Seismic Tomography

what resolution can be achieved

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Plan of the seminar

- Part I – Principles
  - back-projection
  - non uniqueness
  - tomography as an inverse problem
  - examples
- Part II – Resolution
  - a "pinch" of mathematics
  - synthetic experiment
  - granite rock sample
- Concluding remarks

Tomography ...

The term tomography comes from the Greek word *tomos*, that is, slice, and means forming an image of an object from measurements made from slice (rays) trough it.

The waves passing through the object accumulate information on the spatial distribution of its physical properties and, recorded outside of the object's body, are used to infer this distribution.

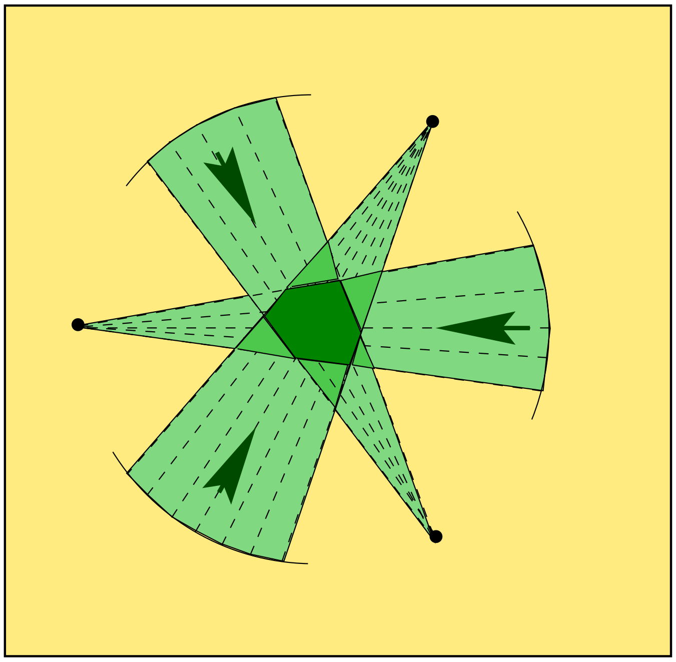
Basic Assumption: the wavelengths of the used (recorded) waves are smaller than the size of heterogeneities of the studied parameters; the waves propagate along lines and sample the medium only in points through which the rays pass.

Example (velocity tomography)

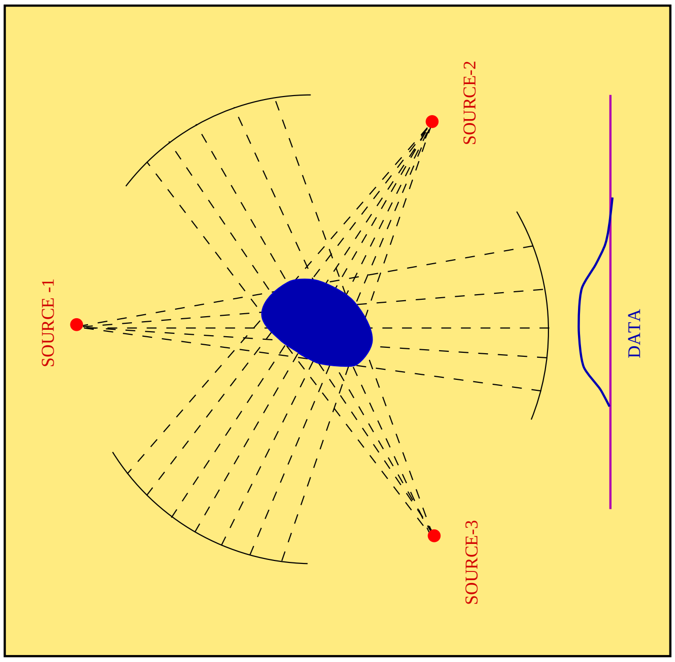
$$t = \int_{ray} \frac{ds}{v(s)}$$

"Back-projection"

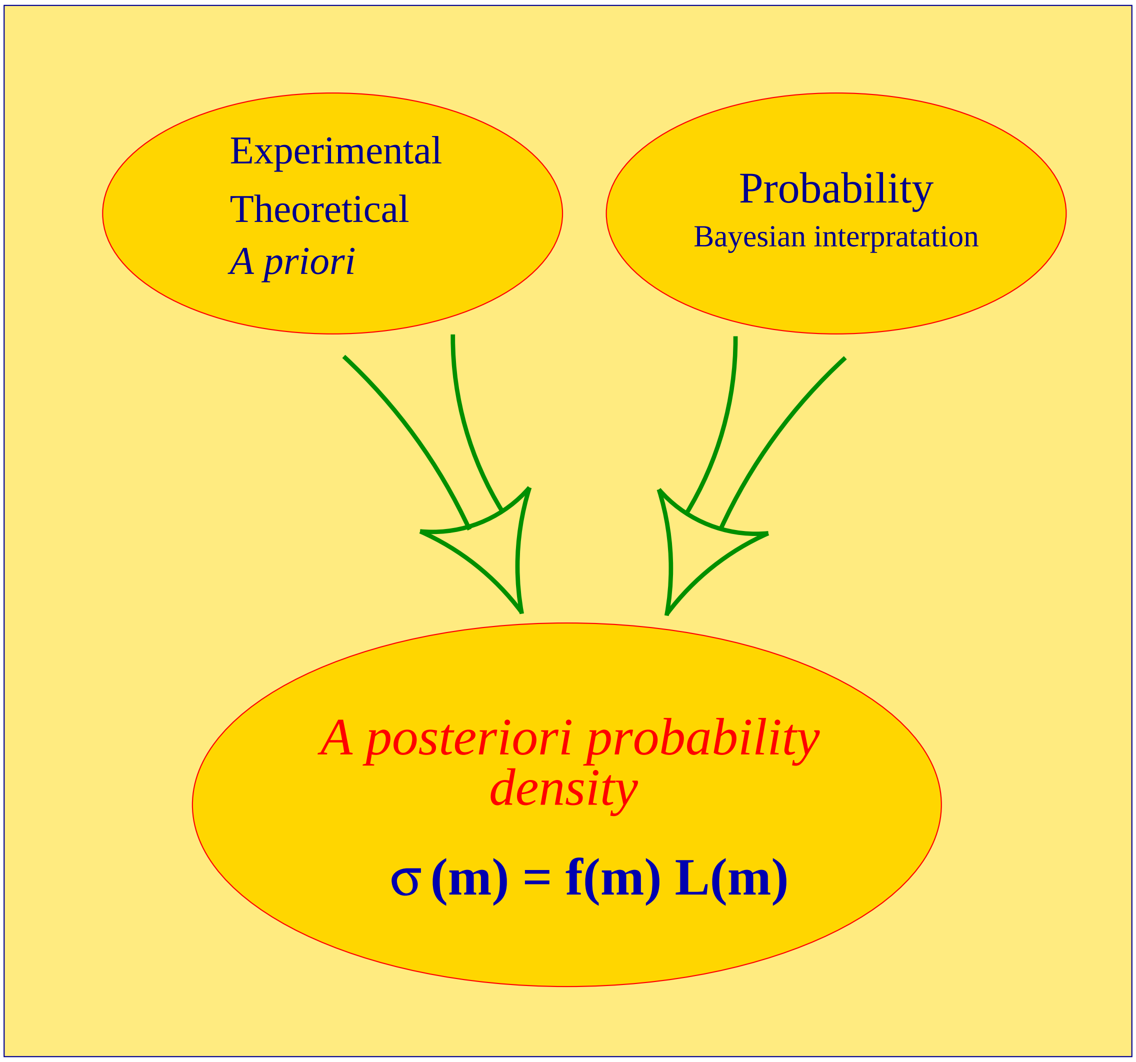
"BackProjection"



