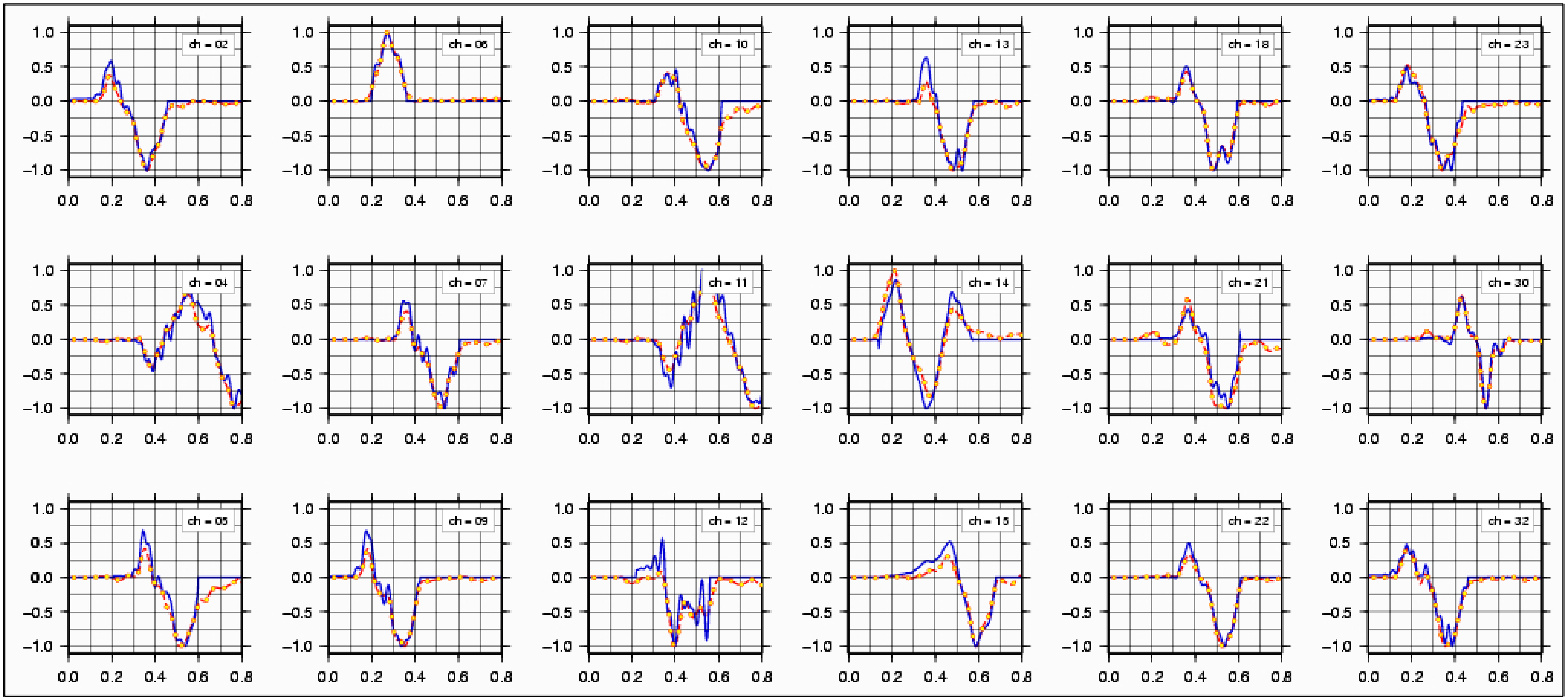
All channels



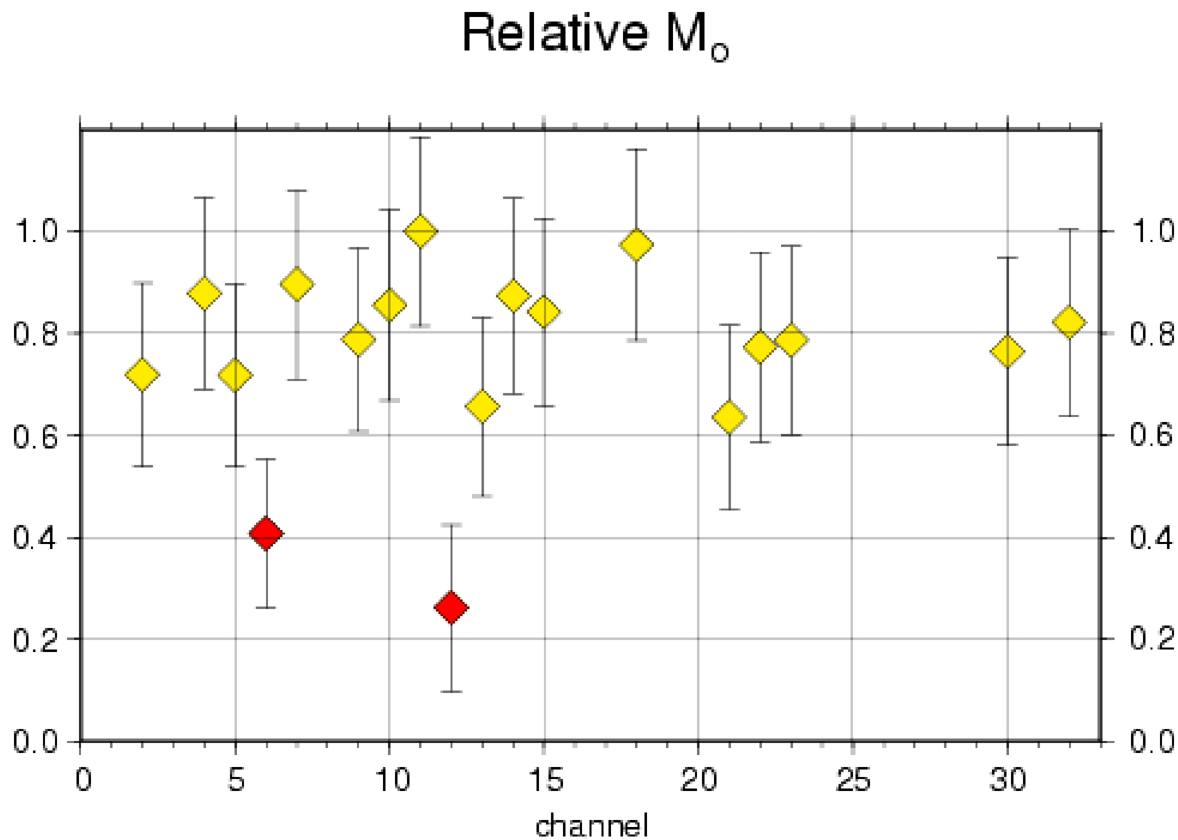
STF- errors


ch = 02
ch = 06
ch = 10
ch = 13
ch = 18
ch = 23
ch = 04
ch = 07
ch = 11
ch = 14
ch = 21
ch = 30
ch = 05
ch = 09
ch = 12
ch = 15
ch = 22
ch = 32

