Physics of seimic sources
an overview

Wojciech Dębski

IGF PAS, Warsaw

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Seismological inference

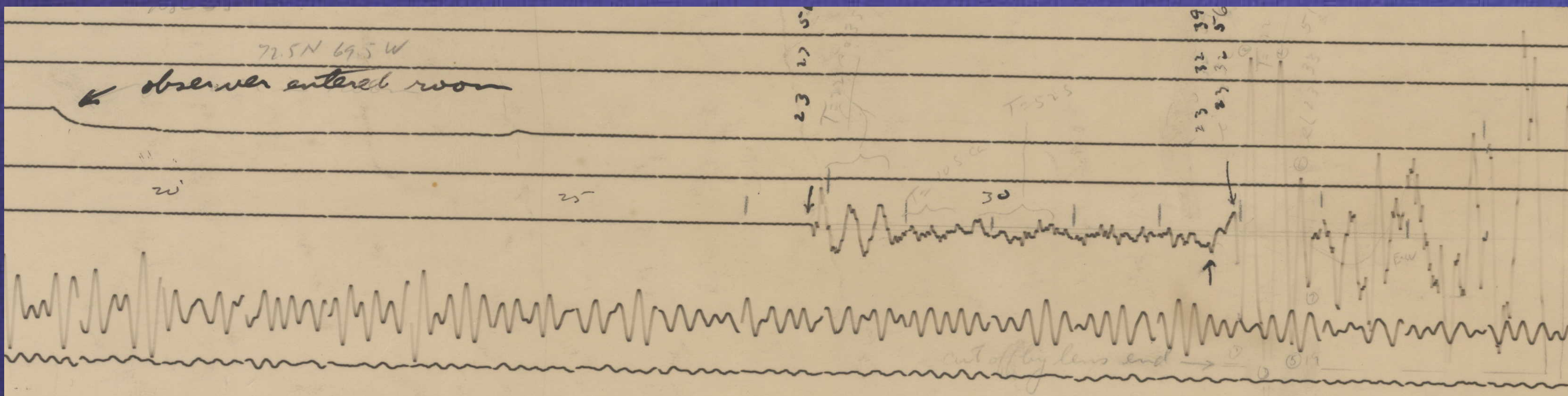
Seismology deals with two broad classes o problems:

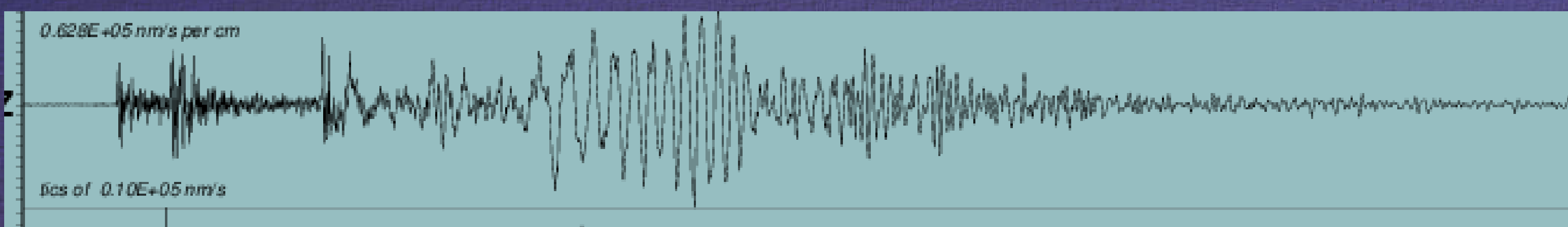
- Structure and composition of the Earth in various scales
  global, regional ($\sim$ 2000 km), local ($\sim$ 100 km), etc.
- Earthquakes: their origins, physics, effects and (possible) prediction

Inference is always based on records made on surface (space)
There is no possibility of methodological direct measurements
(especially seismic sources).

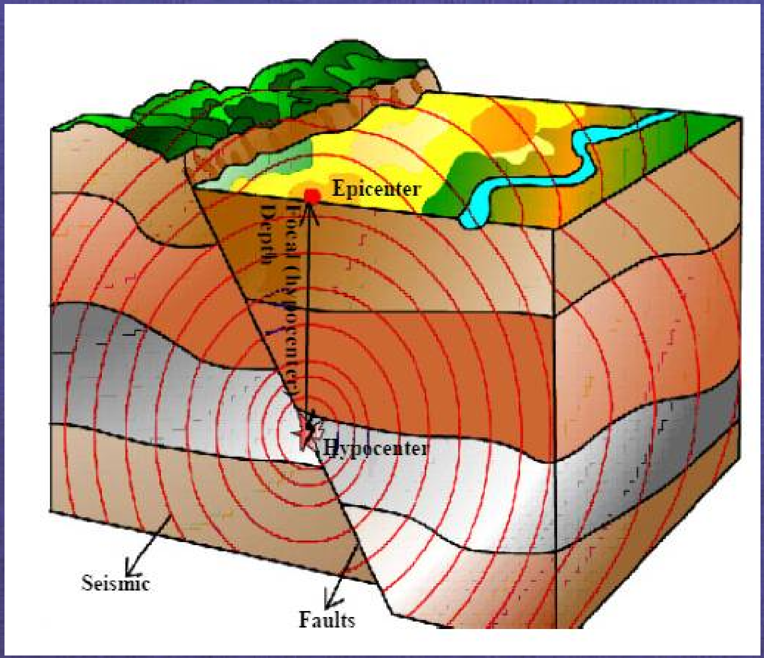
Seismic waves - primary source of information