Projection

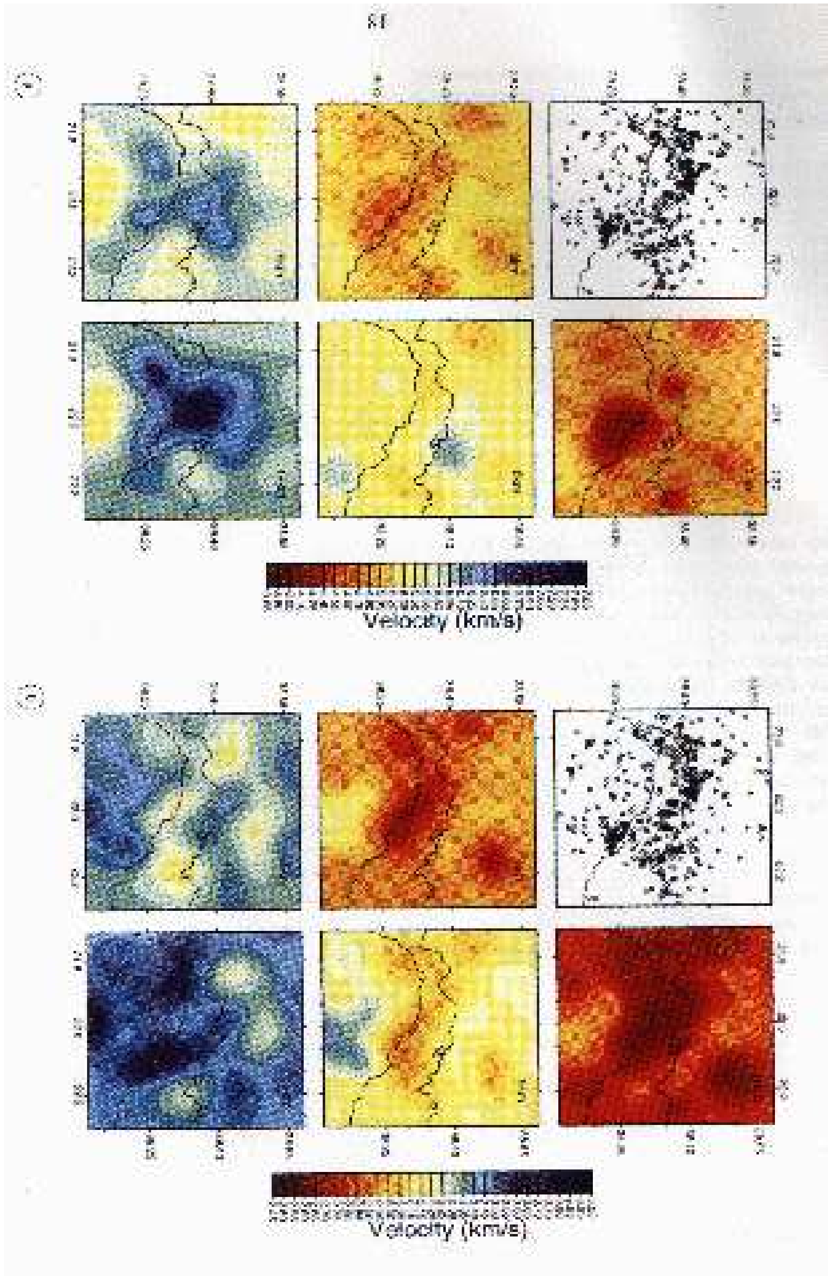


Solving Tomography Problem



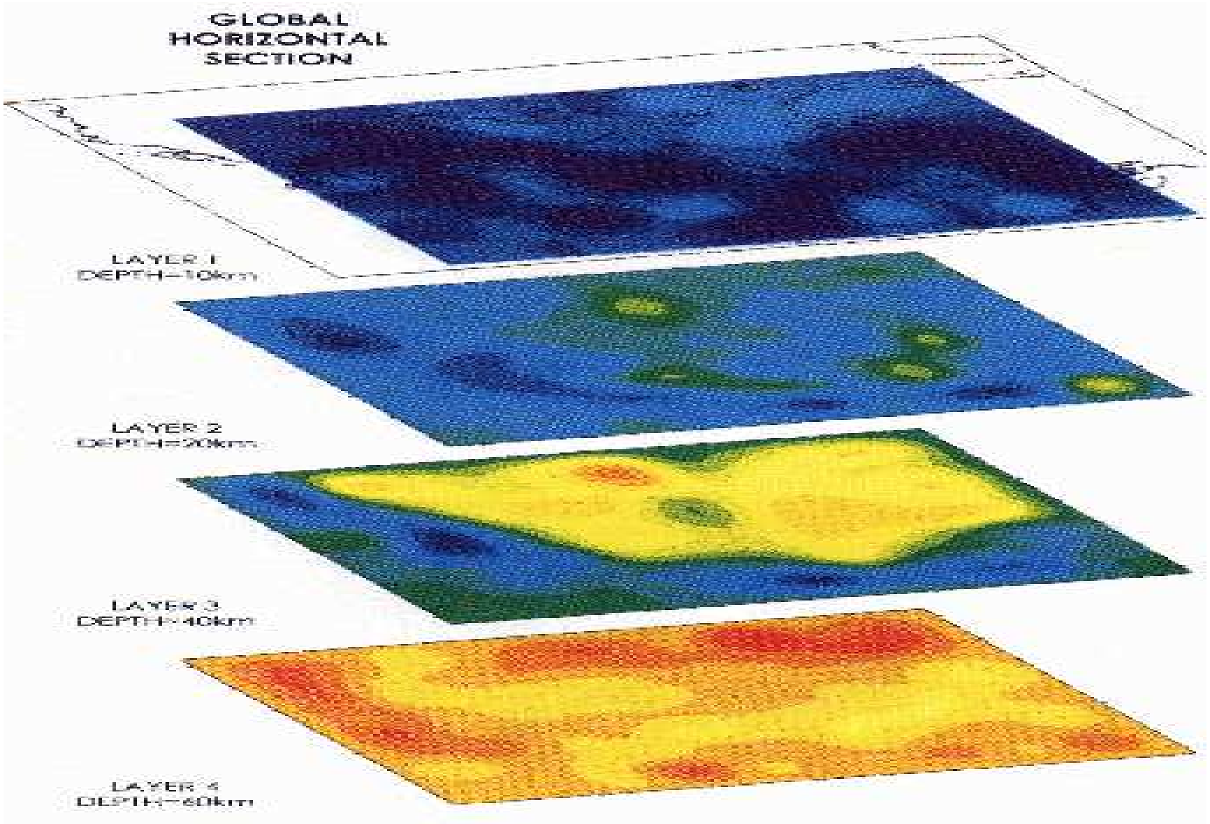
Example-1

Velocity tomography



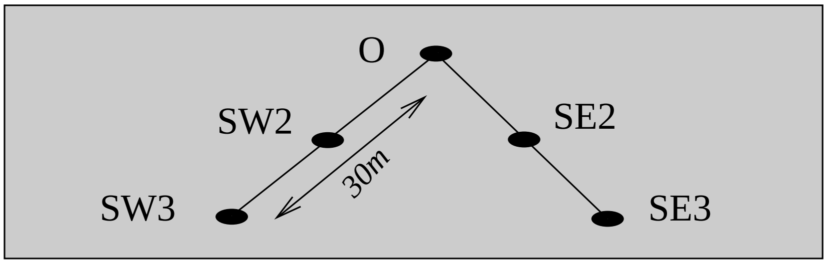
Example-2

Velocity tomography



Example-3

Attenuation tomography



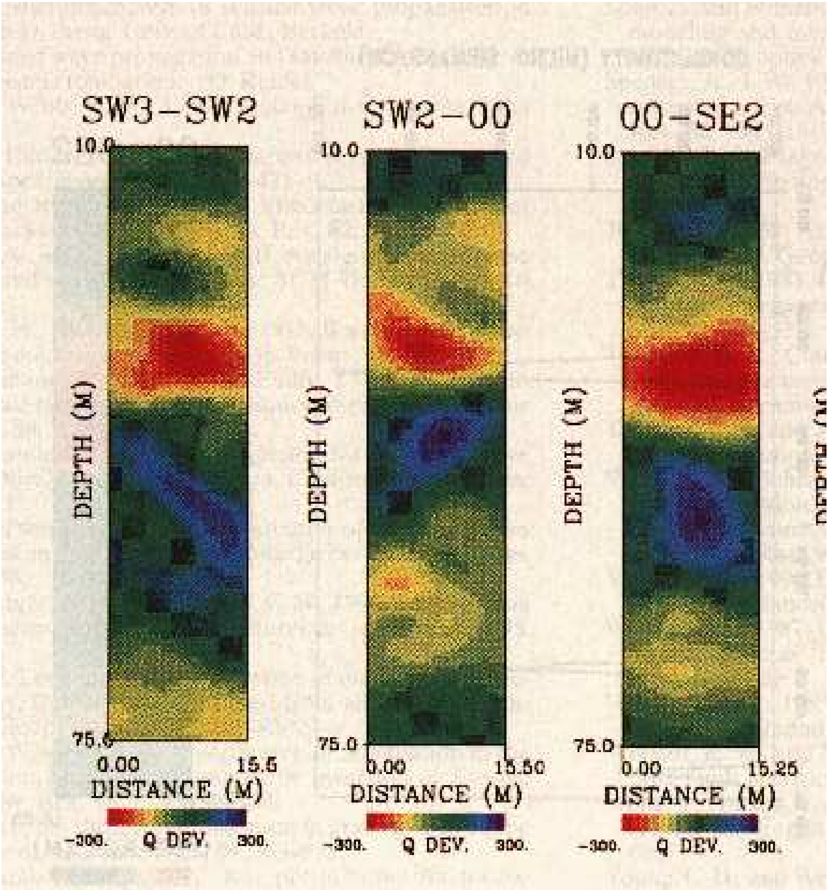


FIG. 20. LSQR inverse solutions for Q in the four Raymond cross-

Mathematics of tomography

- linear tomography

$$\mathbf{d} = \mathbf{G} \cdot \mathbf{m}$$

- $l2$ - least square solution

$$\mathbf{m}^{ML} = \mathbf{m}_{apr} + \mathbf{H}^{-1} \cdot \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot (\mathbf{d}_{obs} - \mathbf{G} \cdot \mathbf{m}_{apr})$$

where

$$\mathbf{H} = \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot \mathbf{G} + \mathbf{C}_M^{-1}$$

- noise free data

$$\mathbf{d}_{obs} = \mathbf{G} \cdot \mathbf{m}_{true}$$

Resolution Operator (matrix)

$$\mathbf{m}^{ML} - \mathbf{m}_{apr} = \mathbf{R} \cdot (\mathbf{m}_{true} - \mathbf{m}_{apr})$$

$$\mathbf{R} = \left\{ \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot \mathbf{G} + \mathbf{C}_M^{-1} \right\}^{-1} \cdot \mathbf{G}^T \cdot \mathbf{C}_D^{-1} \cdot \mathbf{G}$$