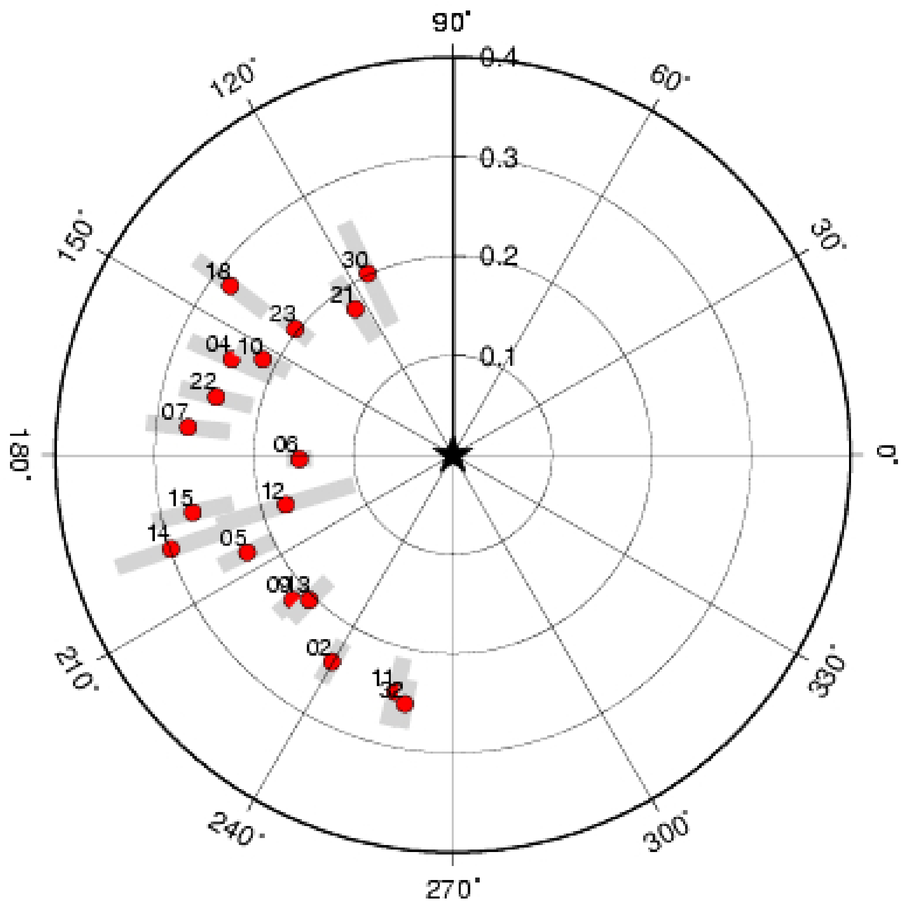
Synthetic vs. recorded seismograms



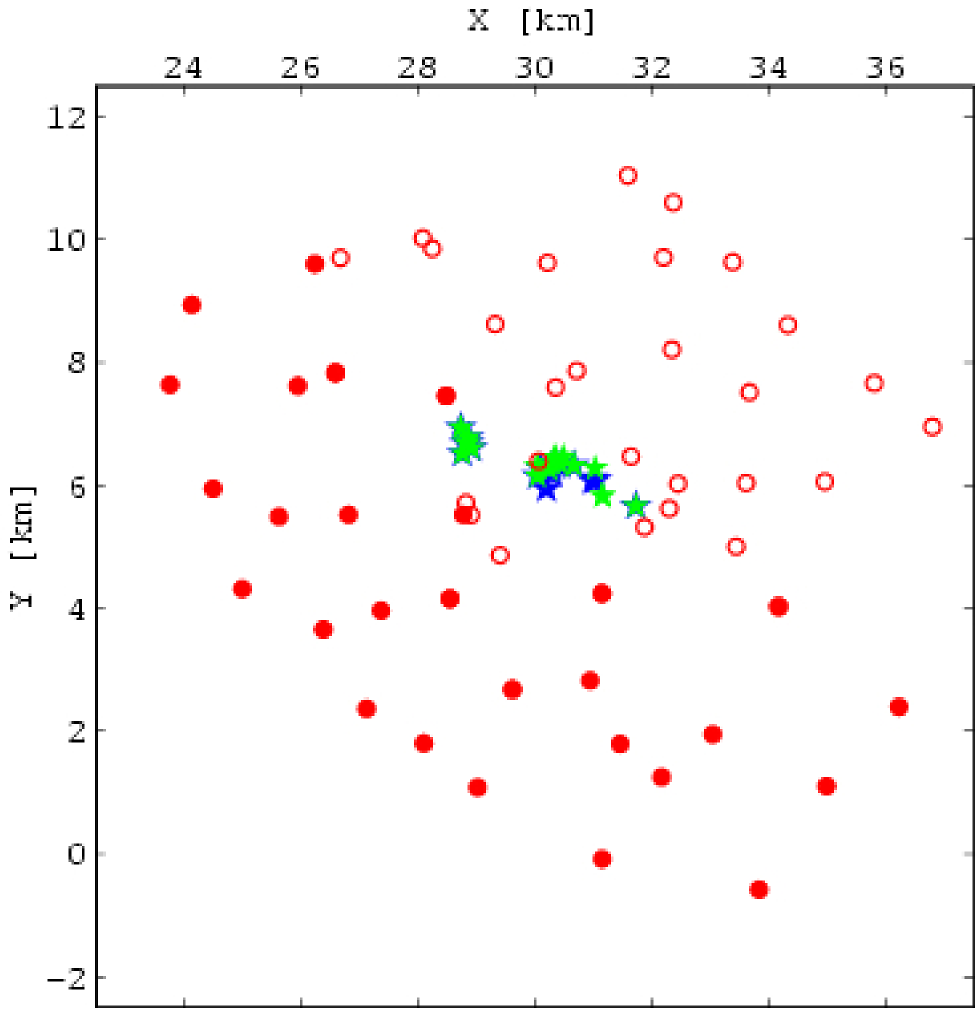