Seismograms: records of seismic waves



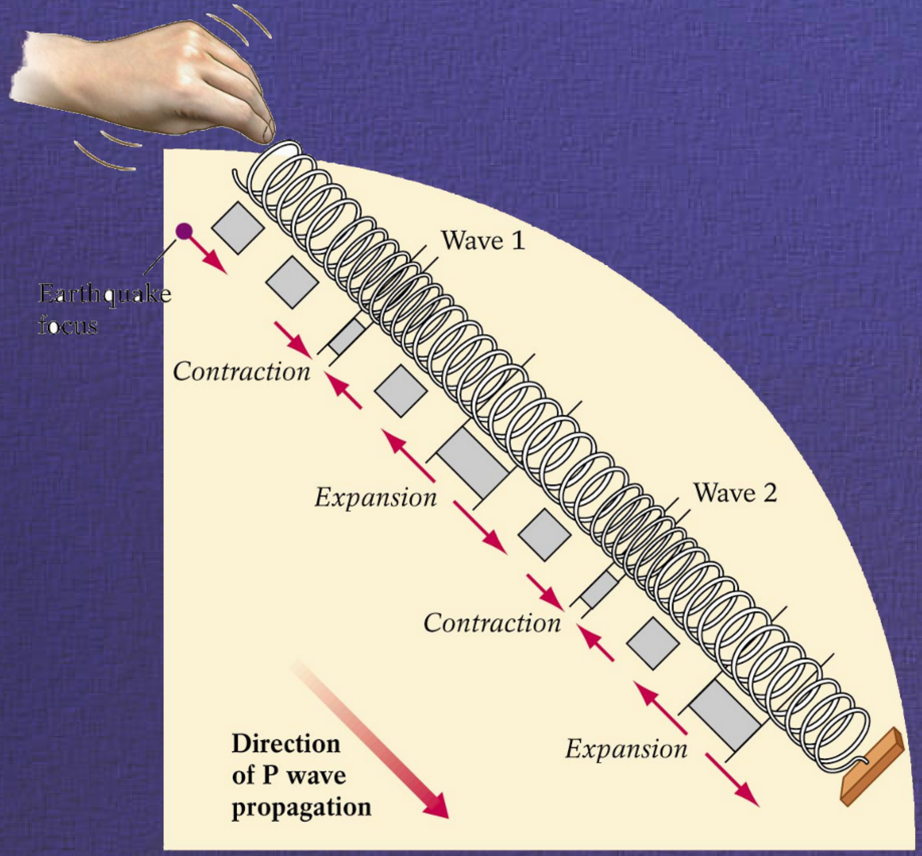


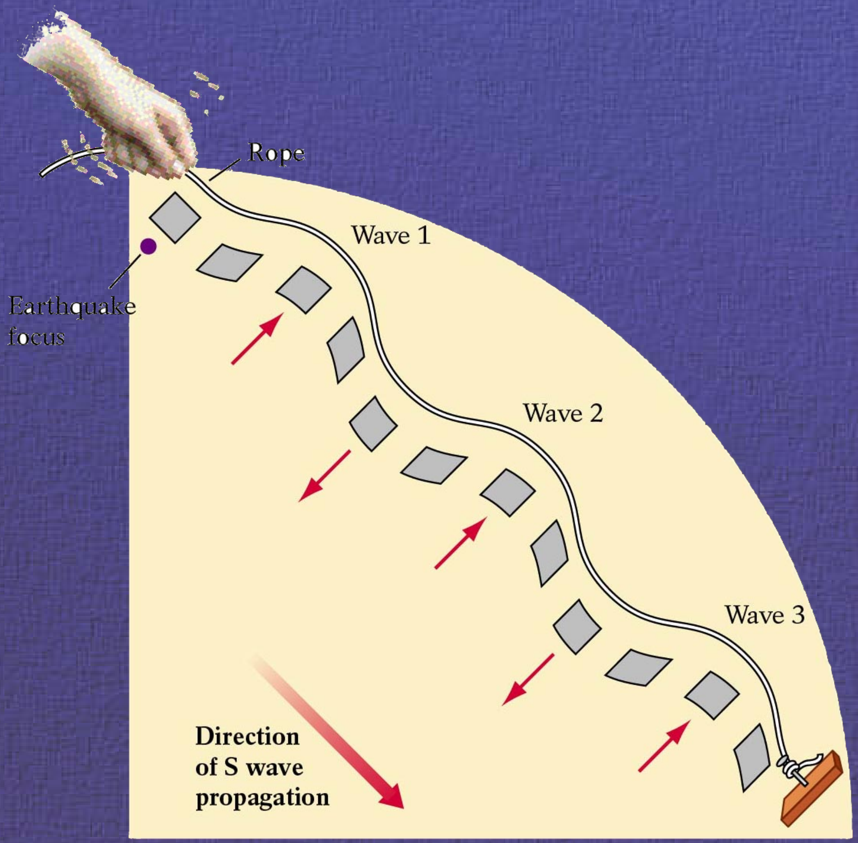
Seismic waves - information contents



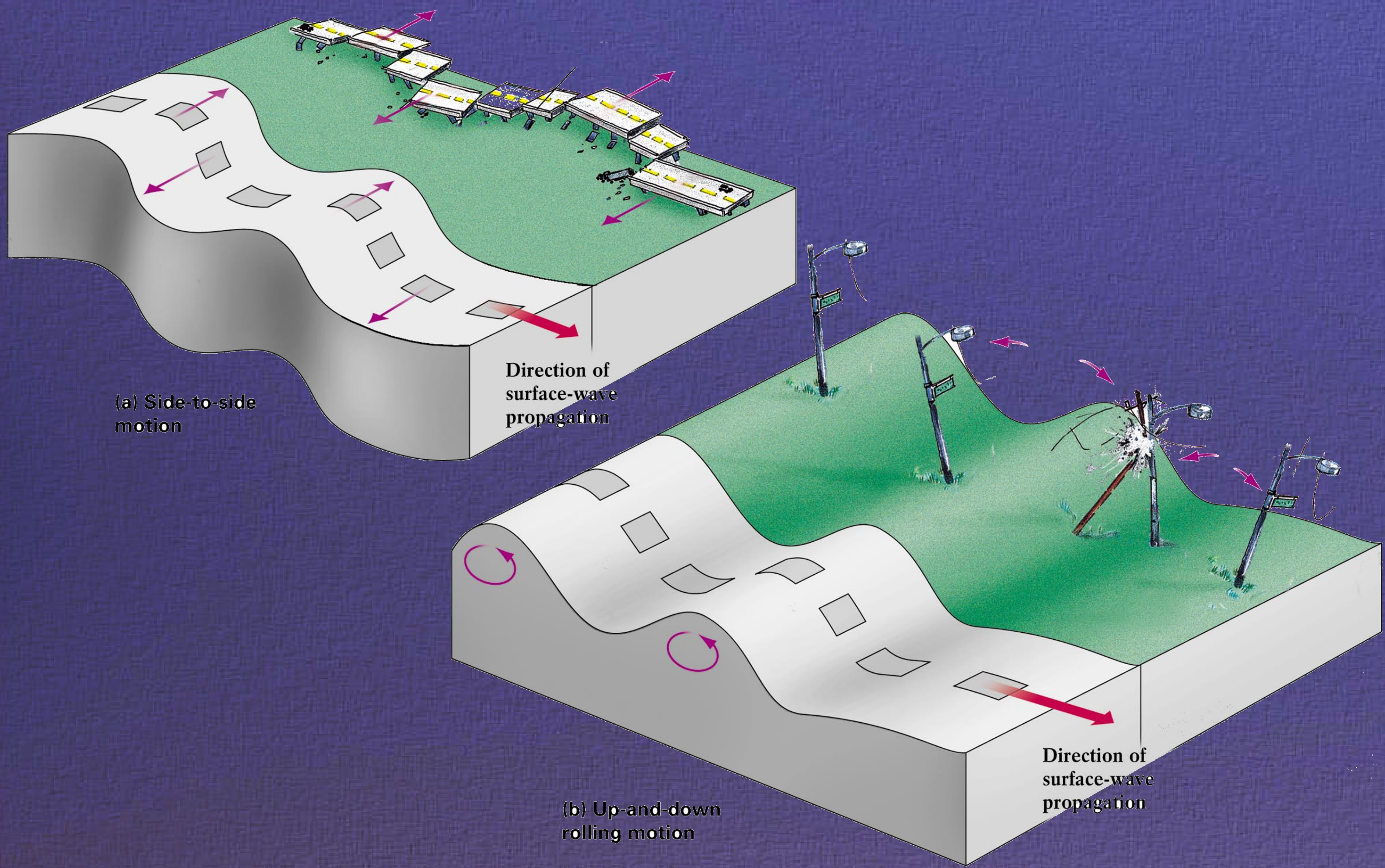
$$\mathbf{u}(\mathbf{r},t) = \int_{R',T'} G(\mathbf{r},\mathbf{r}',t-t')\; S(\mathbf{r}',t')\, \mathrm{d}\mathbf{r}'\mathrm{d}t'$$