$$\mathbf{C}_D = \sigma_D^2\, \mathbf{I} \qquad \mathbf{C}_D = \sigma_M^2\, \mathbf{I} \qquad g = \sigma_D / \sigma_M$$

$$\mathbf{R} = \left(\mathbf{G}^T \cdot \mathbf{G} + g^2\, \mathbf{I} \right)^{-1} \cdot \mathbf{G}^T \cdot \mathbf{G}$$

Properties

- Ideal case: $\mathbf{R} \approx \mathbf{I}$
  never achieved due to a limited number of observations $d^i_{obs} \quad i = 1 \ldots N_{obs}$

- Normalized diagonal elements:

$$0 \leq \mathbf{R}_{ii} \leq 1$$

- When medium is divided into homogeneous blocks $\mathbf{R}$ becomes a 2-D matrix of the size $\mathrm{N}_{cell} \times \mathrm{N}_{cell}$

- $\mathbf{R}_{ii} = 0$ only for cells probed by no rays

- Average (minimal) values of $\mathbf{R}_{ii} \quad ii = 1 \ldots \mathrm{N}_{cell}$ strongly depends on:
  - source-receiver configuration
  - cell size

Numerical Experiment (1)

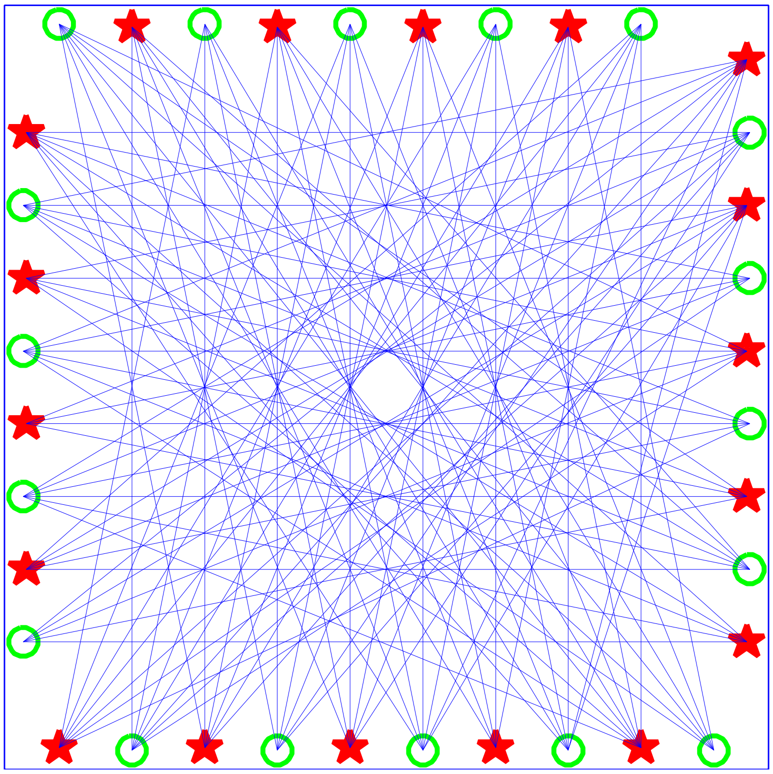


Figure 1: An example of the source-receiver configuration used in a pillar or rock sample tomography imaging.

Numerical Experiment (2)

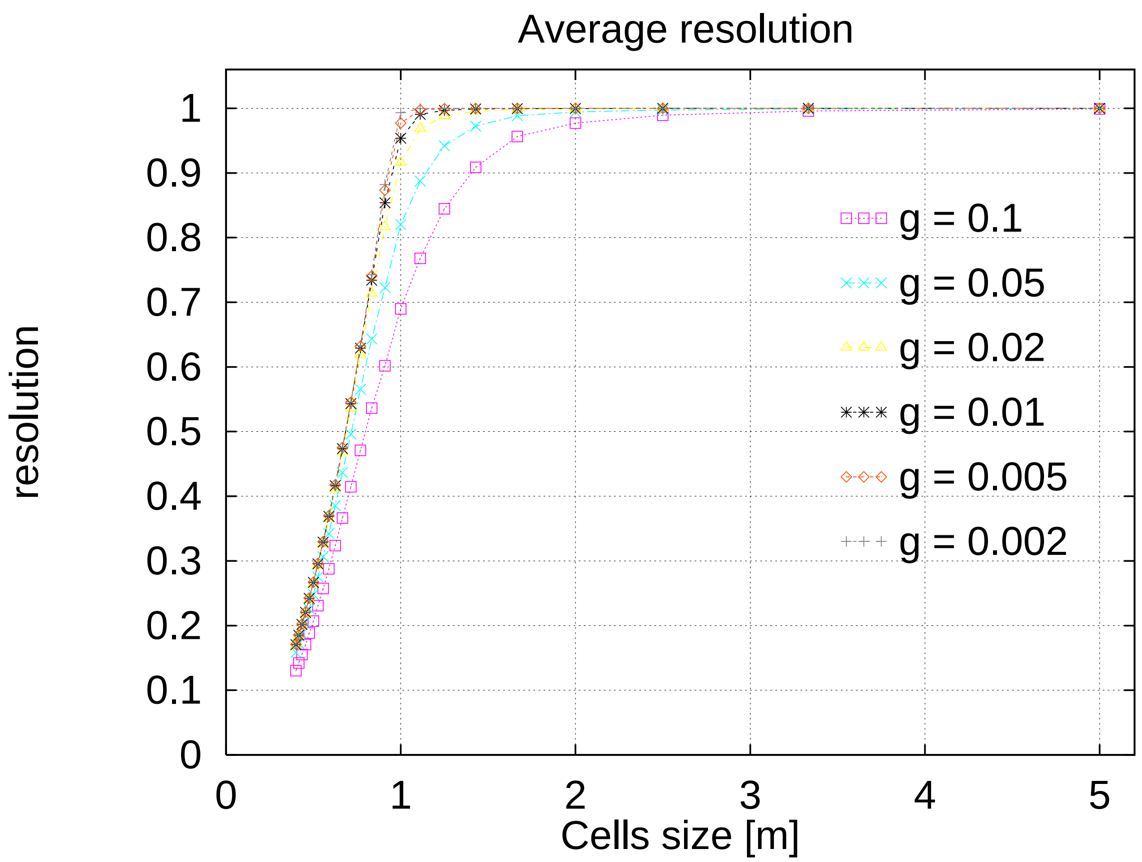


Figure 2: Average resolution as a function of cells size. The square cells were considered only