Moment sejsmiczny



Rozkład kątowy



Station distribution



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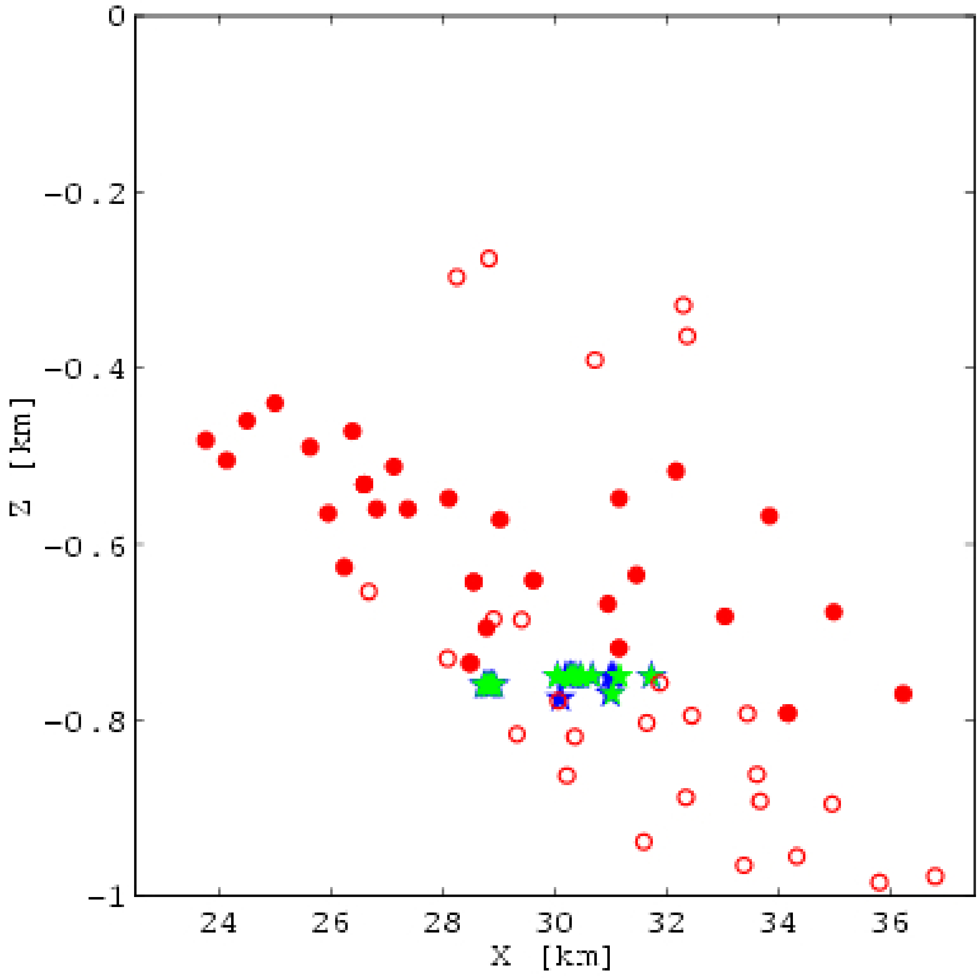
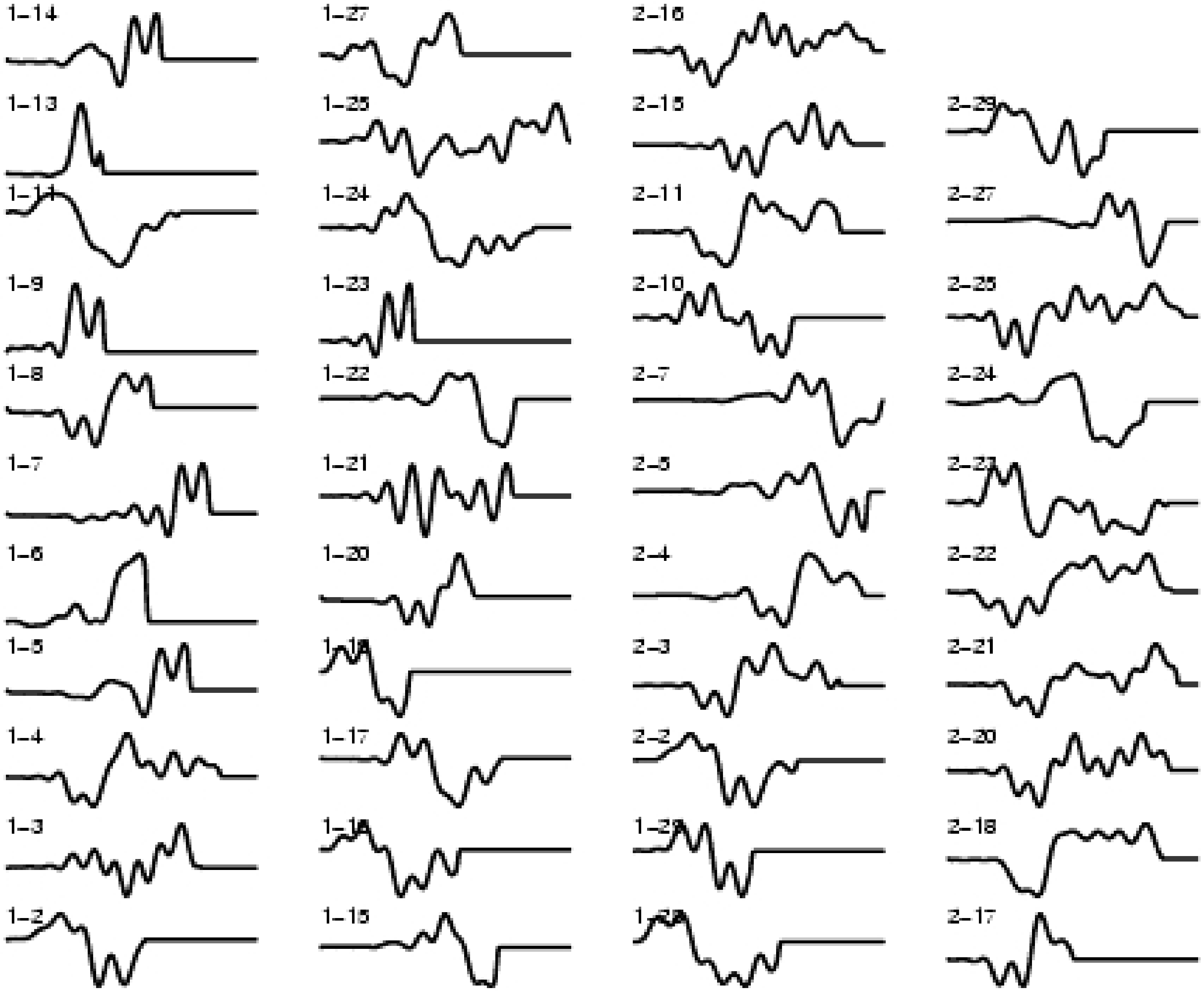
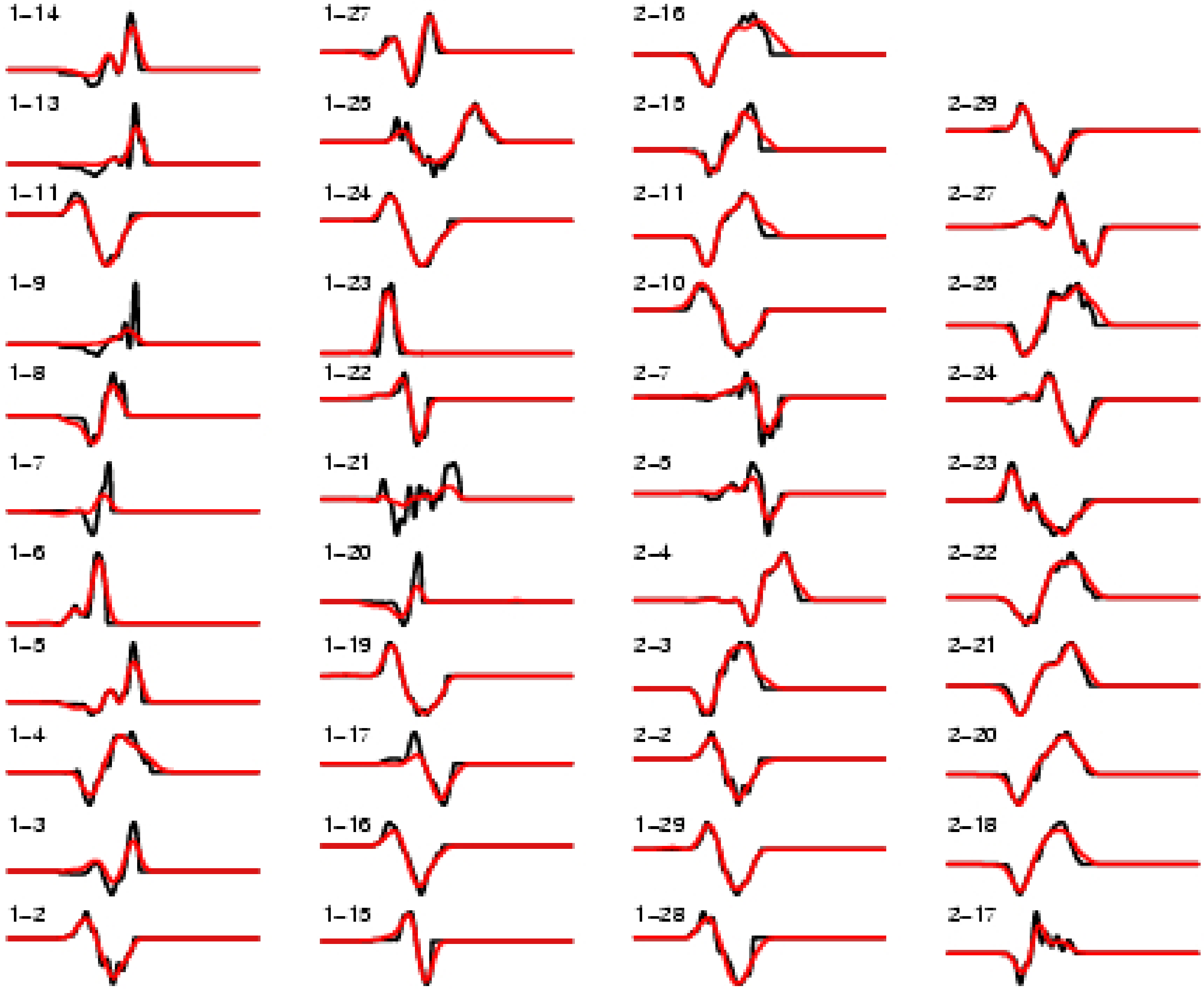