Seismic waves - P and S waves

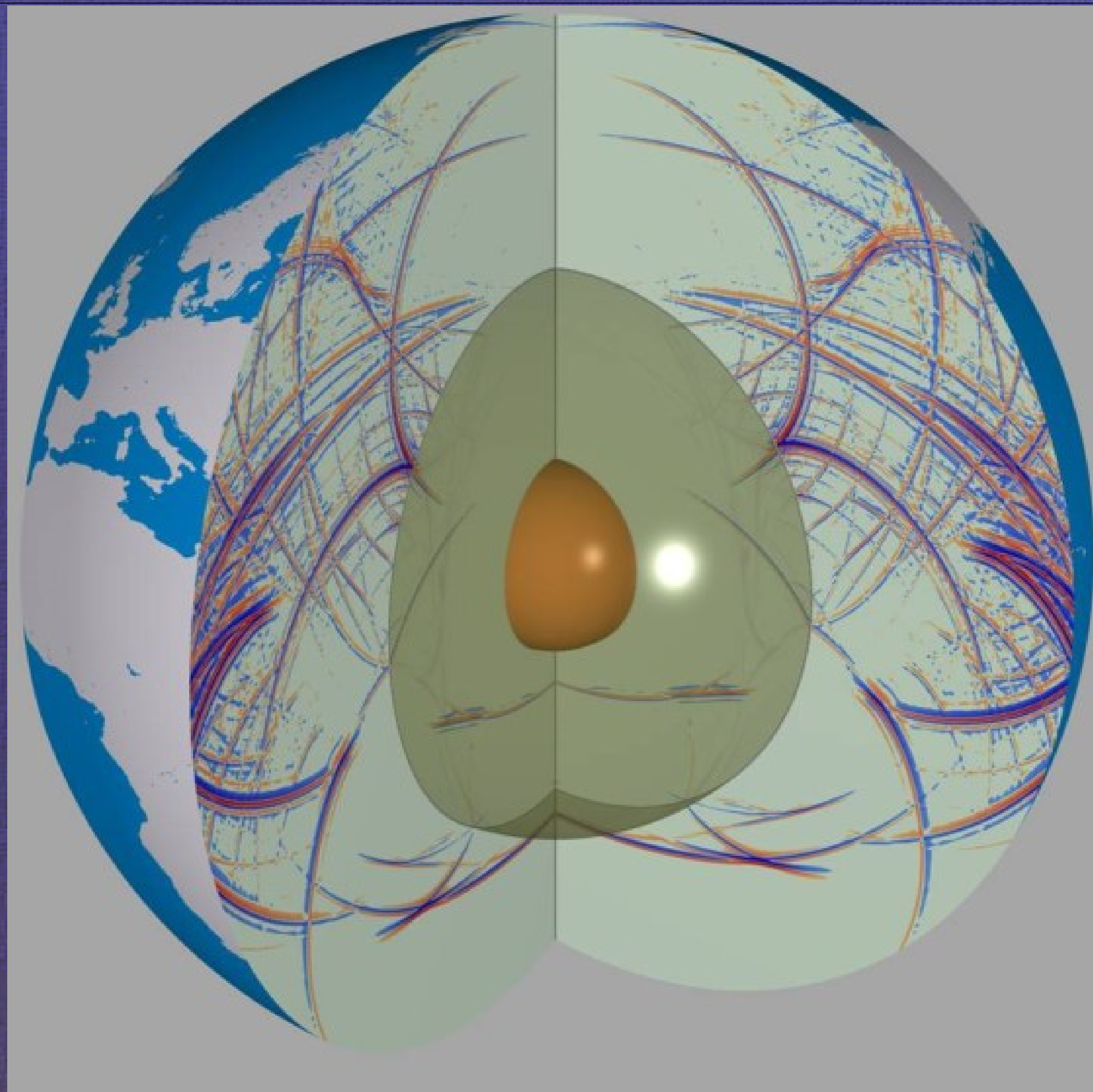




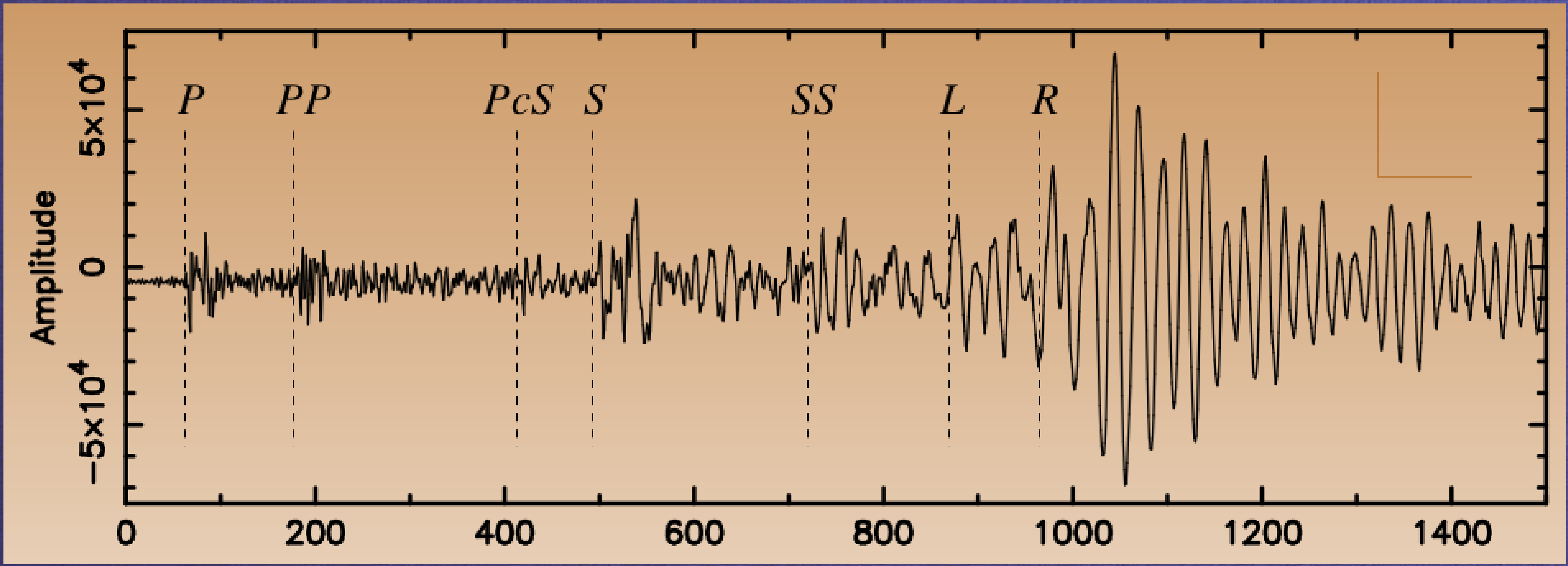
Seismic waves - surface waves



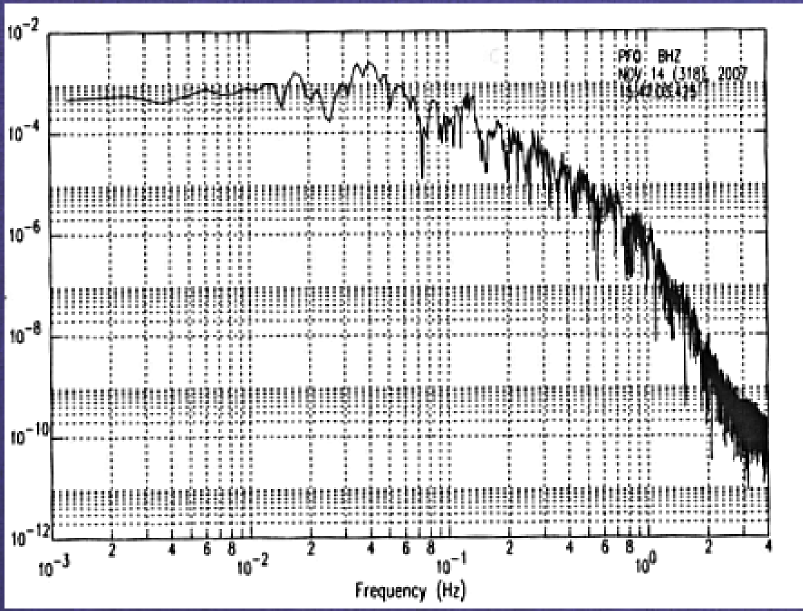
Seismic waves - propagation effects

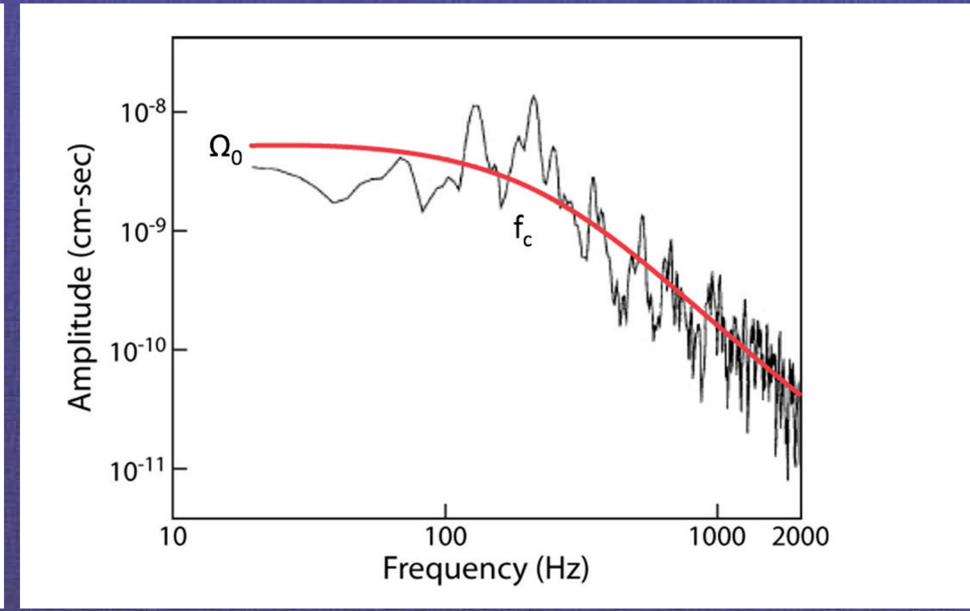


Seismic info - seismic phases

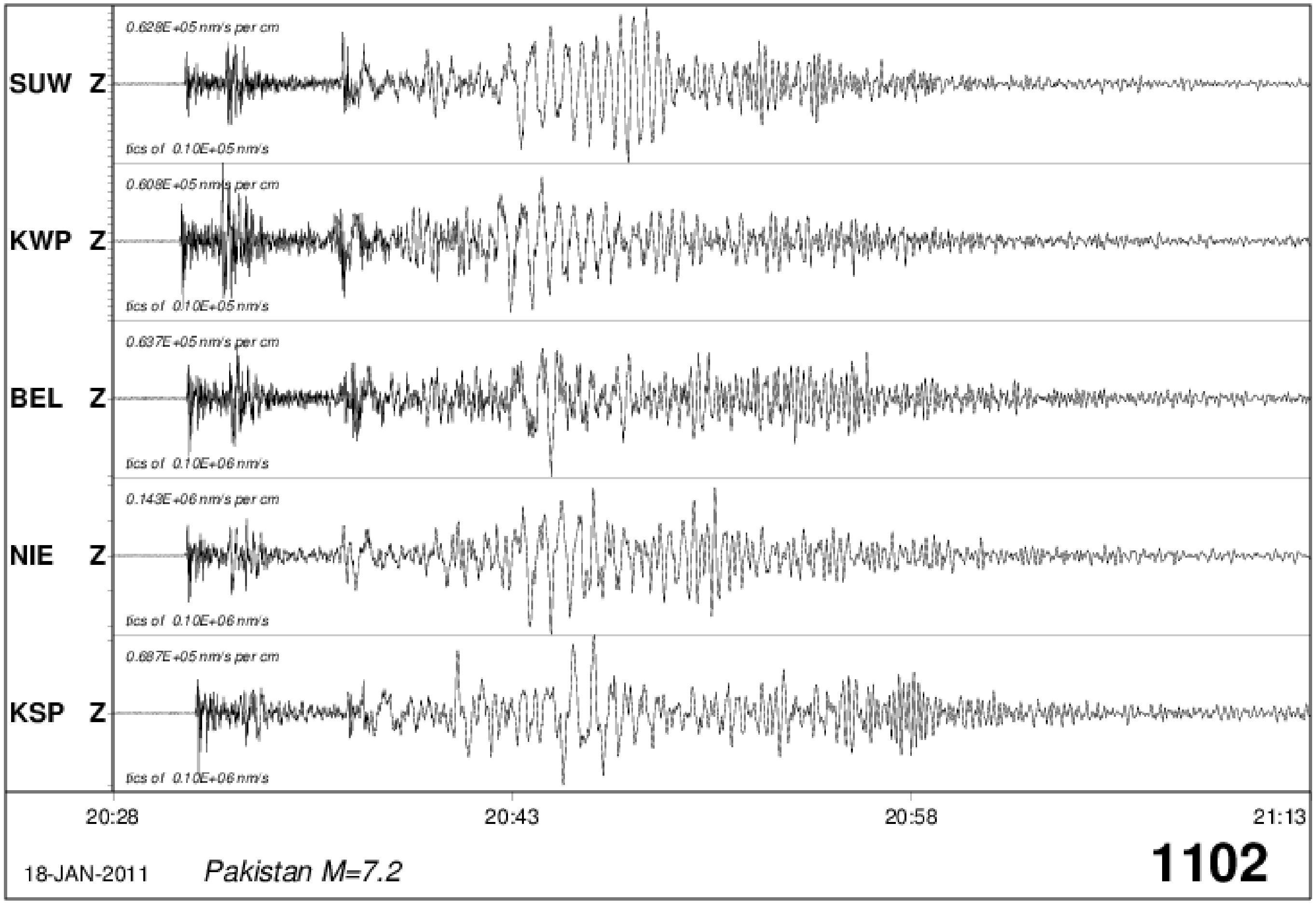


Seismic info - spectrum

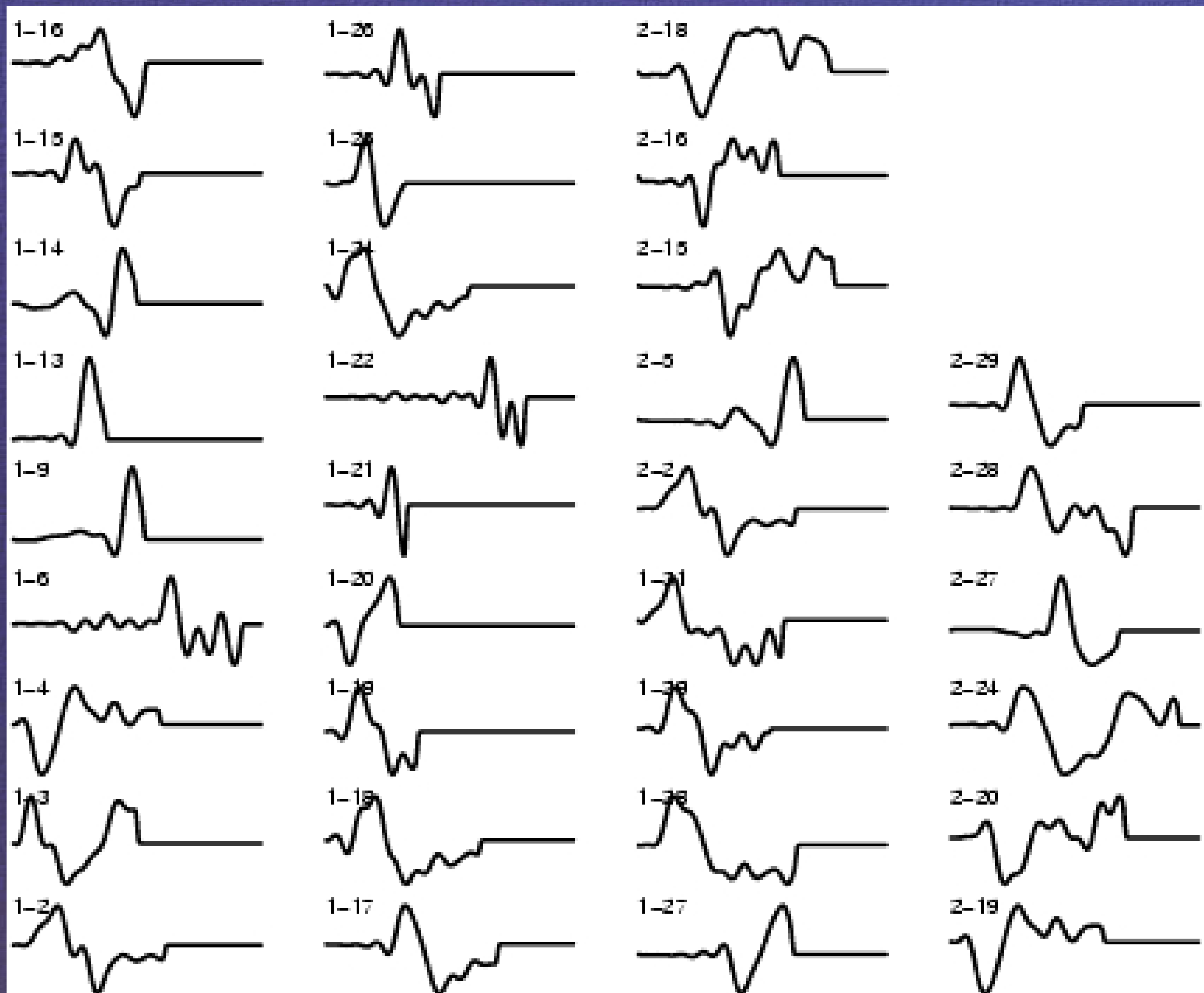




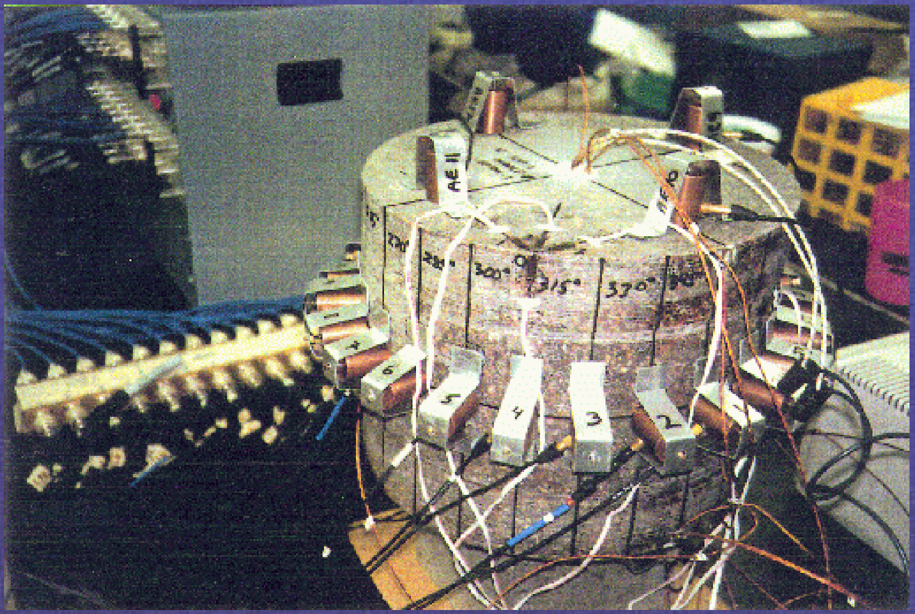
Seismic info - spatial variation (teleseismic event)



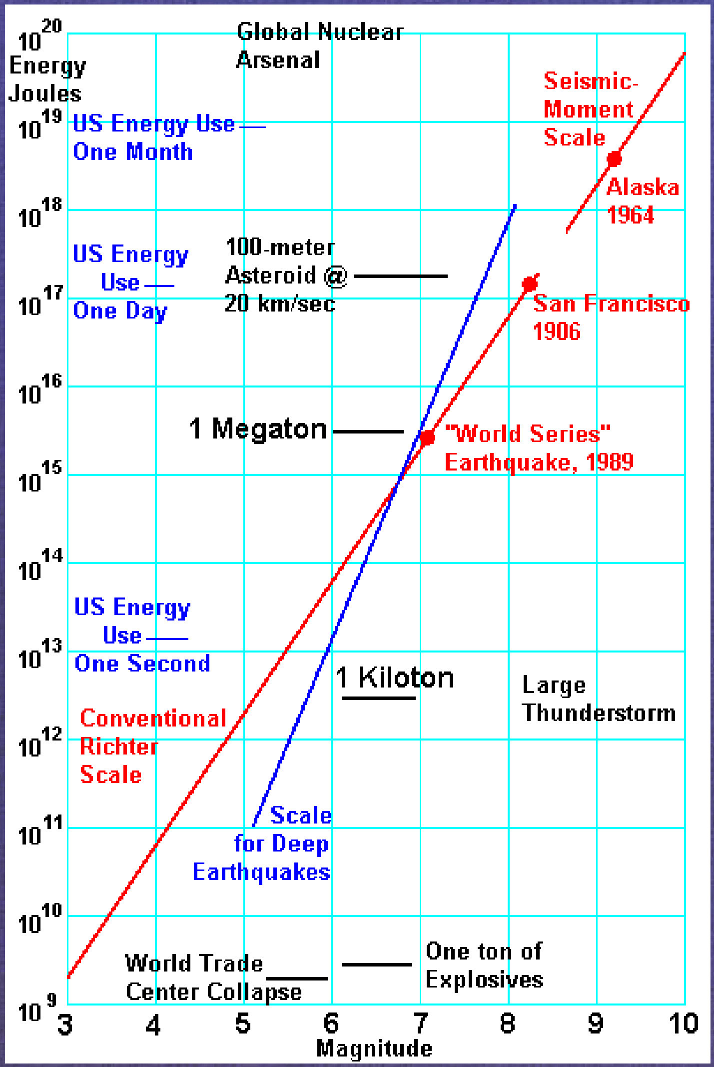