Rock Sample - Layout

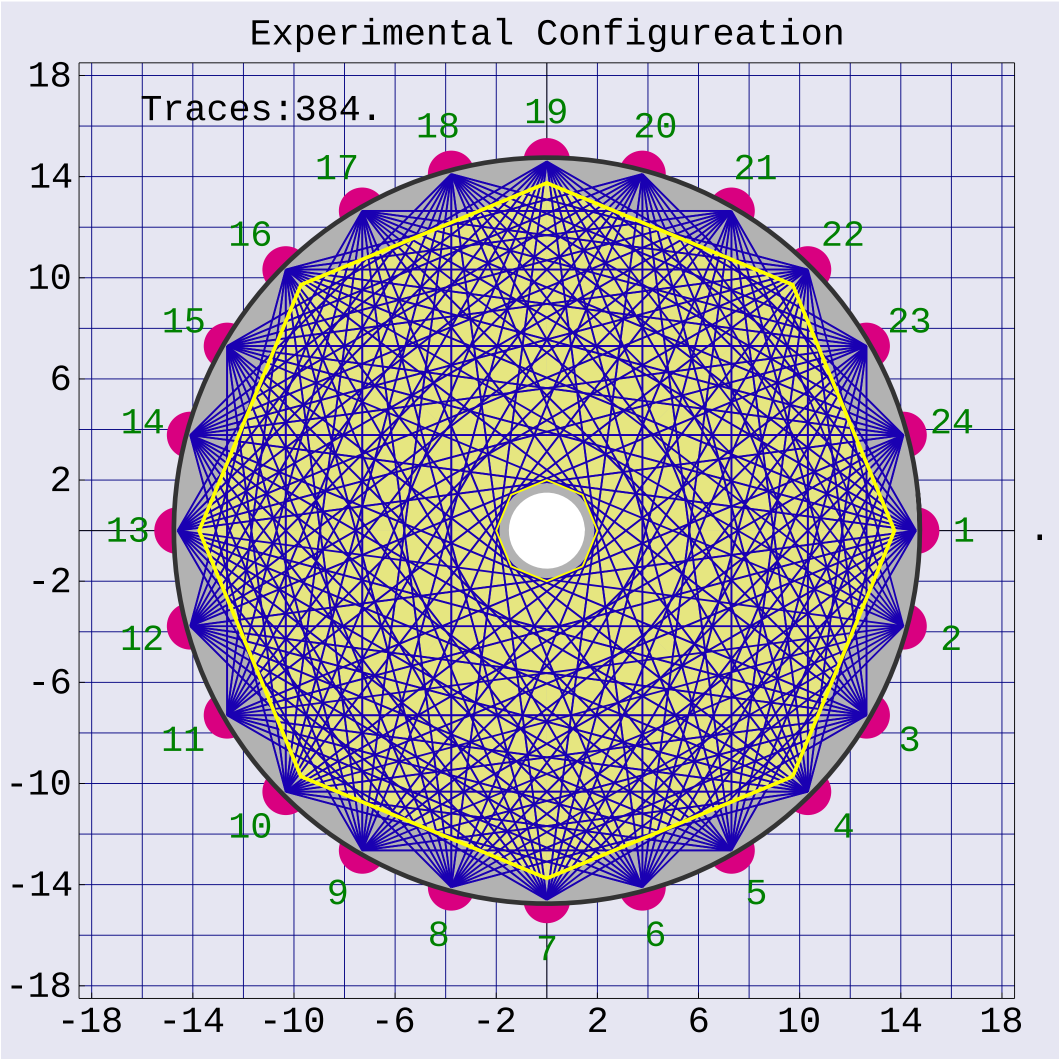


Figure 3: Layout of transducers and ray paths

Ray path coverage

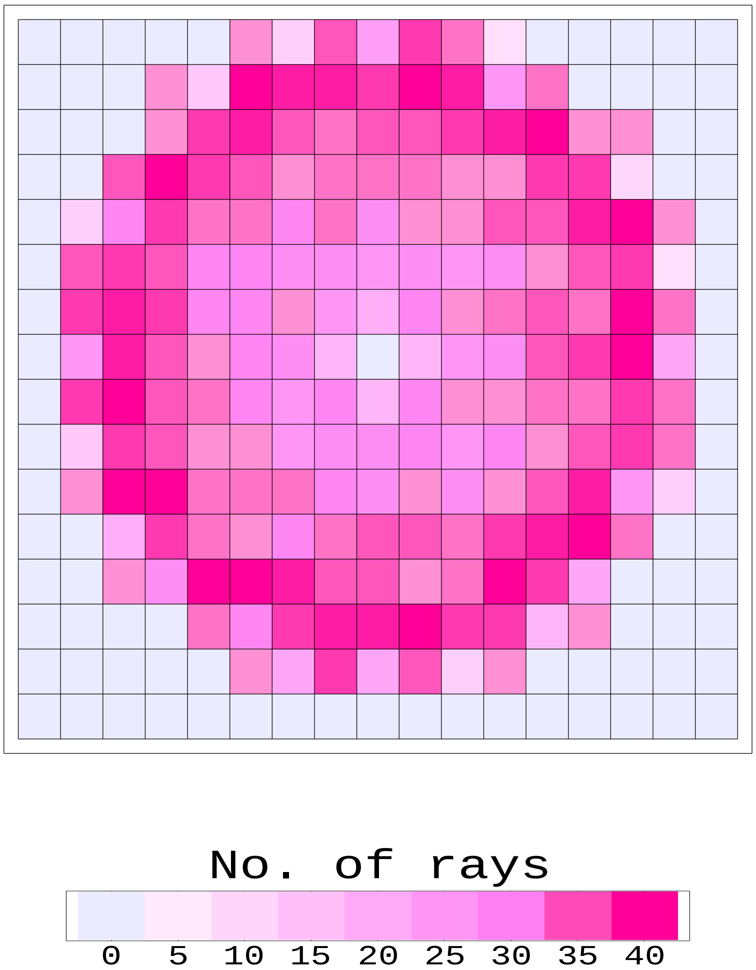


Figure 4: Ray path coverage

Resolution Curve

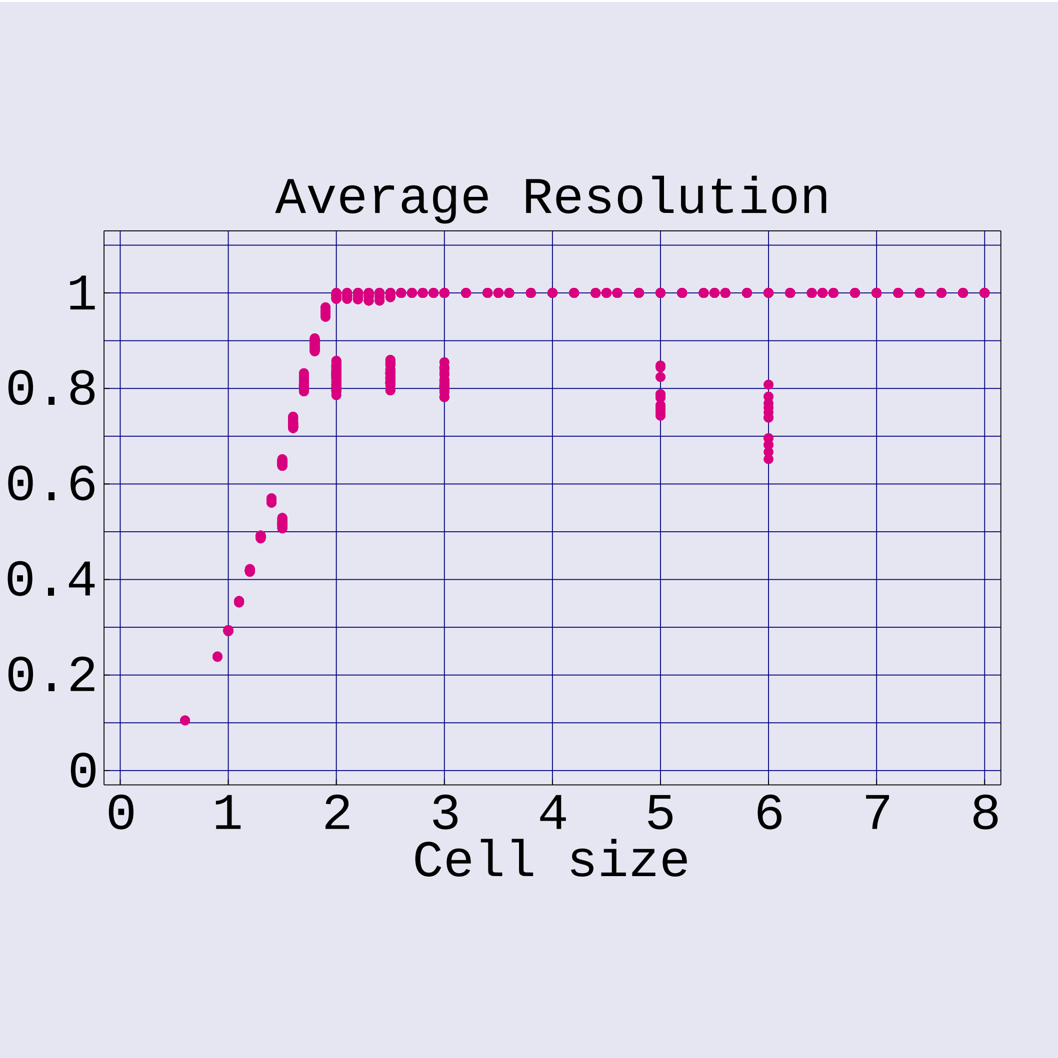
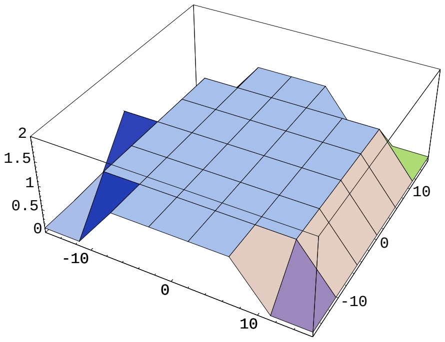
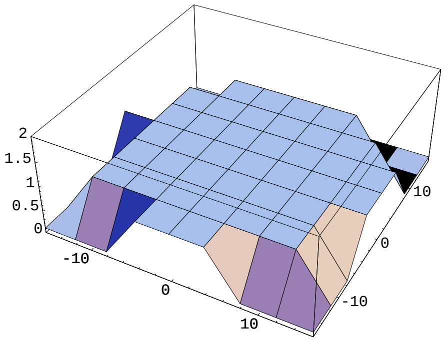