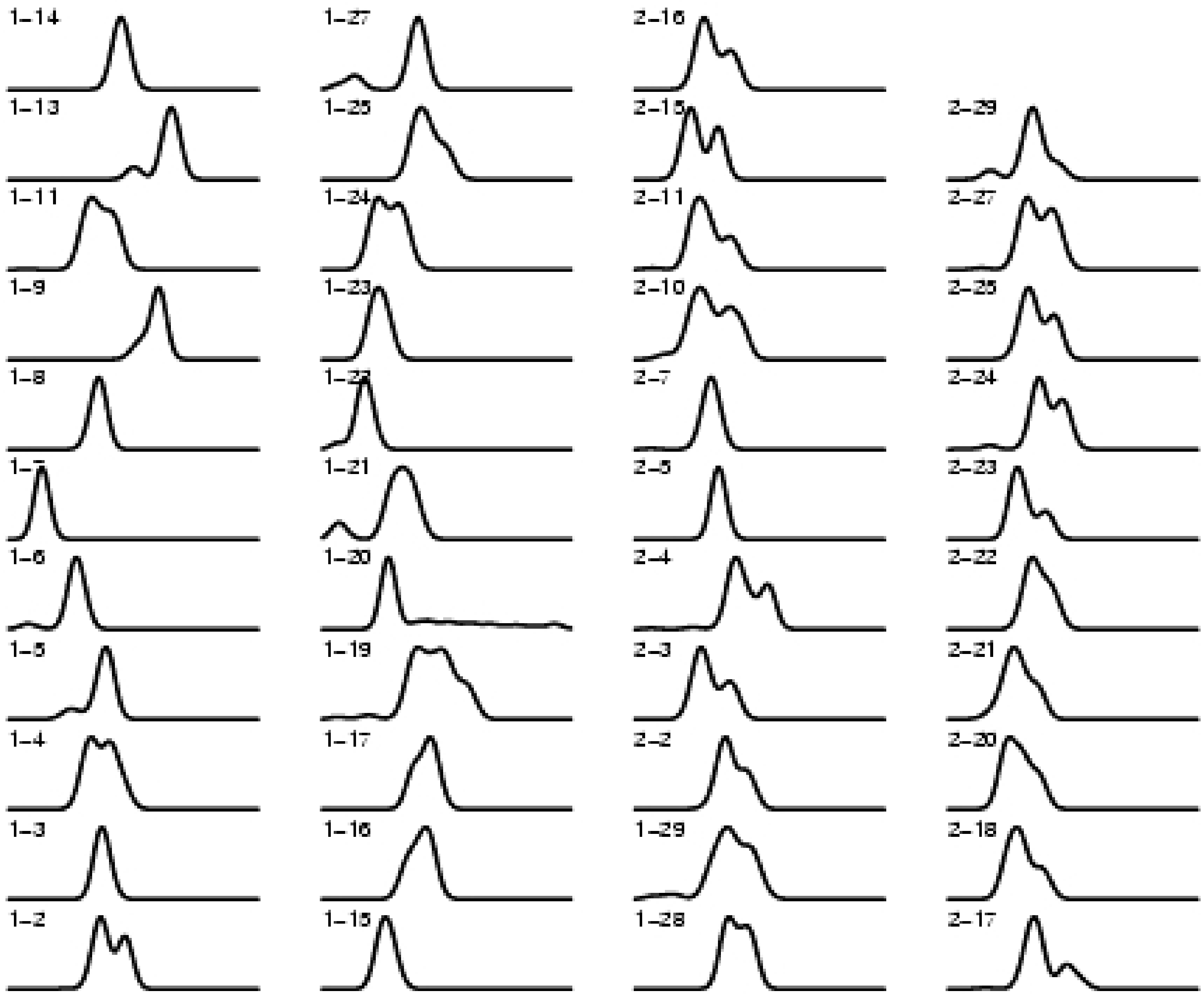
Empirical Green Functions



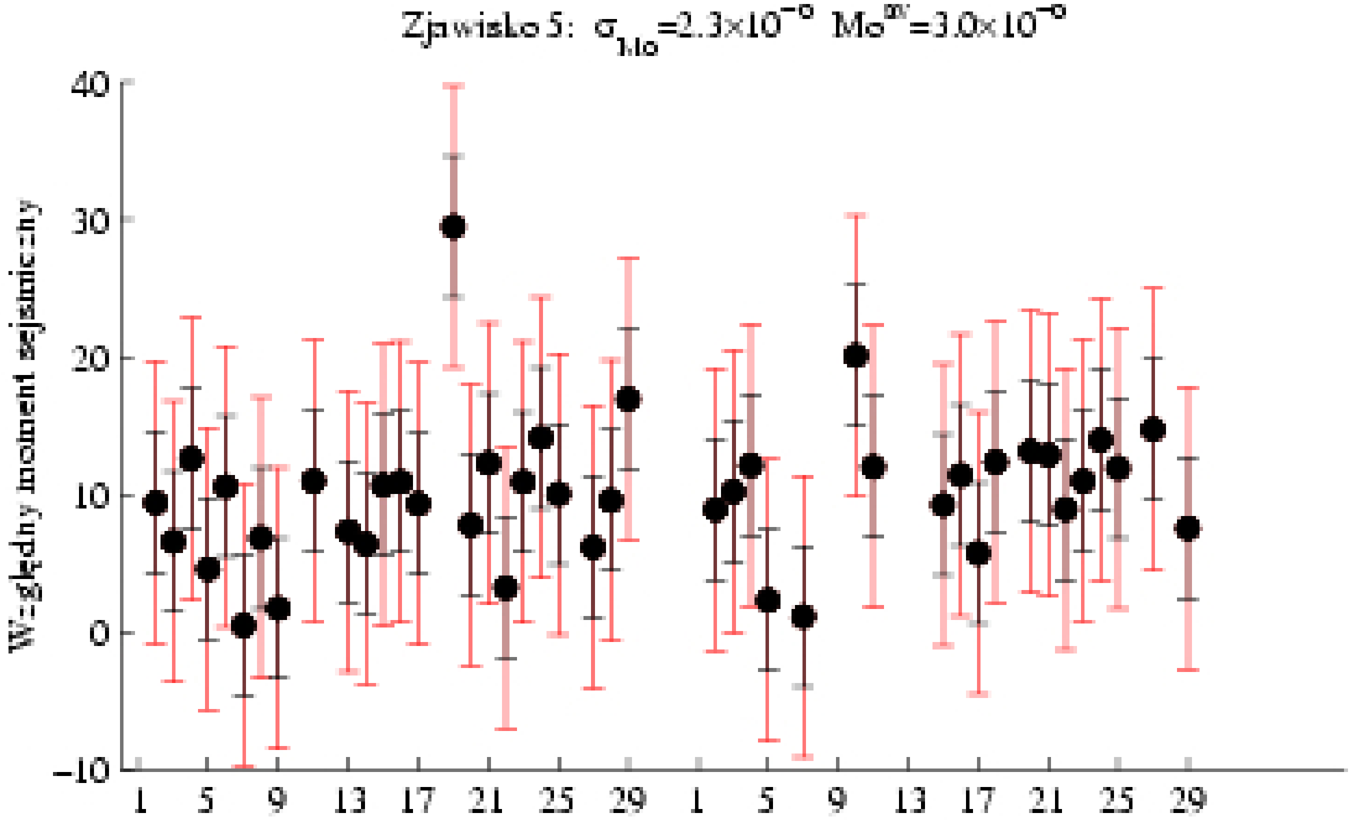
Synthetics/Main