Seismic info - spatial variation (mining event)



From Earth to laboratory scale


From Earth to laboratory scale - energy range



Earthquakes

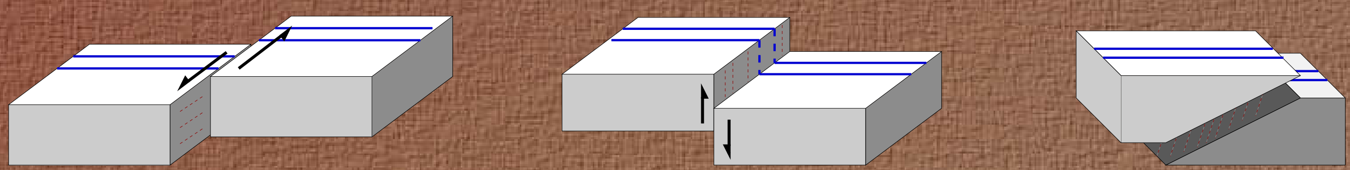
Earthquakes

Earthquakes

Earthquakes

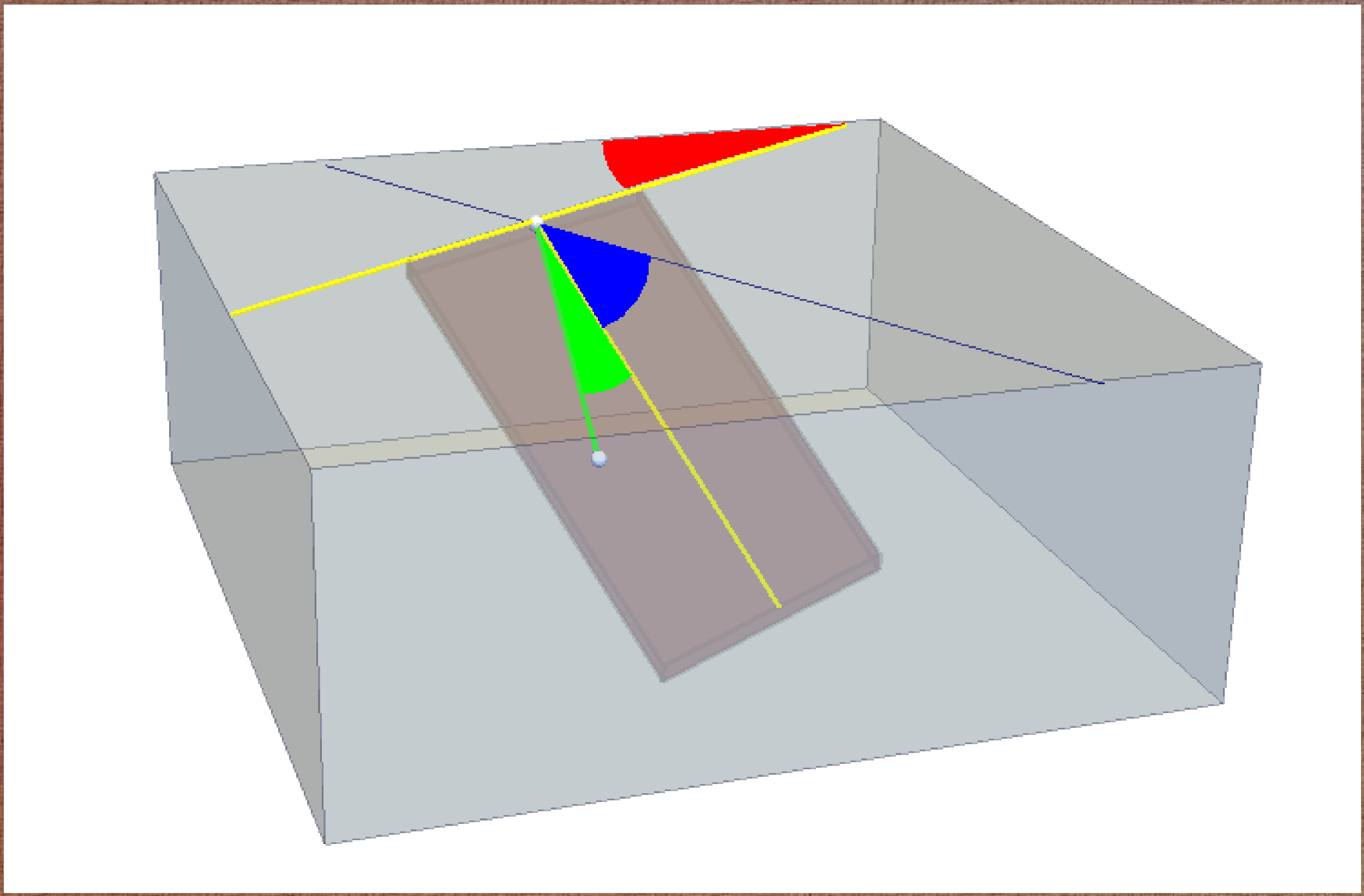
Seismic sources - geometry

Almost all natural, and most of human induced seismic events are shearing type events resulting from a rocks slip along given (fault) plane

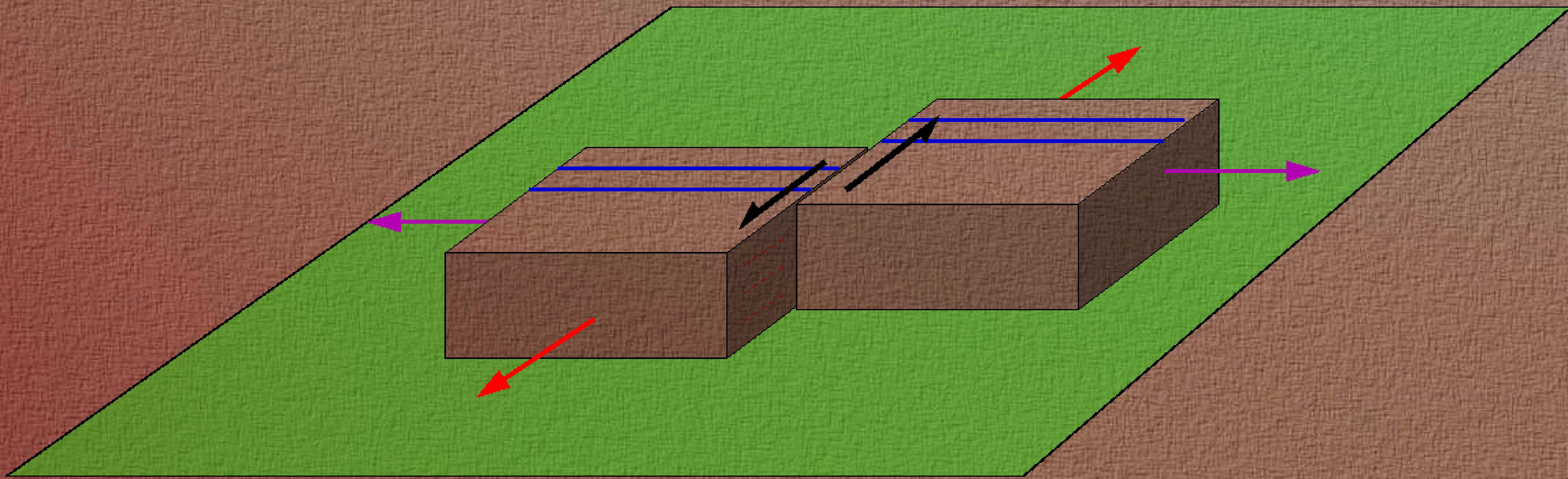


How to describe phenomenologically such sources?