Figure 5: Monte Carlo simulation of the resolution curve

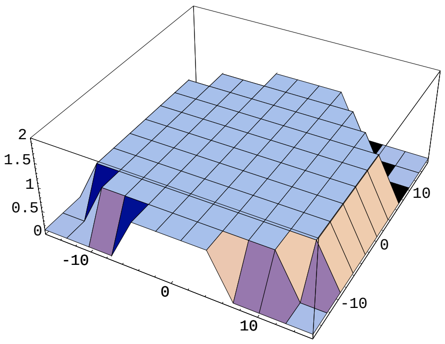
Synthetic images



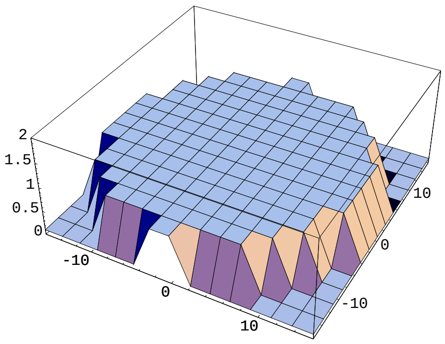
5cm



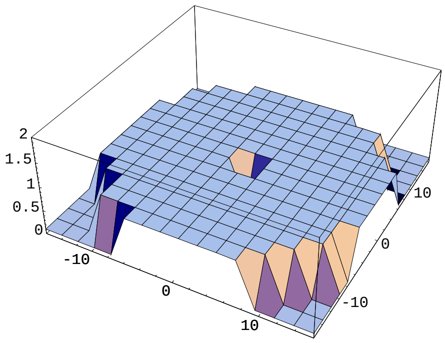
4cm



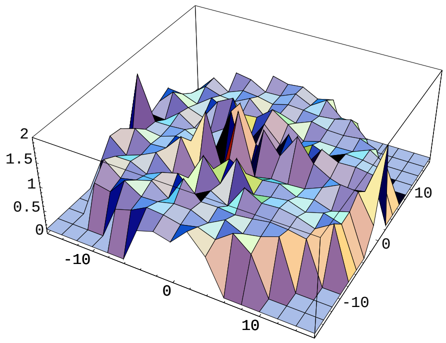
3cm