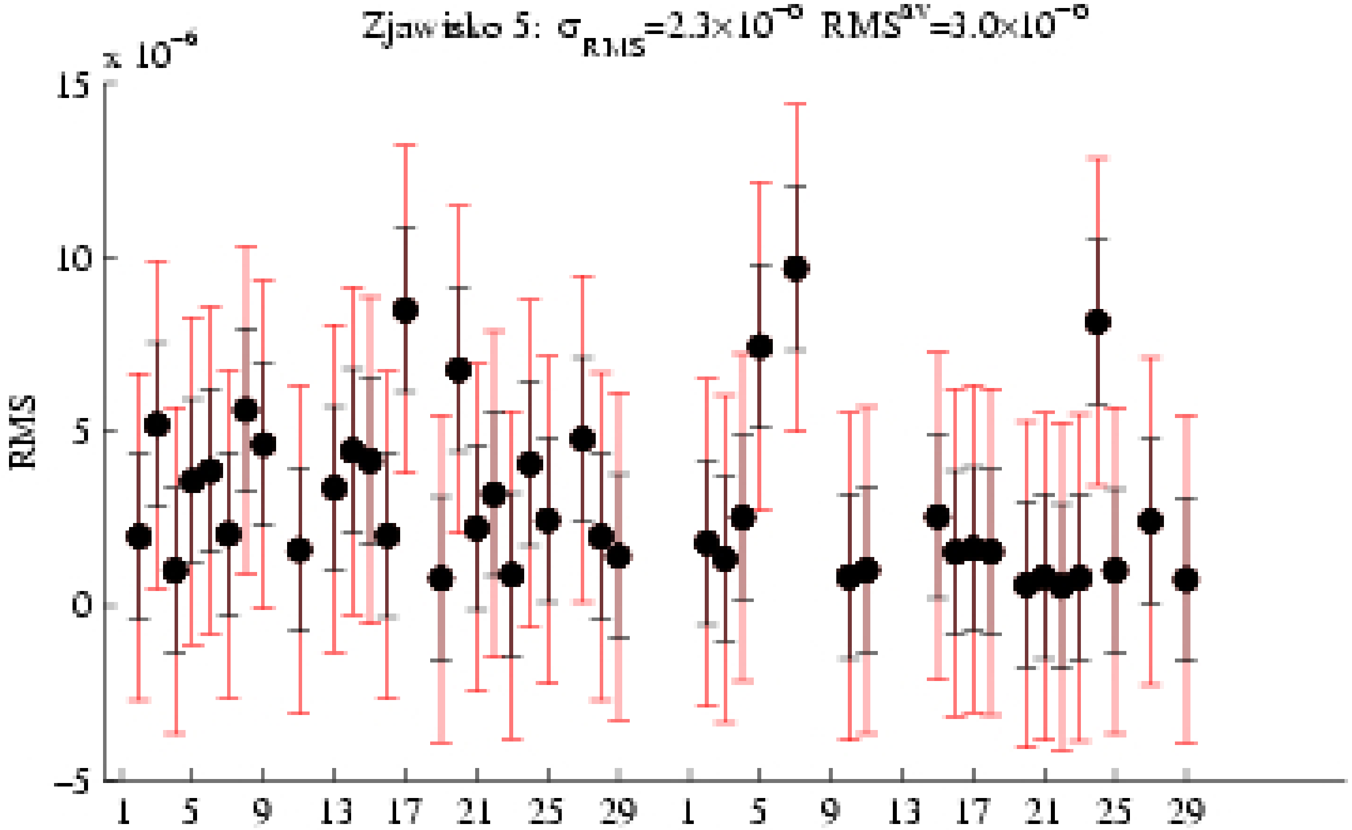
Source Time Function



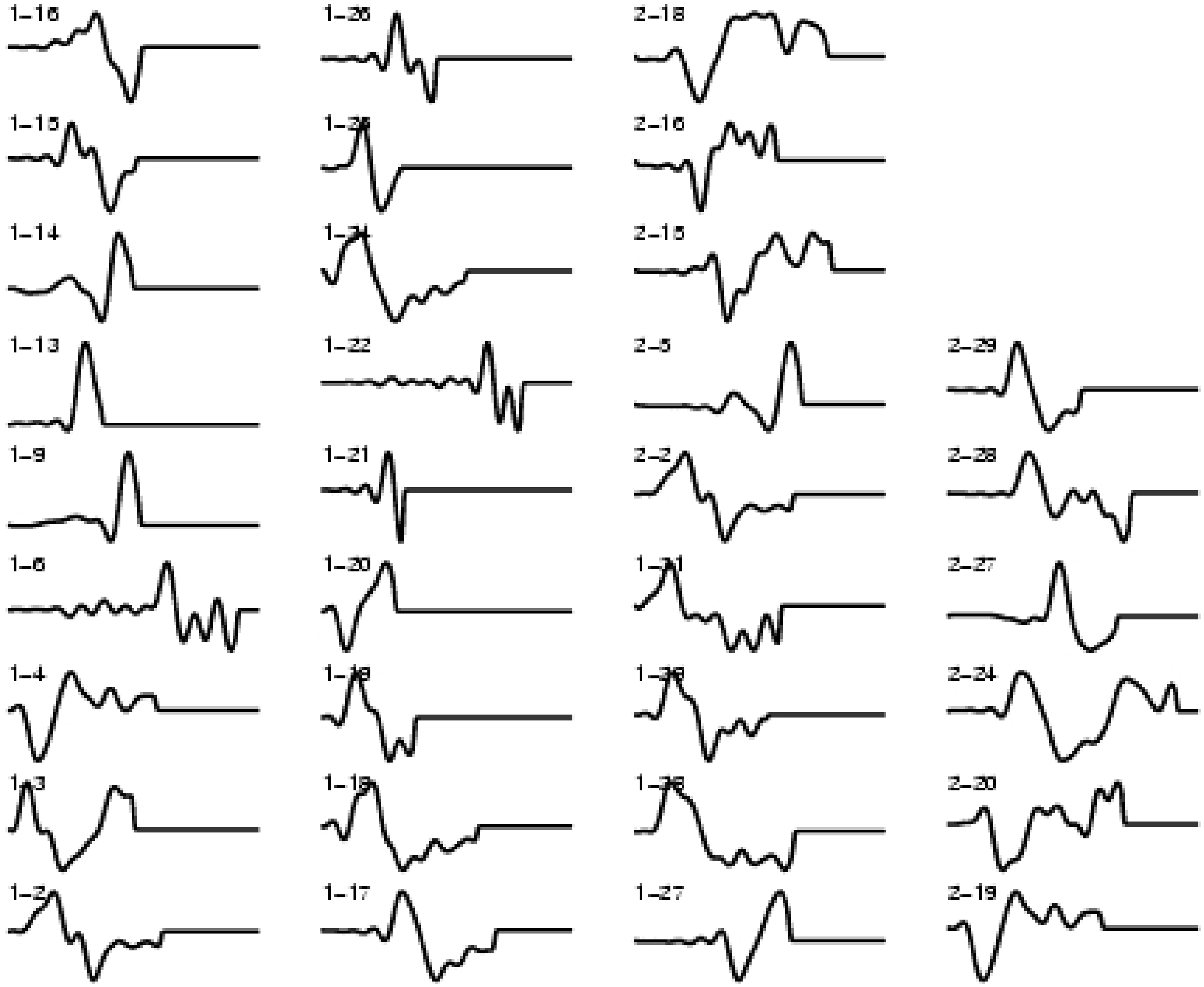
Scalar Moment