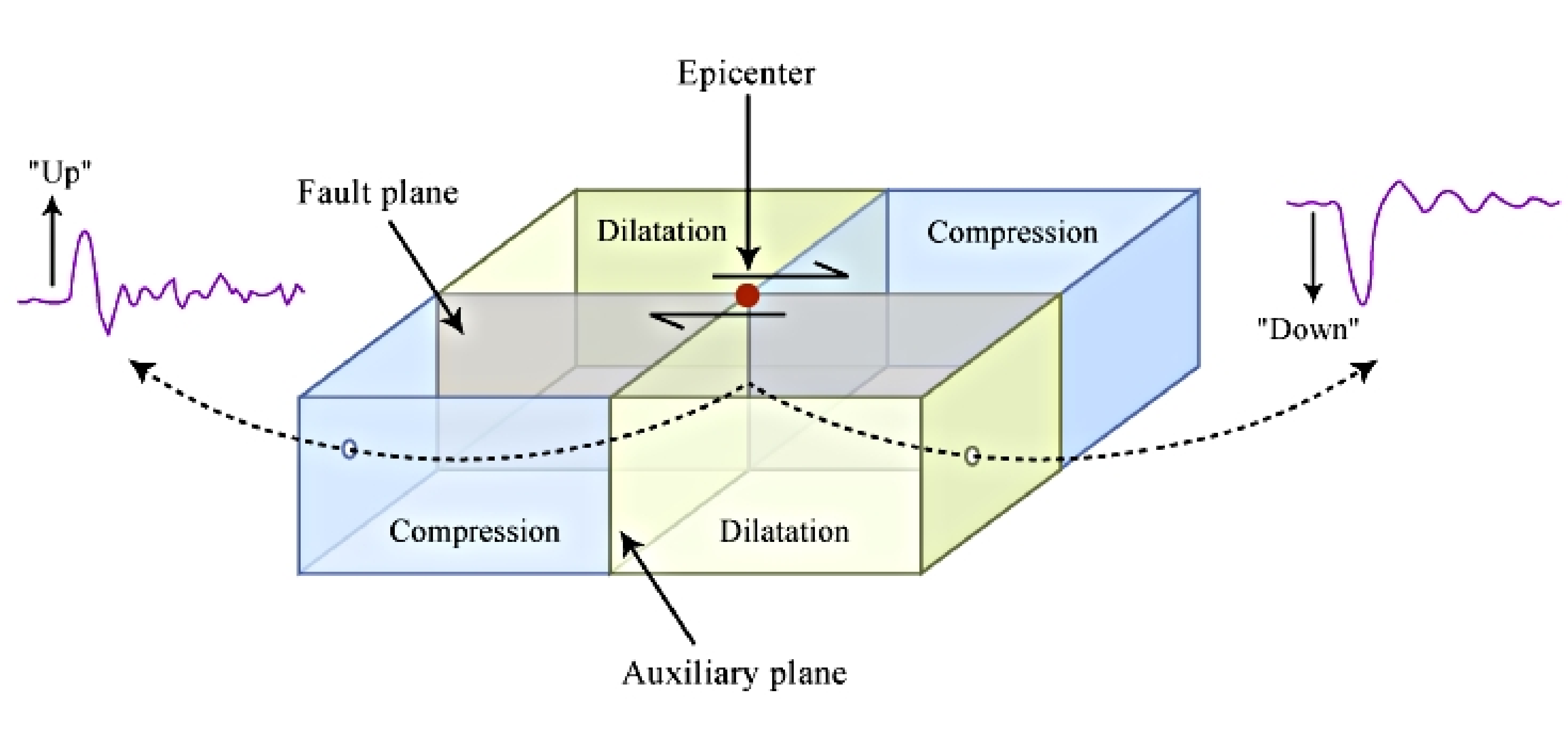
Rapture plane orientation



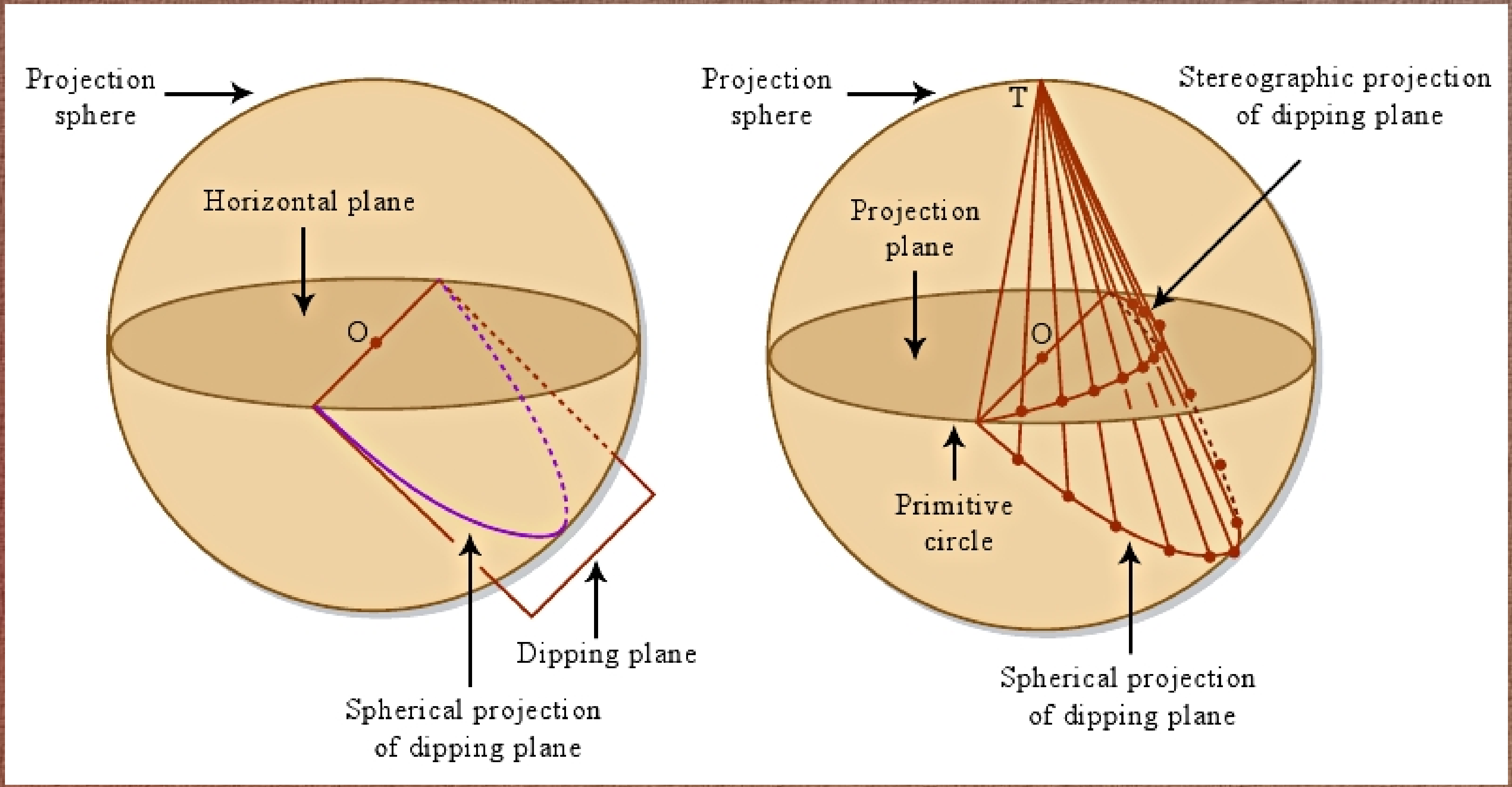
Equivalent forces - double couple



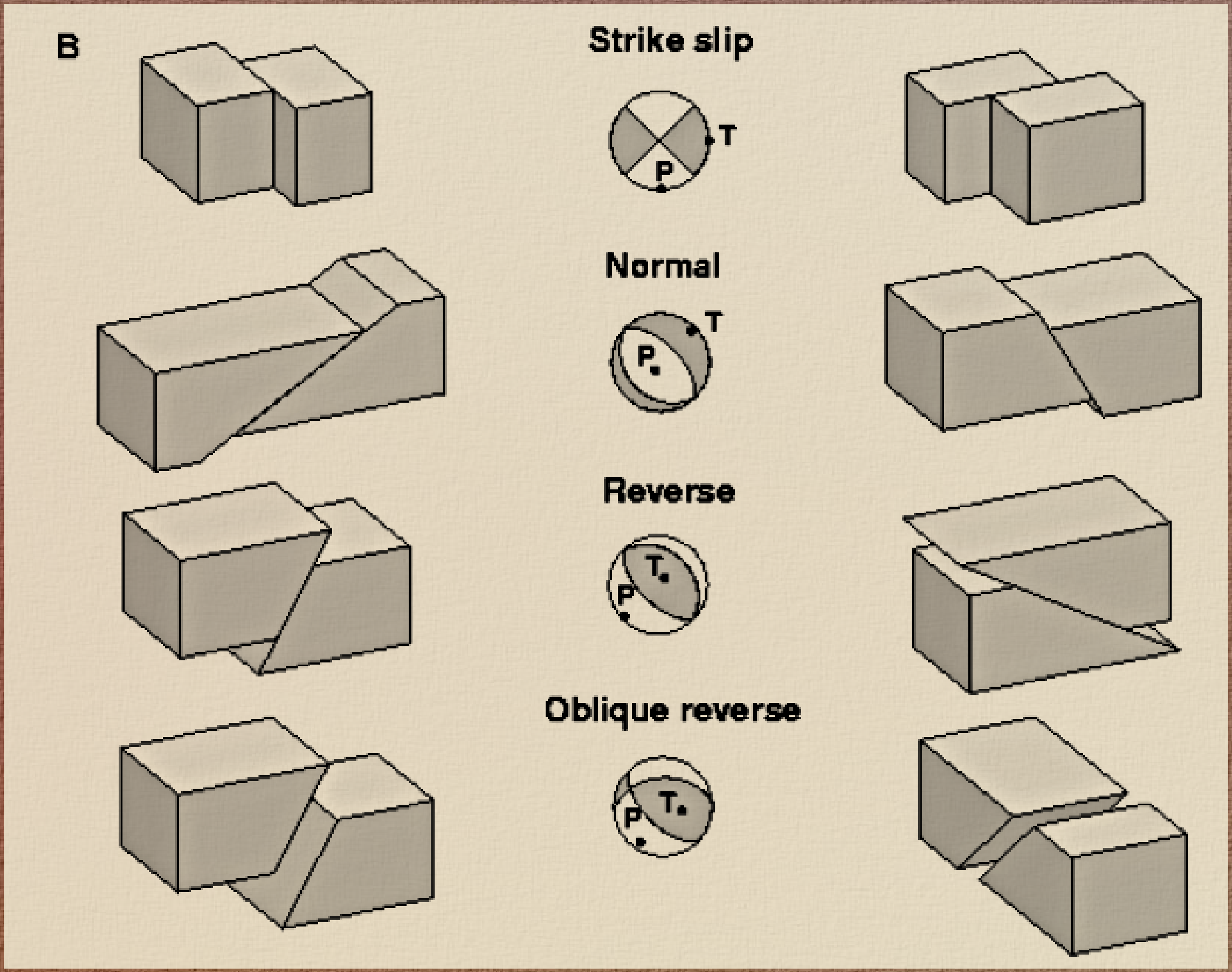
Shearing process - mechanism



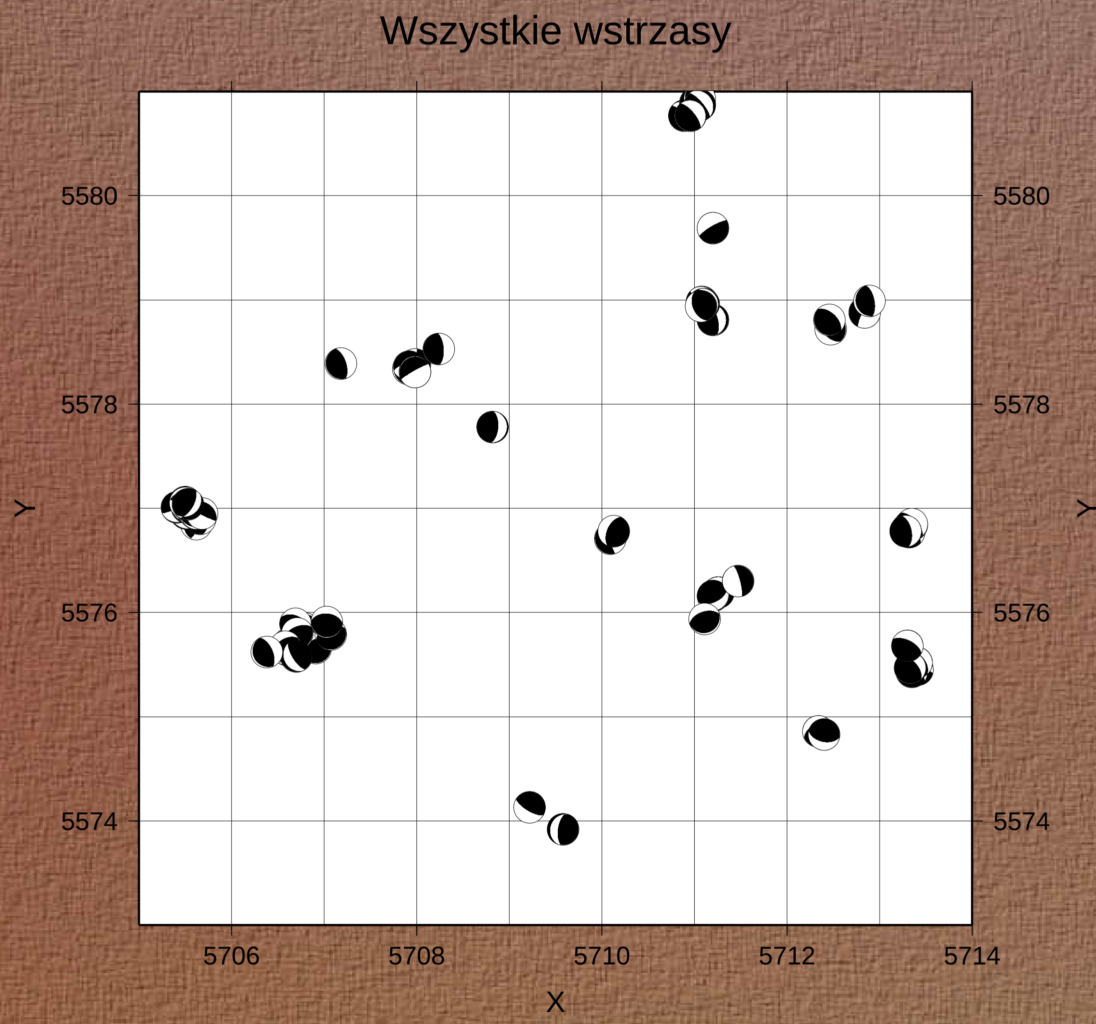
Fault plane solutions