2.2cm

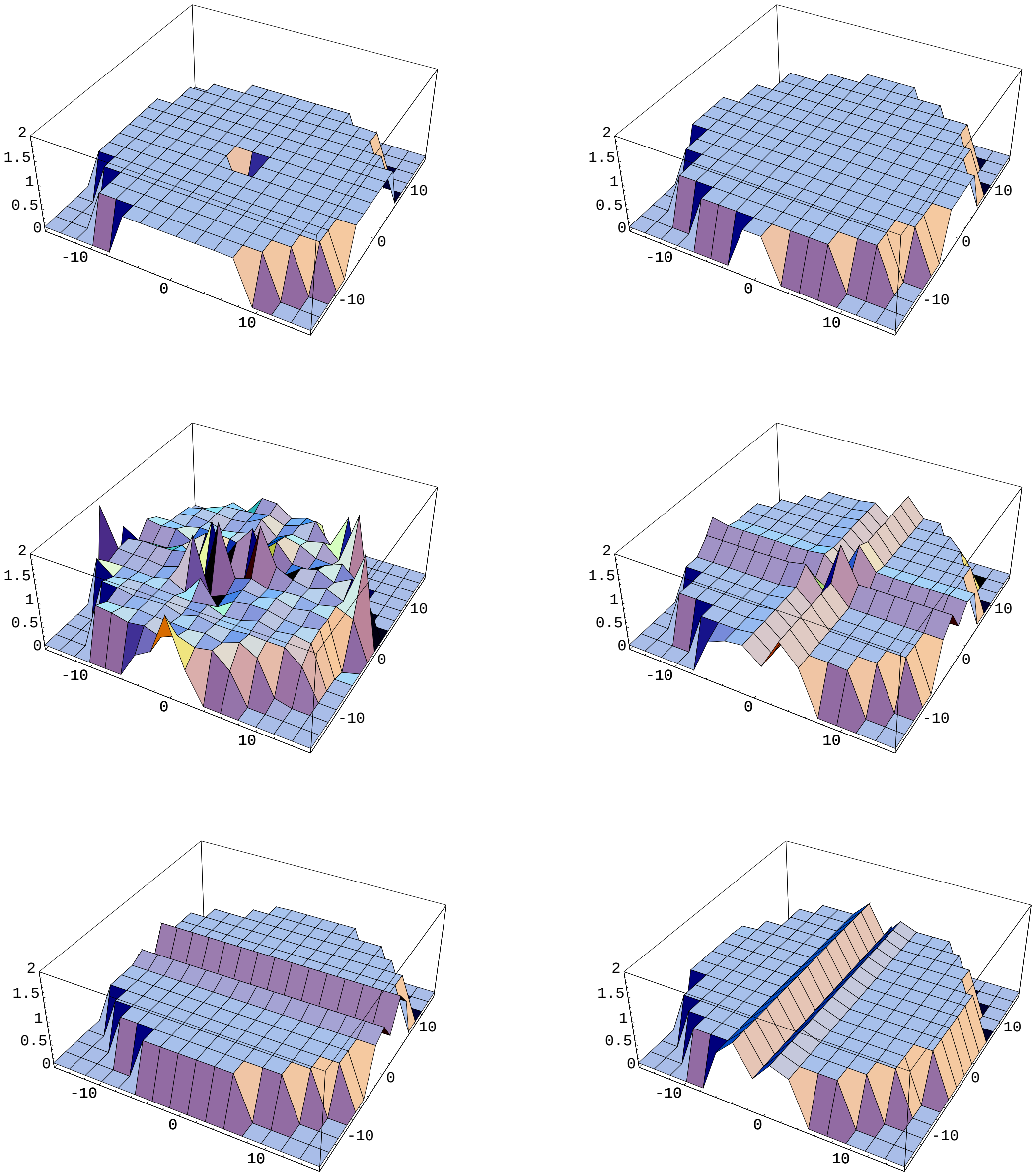


2cm

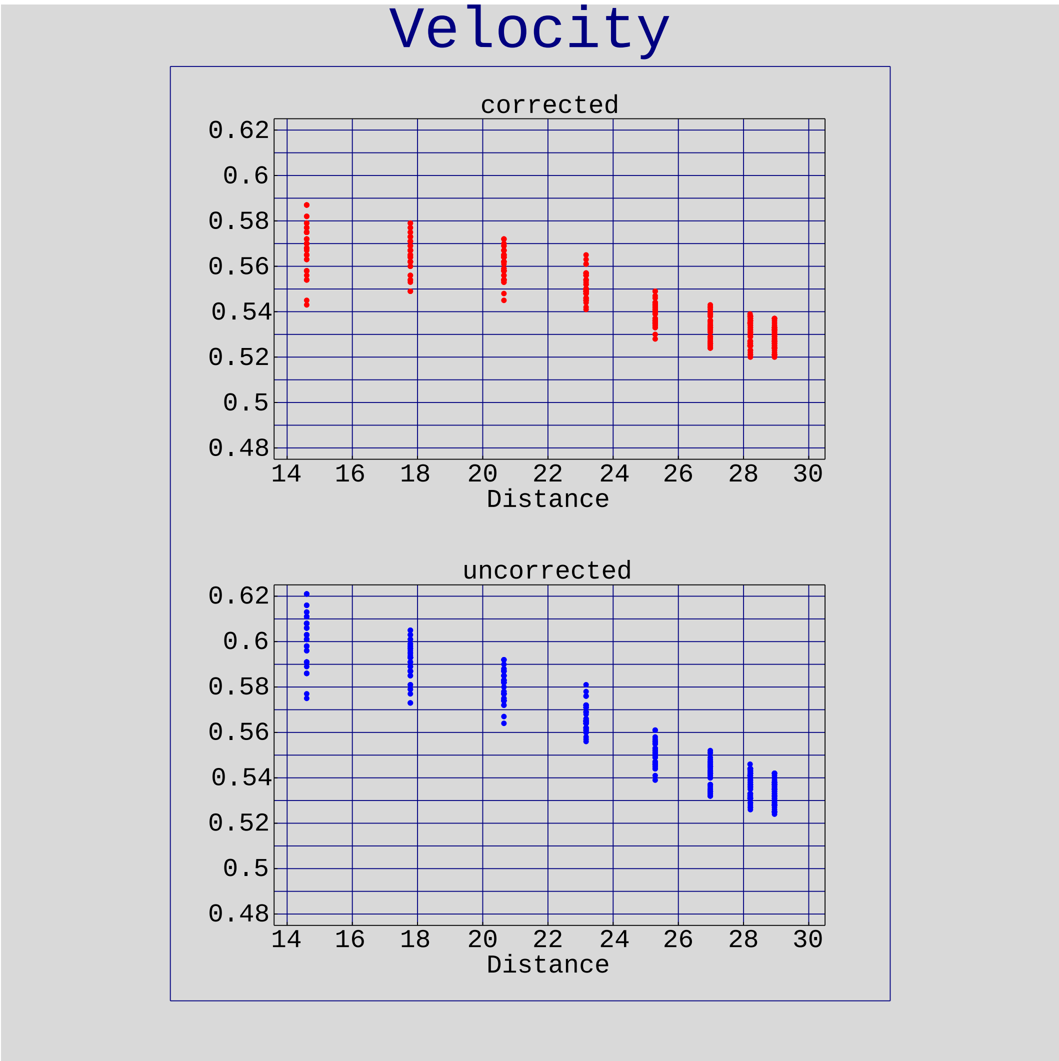


1.9cm

Synthetic images (2cm cells)



Tomography data



Tomography data cd...

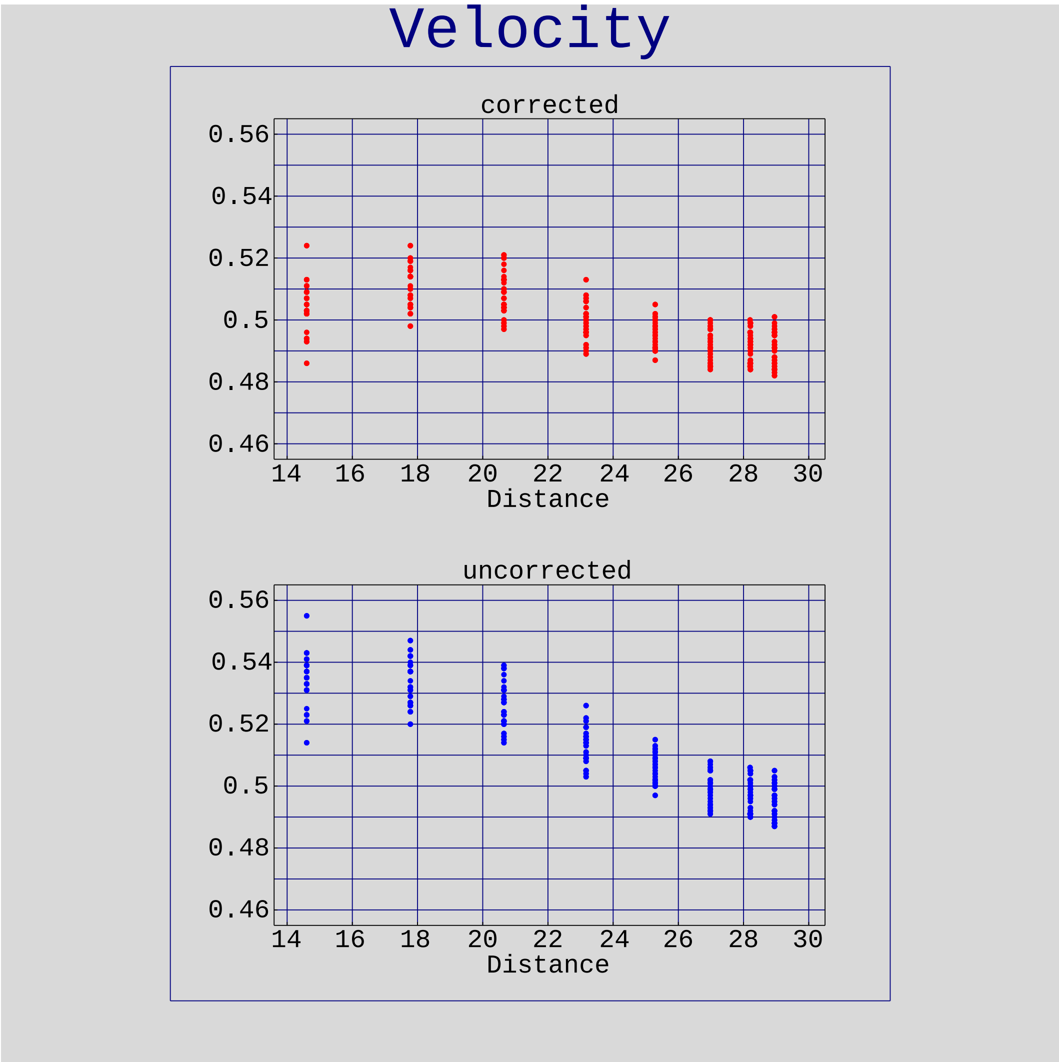
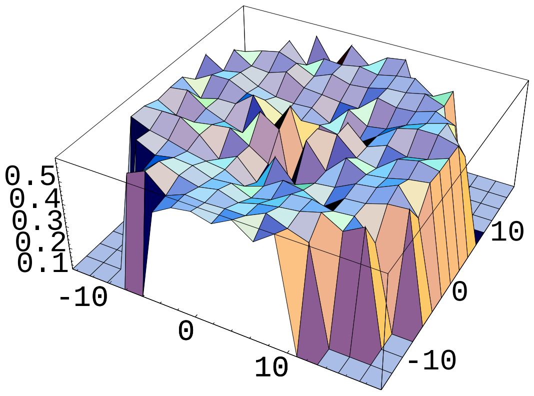