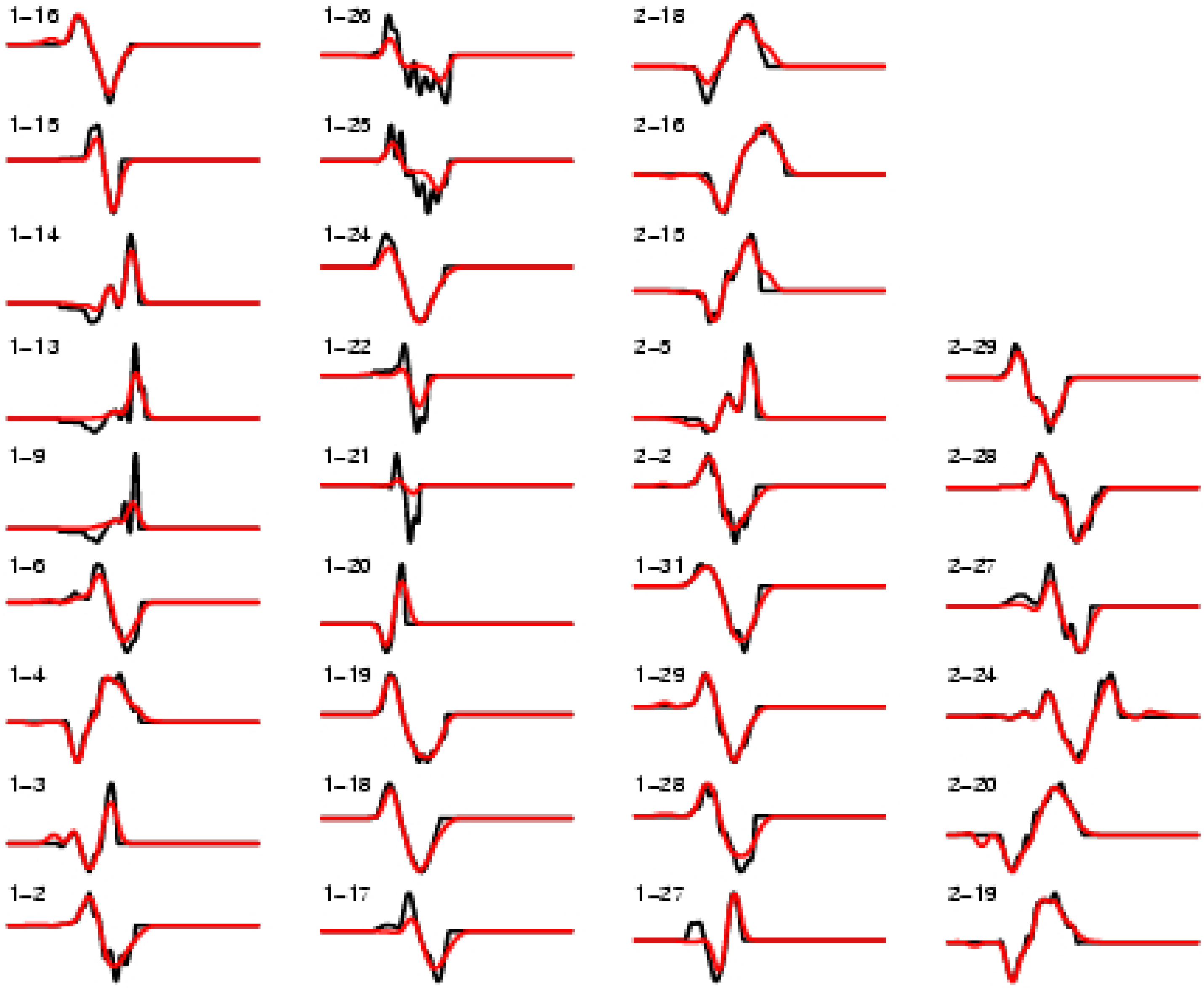
Root Mean Squares Residua



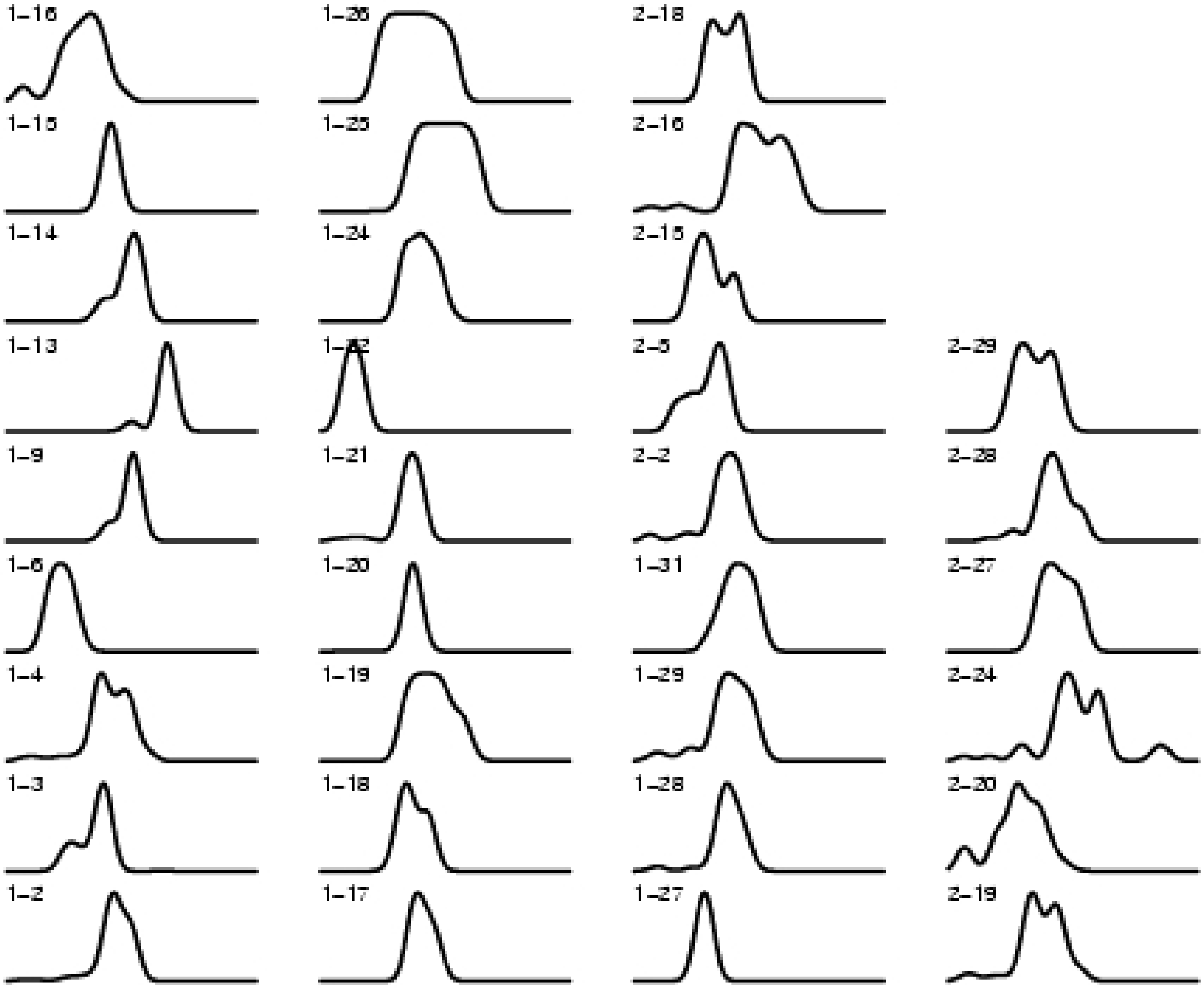
Empirical Green Functions