Fault plane solutions - examples



Fault plane solutions - Rudna copper mine

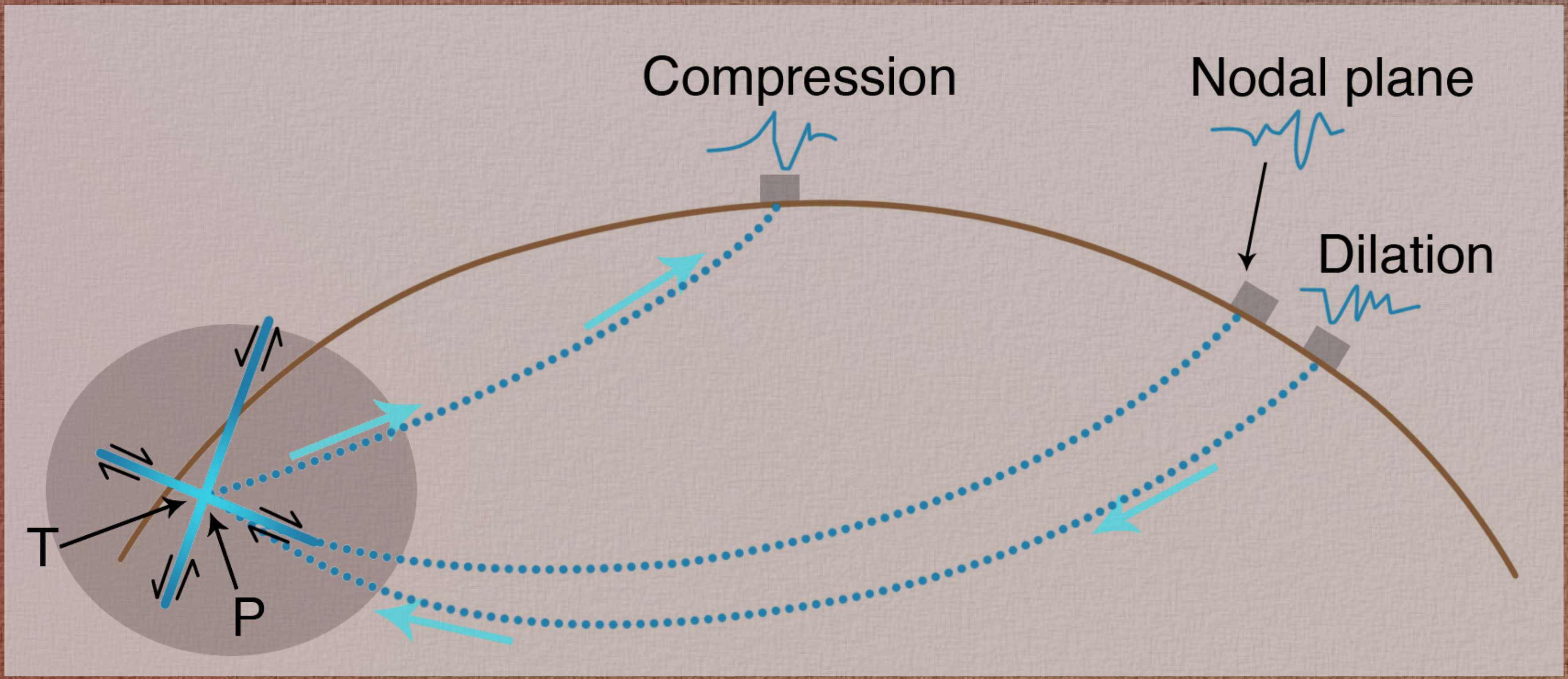


Fault plane solutions - methods

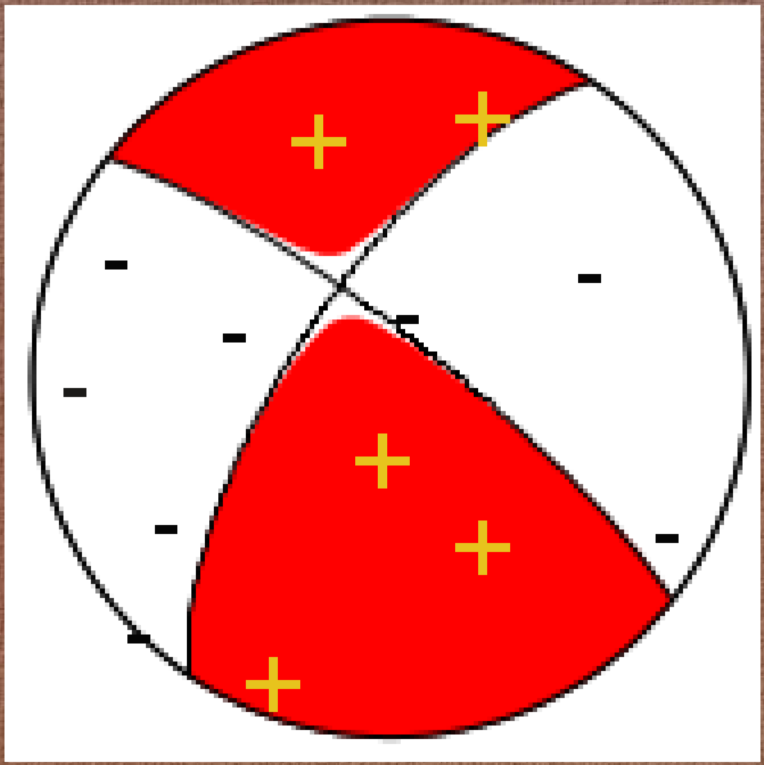
- Polarity of P-waves onsets
- Moment tensor inversion
- Full waveform inversion

- Time reversal ???