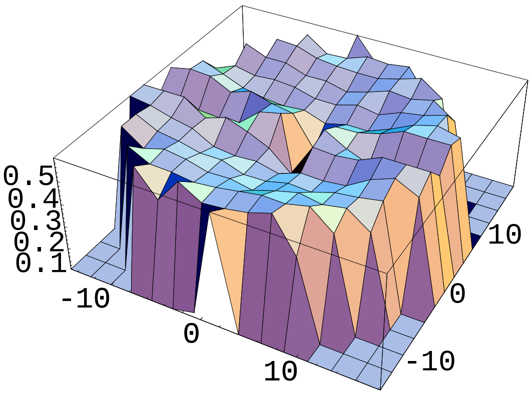
Figure 6: First peak arrival time based velocity data

Velocity (3D) Images

(first arrival)



2cm



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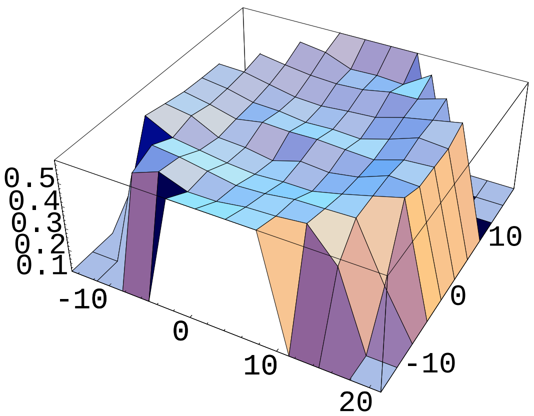
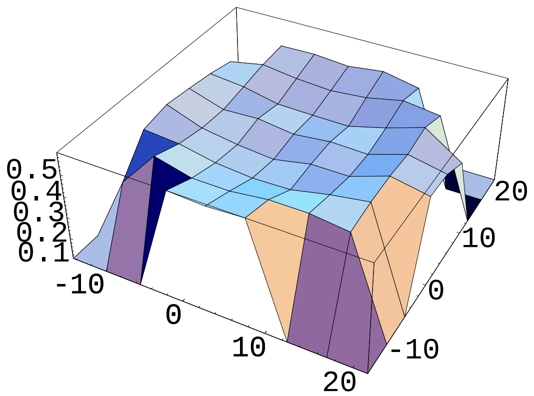


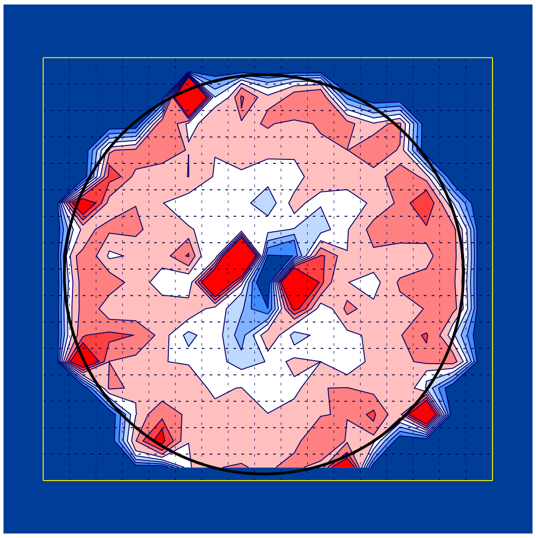
3cm



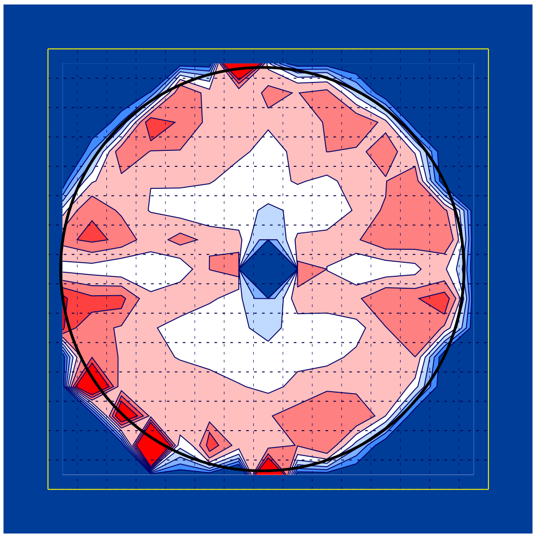
4cm

Velocity (2D) Images

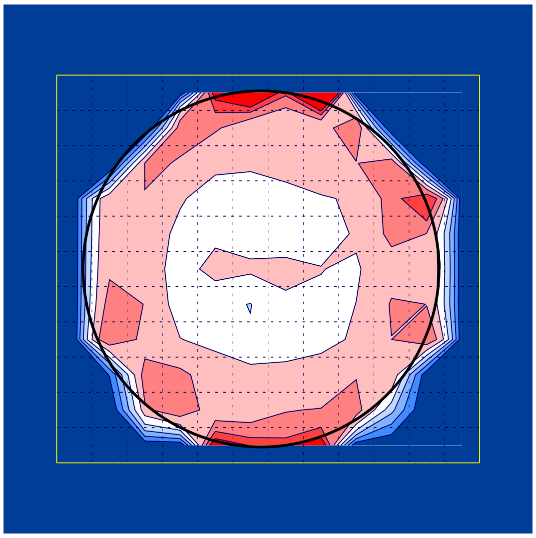
(first arrival)



2cm



2.2cm



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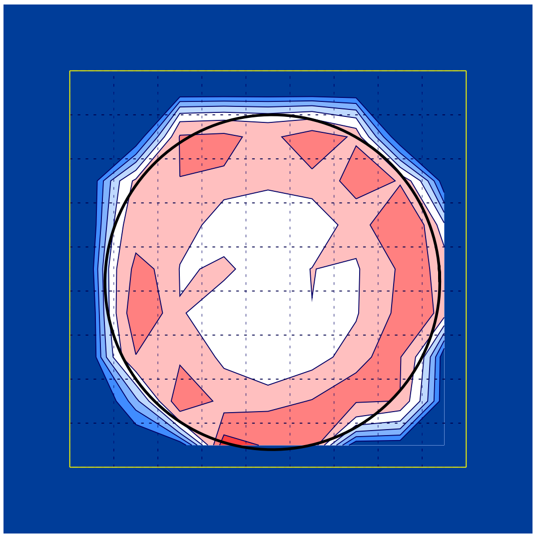
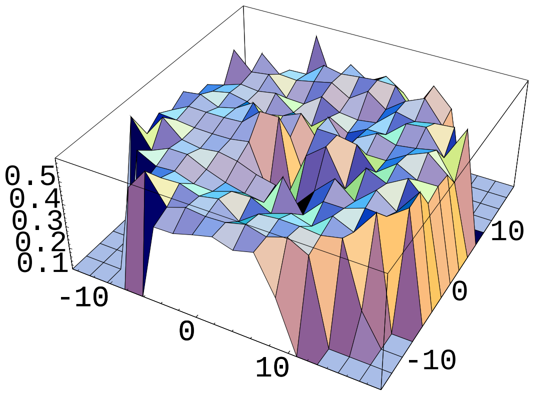


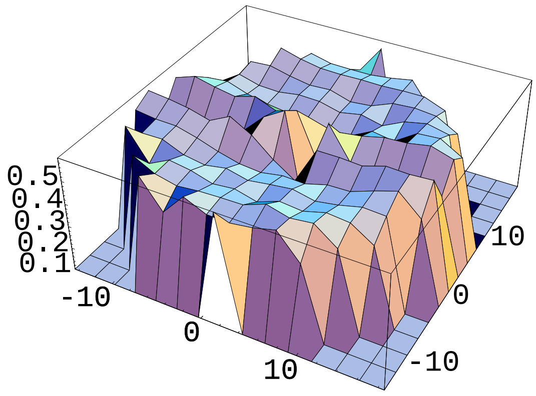
4cm

Velocity (3D) Images

(peak arrival)