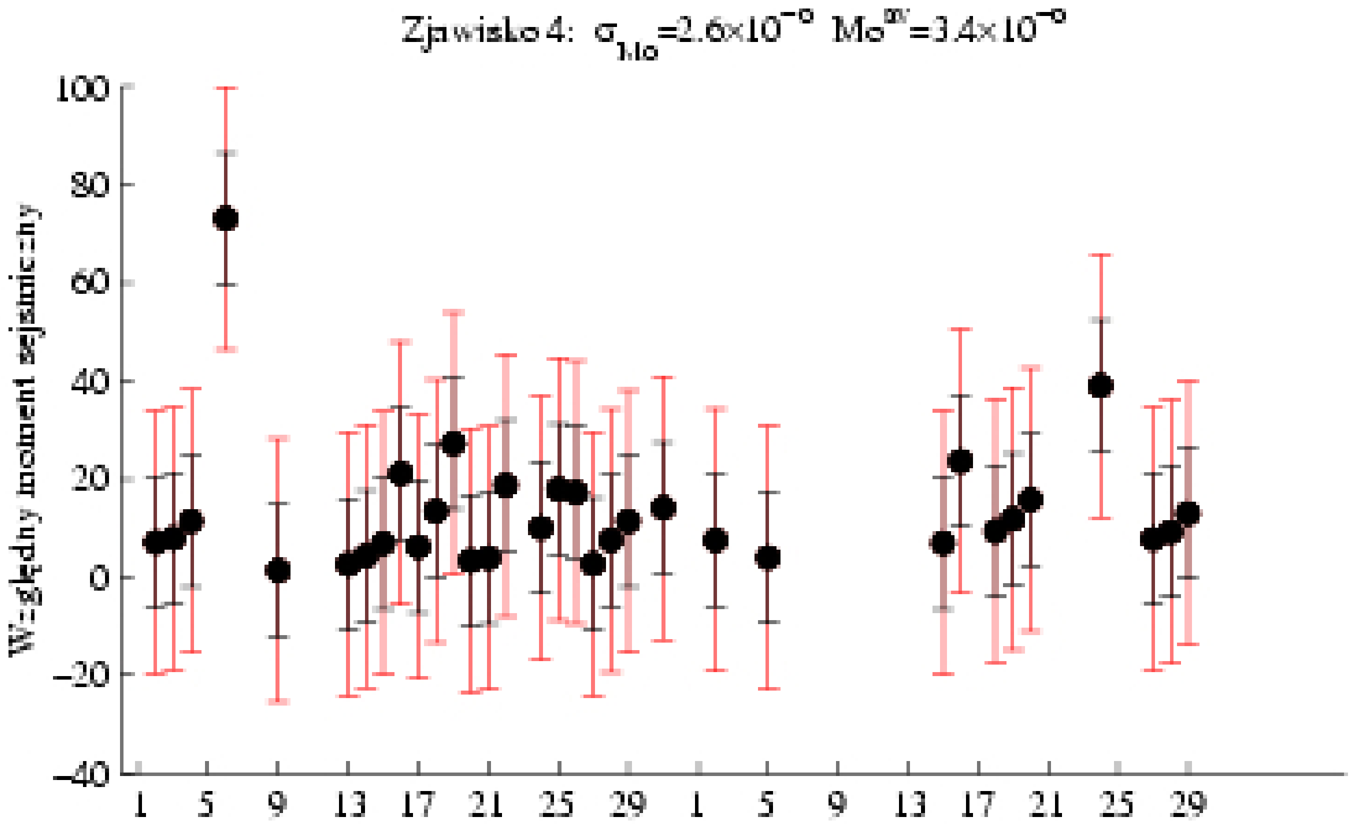
Synthetics/Main



Source Time Function



Scalar Moment



Root Mean Squares Residua