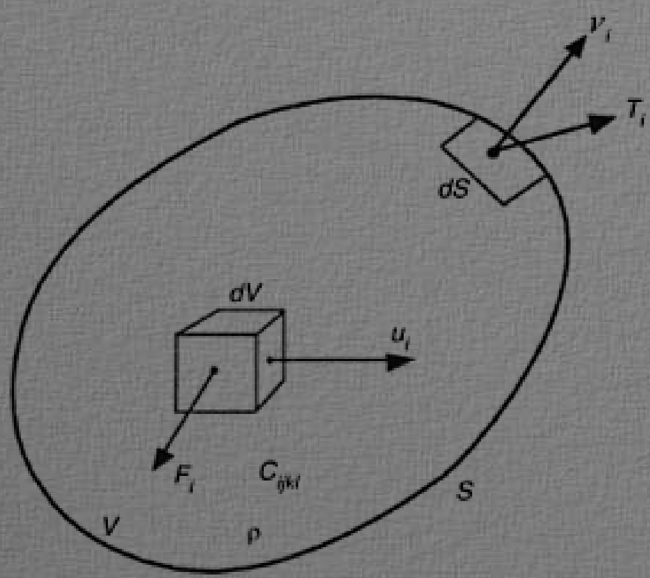
P waves polarity method (1)



P waves polarity method (2)



Mathematics of elastic waves



$$T_i = \tau_{ij} n_j$$

$$\epsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$

$$\frac{d}{dt} \int_V \rho v_i(x,t) \mathrm{d}V = \int_S T_i(x,t) \mathrm{d}S + \int_V F_i(x,t) \mathrm{d}V$$

Mathematics of elastic waves

Using Stock's theorem

$$\int_V \rho \frac{\partial^2 u_i}{\partial t^2} \mathrm{d}V = \int_V \frac{\partial \tau_{ij}}{\partial x_j} \mathrm{d}V + \int_V F_i(x,t) \mathrm{d}V$$

$$\rho \ddot{u}_i = \partial_j \tau_{ij} + F_i$$

Homogeneous isotropic elastic medium

Hook's law: $\tau_{ij} = c^{ijkl}\,\epsilon_{kl}$

Isotropic materials: $c^{ijkl} = \lambda\delta_{ij}\delta_{kl} + 2\mu\delta_{ik}\delta_{jl}$

Homogeneous medium: $\lambda, \mu = \text{const.}$

$$\rho\ddot{\mathbf{u}} = (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} + \mathbf{F}$$

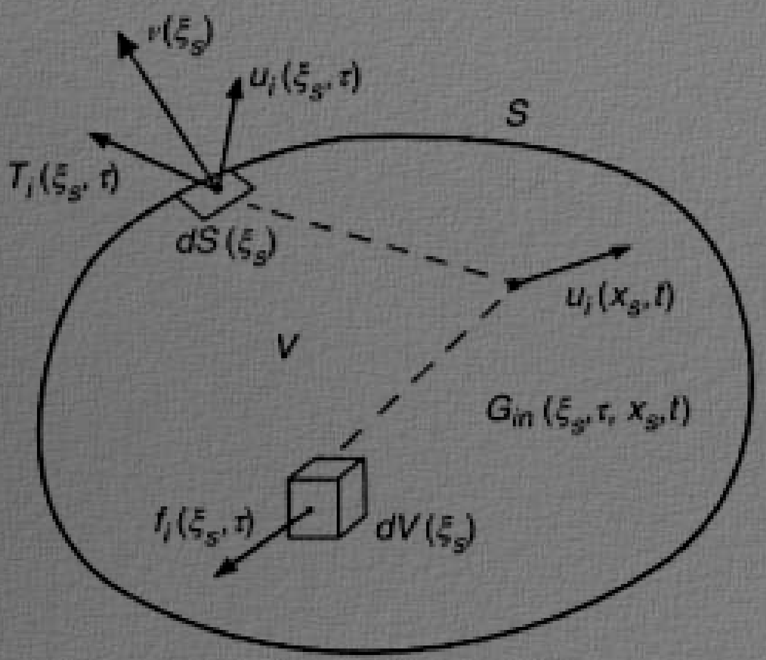
Representation theorem

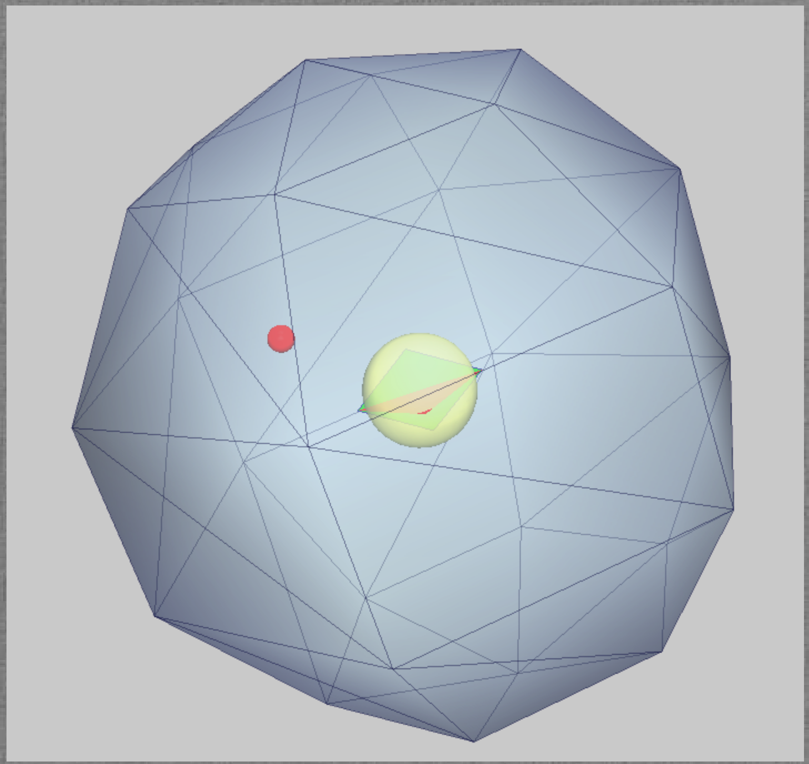
Desired explicite solution for given source term (F_i)

Green's function: $\left(\rho\delta_{ik}\partial_t^2 + \partial_j\tau_{ij}\right) G_{ik} = e_k\delta(\mathbf{r},\mathbf{t})$

$$\begin{aligned} u_n(x_s,t) &= \int_{-\infty}^{\infty} \mathrm{d}t' \int_V f_i(\xi,t') G_{ni}(\xi,t';x_s,t)\mathrm{d}\xi \\ &+ \int_{-\infty}^{\infty} \mathrm{d}t' \int_S G_{ni}(\xi,t';x_s,t) T_i(\xi,t')\mathrm{d}\xi \\ &- \int_{-\infty}^{\infty} \mathrm{d}t' \int_S C_{ijkl}\, u_i(\xi,t') G_{nk,l}(\xi,t';x_s,t) n_j(\xi)\mathrm{d}\xi \end{aligned}$$

Interpretation - forward problem





Given $f_i, T_{|\Sigma}, u_{|\Sigma}$ one can uniquelly determine $u(\mathbf{x})$

Interpretation - inverse problem

Given $u(\mathbf{x}, t)$ can source be determined uniquelly ?

NO!

Three **equivalent** class of source models:

1. f_i - body force (intuitive)
2. $u_{|\Sigma}$ - kinematic (disslocation source)
3. $T_{|\Sigma}$ - dynamic (stress drop, friction, etc.)

Equivalence: Body force model

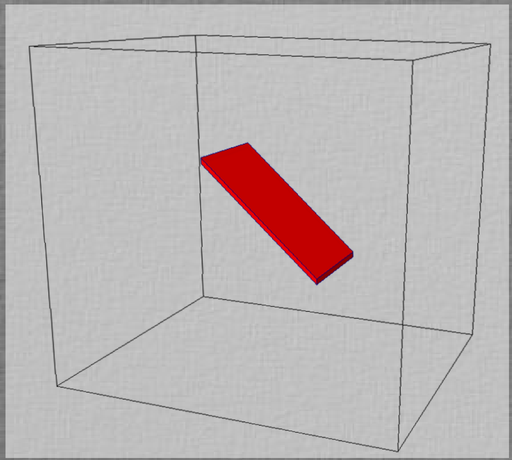
Point source approximation: $L << \lambda$, $L << R$

$$u_n(x_r, t) = \int_{-\infty}^{\infty} f_i(x_s, t') G_{ni}(x_s, t'; x_r, t) \mathrm{d}t'$$

For DC source system (4 elementary sources)

$$u_n(x_r, t) = (e_k e_j + e_j e_k) \int_{-\infty}^{\infty} M(t') \frac{\partial G_{nk}(x_s - x_r, t - t')}{\partial x_j} \mathrm{d}t'$$

Equivalence: Kinematic source model



$$u_n(x_s,t) = \int_{-\infty}^{\infty} \mathrm{d}t' \int_S C_{ijkl}\, \Delta u_i(\xi,t') G_{nk,l}(\xi,t';x_s,t) n_j(\xi) \mathrm{d}\xi$$

Equivalence: Kinematic source model

Assumptions:

- point source, homogeneous medium, non-rotating rupture
- $\mathbf{e}^s$ - slip unit vector ($\Delta\mathbf{u} = \mathbf{e}^s \Delta u$)
- $\mathbf{e}^n$ - vector normal to rupture plane
- $\mathbf{e}^n \cdot \mathbf{e}^s = 0$ - pure shearing

$$u_n(x_s, t) = \int_{-\infty}^{\infty} S\Delta u \left(\lambda e_l^s e_l^n \delta_{ij} + \mu(e_i^s e_j^n + e_j^s e_i^n)\right) G_{ni,j}(x_s, t'; x_r, t) \mathrm{d}t'$$

Equivalence:

Kinematic model:

$$u_n(x_s,t) = \int_{-\infty}^{\infty} (e^s_k e^n_j + e^s_j e^n_k) \underbrace{\mu S \Delta u(t')}_{M_o(t')} G_{nk,j}(x_s,t';x_r,t)\mathrm{d}t'$$

Body force model:

$$u_n(x_r,t) = (e_k e_j + e_j e_k) \int_{-\infty}^{\infty} M(t') G_{nk,j}(x_s,t';x_r,t)\mathrm{d}t'$$

Doble couple $\equiv$ shear dislocation source

Far field solution - homogeneous medium

Moment tensor

$$M_{ij}(t) \; = \; (e^s_k e^n_j + e^s_j e^n_k)\mu S \Delta u(t) = (e^s_k e^n_j + e^s_j e^n_k) M_o(t)$$

Source time function

$$M_{ij}(t) \; = \; m_{ij} \, M_o S(t)$$

Solution

$$u_i(x_s, t) = \frac{M_o}{4\pi\rho v_p^3 r} \gamma_i \gamma_j \gamma_k m_{jk} \dot{S}(t - r/v_p)$$

Moment tensor - generalisation

Three equivalent class of source models:

1. f_i - body force (intuitive)
2. $u_{|\Sigma}$ - kinematic (disslocation source)
3. $T_{|\Sigma}$ - dynamic (stress drop, friction, etc.)