2cm



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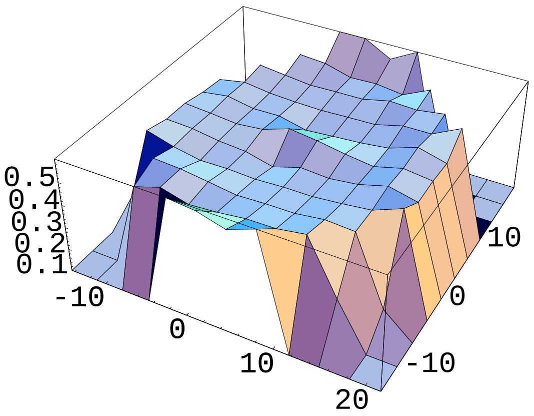
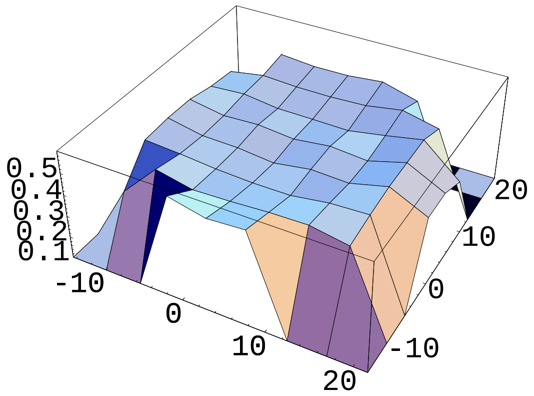


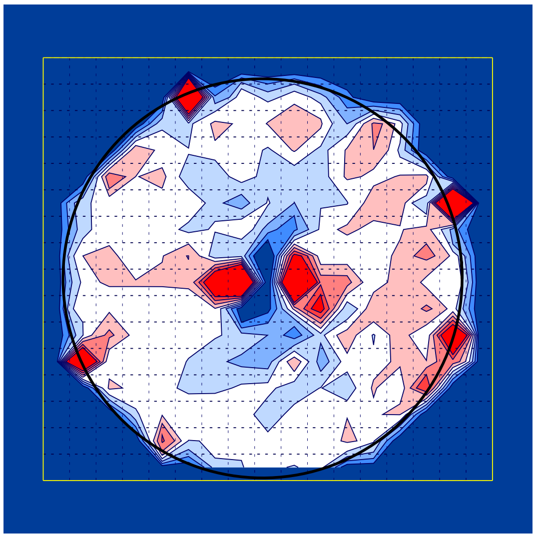
3cm



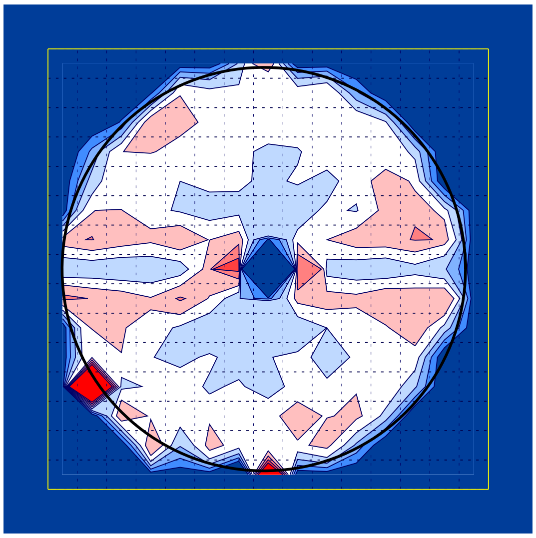
4cm

Velocity (2D) Images

(peak arrival)



2cm



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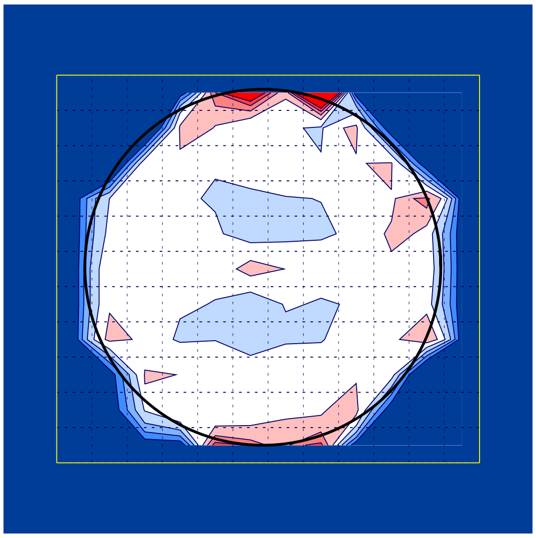

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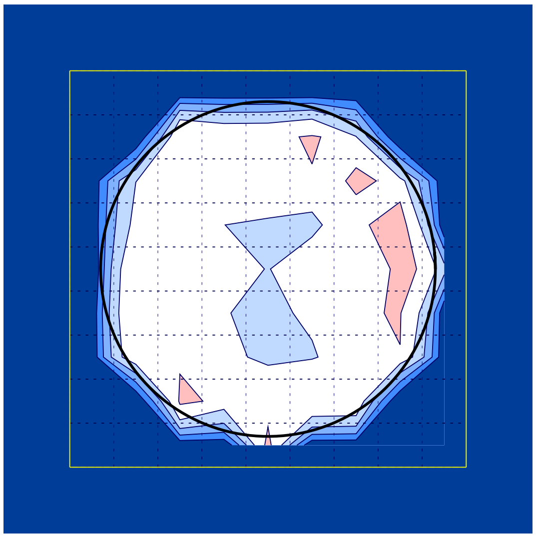

4cm

Conclusions

Local vs. Global Optimization

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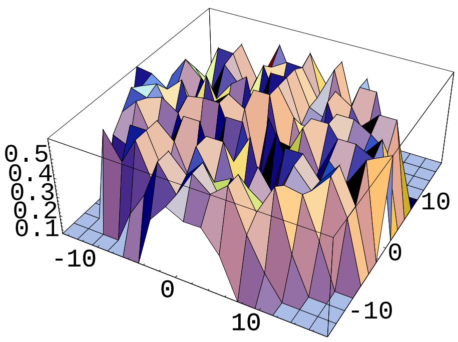
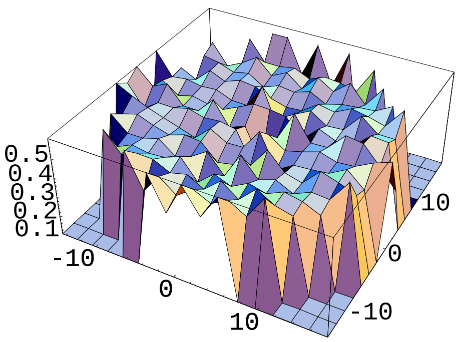
14 sec.</th><th>Genetic (l_C-norm)
37 hours</th></tr>
<tr><td>
</td><td>
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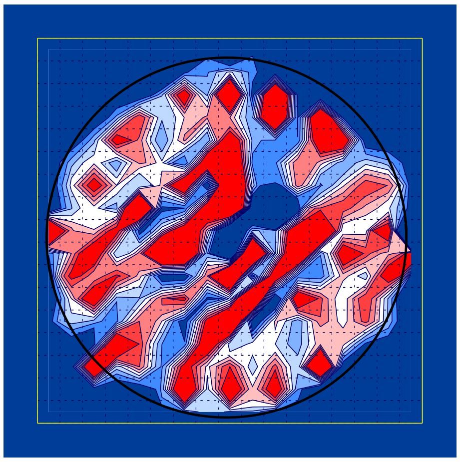
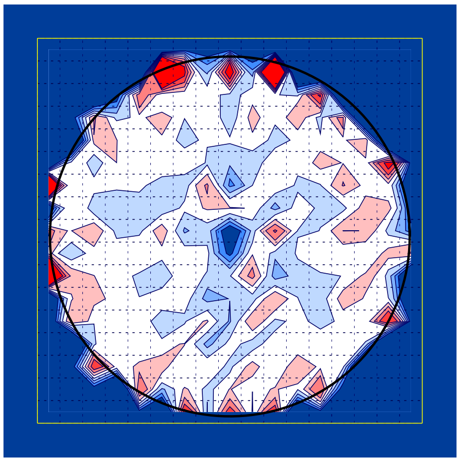
<tr><td></td><td></td></tr>
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