Elastic wave equation

$$\rho \ddot{u}_i = \partial_j \tau_{ij} + F_i$$

Moment tensor - generalisation

Asumptions:

- source - a non-elastic process in a finite volume
- net sum of forces and moments in source is null
- absence of body forces ($F_i = 0$)

Total stress σ_{ij}

$$\sigma_{ij} = \tau_{ij} + m_{ij}$$

Moment tensor - generalisation

Wave equation

$$\rho \ddot{u}_i \;=\; \partial_j \, \tau_{ij}$$

Generalise (postulate)

$$\rho \ddot{u}_i \;=\; \partial_j \, \sigma_{ij}$$

$$\rho \ddot{u}_i \;=\; \partial_j \, \tau_{ij} - \underbrace{\partial_j \, m_{ij}}_{F_i}$$

Moment tensor represents the stresses responsible for non-elastic processes in the source volume

Moment tensor - generalisation

Moment tensor m_{ij} represents the general point-like source

$$u_i \;=\; \int\limits_{-\infty}^{\infty} \mathrm{d}t' \int\limits_{V_o} F_i(\xi, t') G_{ni}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

Taylor expansion:

$$G_{ik}(\xi_n) \;=\; G_{ik}(0) + \xi_j \frac{\partial G_{ik}}{\xi_j} + O(\xi^2)$$

Moment tensor - generalisation

$$u_i = \int_{-\infty}^{\infty} \mathrm{d}t' \int_{V_o} \xi_j F_k G_{ik,j}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

null net forces:

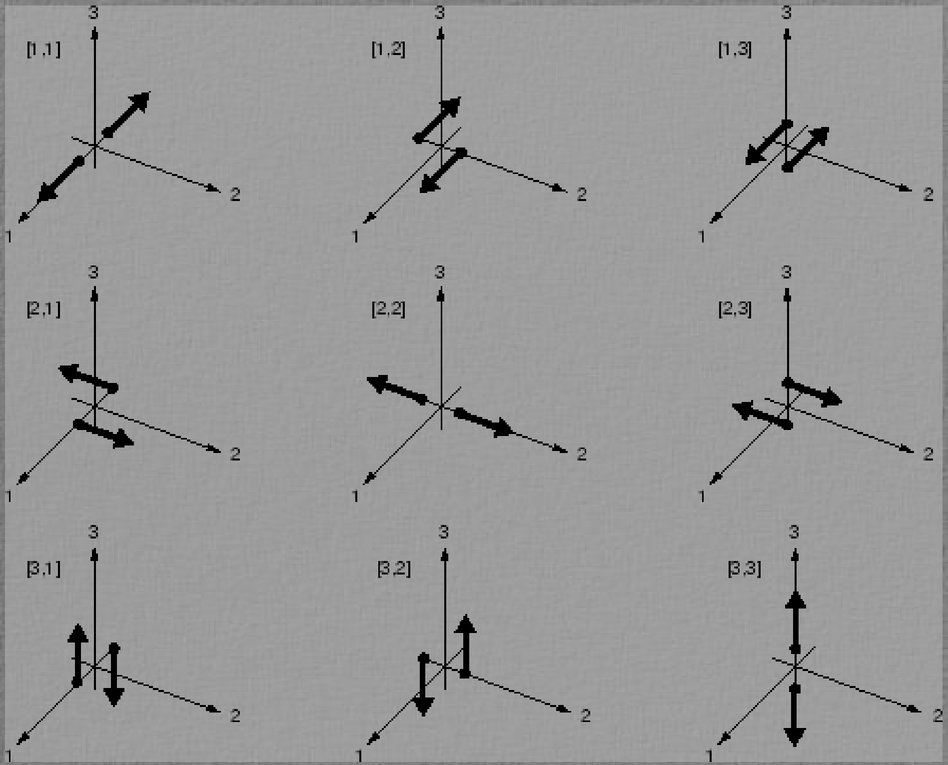
$$\int_{V_o} F_k(0) G_{ik}(0, t'; x_s, t) \mathrm{d}V_\xi = 0$$

$$u_i = \int_{-\infty}^{\infty} \mathrm{d}t' \int_{V_o} \xi_j F_k G_{ik,j}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

$$m_{ij} = \xi_i F_j$$

Moment tensor - physical interpretation

$$M_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$



Moment tensor - principle stresses

m_{ij} real, symmetric - can be diagonalized

$$(m_{ij} - \delta_{ij}\sigma) \cdot \vec{\mathbf{v}}_j = 0$$

$\vec{\mathbf{v}}_i$ - principle axes of inelastic stress in source volume

$$m_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \qquad \sigma_1 > \sigma_2 > \sigma_3$$

Moment tensor - isotropic part

Isotropic part - inelastic dilatation - change of source volume

$$m_{ij}^I = \frac{1}{3} Tr(m_{ij}) \delta_{ij}$$

$$m_{ij}^I = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Deviatoric part - processes with no volume change, e.g. shearing

$$m_{ij}^D \; = \; m_{ij} - m_{ij}^I$$

Moment tensor - deviatoric part

$$m_{ij}^{D} = m_{ij}^{DC} + m_{ij}^{CL}$$

where

$$\det(m_{ij}^{DC}) = 0$$

Double Couple part (pure shearing)

$$m_{ij}^{CL} = m_{ij}^{D} - m_{ij}^{CL}$$

Compensated Linear Vector Dipol (e.g. tensil crack)

$$\mathbf{m} = \mathbf{m}^{ISO} + \mathbf{m}^{DC} + \mathbf{m}^{CL}$$

DC - CLVD partitionig

$$\mathbf{m}^{D} \quad = \quad \mathbf{m}^{DC} \quad + \quad \mathbf{m}^{CL}$$

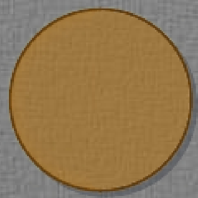
$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\sigma_1 - \sigma_3) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(\sigma_1 - \sigma_3) \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}\sigma_2 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & -\frac{1}{2}\sigma_2 \end{pmatrix}$$

- DC - shearing process
- CLVD - change in μ perpendicular to rupture plane (crack opening, etc.)

Moment tensor - examples

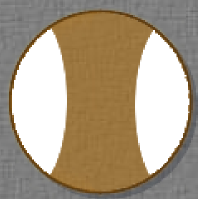
Explosive (ISO)

$$m_{ij} = (\lambda + \frac{2}{3}\mu)\Delta u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



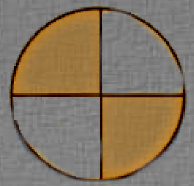
Tensil fracture (CLVD)

$$m_{ij} = A(\lambda, \mu)\Delta u \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Shear fracture (DC)

$$m_{ij} = \mu\Delta u \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Moment tensor inversion

$$m_{ij}(t) \;=\; (e^s_k e^n_j + e^s_j e^n_k) M_o S(t)$$

Moment tensor inversion should comprise

- Nm=6 *"Fault plane solution"* $e^s_k e^n_j + e^s_j e^n_k$
- Nm=1 Seismic scalar moment $M_o = \mu S \Delta u_o$
- Nm=(∞) Source time function $S(t) = \Delta \dot{u} / u_o$

$$u_n(x_r, t) = \int_{-\infty}^{\infty} m_{ij}(t')\, G_{ni,j}(x_s, t'; x_r, t) \mathrm{d}t'$$

Moment tensor inversion software

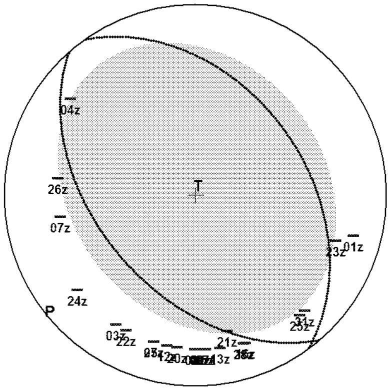
VEBSN Repository: http://www.orfeus-eu.org/

- Foci
- FOCMEC
- ISOLA-GUI
- Seismic Analysis Code (SAC)

Time Reversal Moment Tensor inversion (TRMT)
(under development)

$$u_n(x,t) = \int_{-\infty}^{\infty} M_{kl}(\xi,t')\, G_{nk,l}(\xi,t';x_s,t)\mathrm{d}\mathrm{d}t'$$

Moment tensor inversion - examples



Full solution

M_{11} = **-1,12E+13** [Nm] M_{12} = **-7,81E+12** [Nm] M_{13} = **+4,04E+12** [Nm]
M_{21} = **-7,81E+12** [Nm] M_{22} = **-1,50E+13** [Nm] M_{23} = **+1,37E+12** [Nm]
M_{31} = **+4,04E+12** [Nm] M_{32} = **+1,37E+12** [Nm] M_{33} = **+4,54E+13** [Nm]

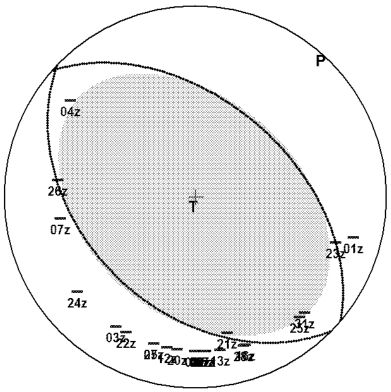
Rupture time: 50,7[ms]
Seismic moment: 3,59E+13[Nm]
Total seismic moment: 4,06E+13[Nm]
Error: 6,77E+11[Nm]
Seismic moment magnitude: 3,0

Decomposition:
EXPL: 18,8 CLVD: 33,9 DBCP: 47,4

Nodal planes:
1: 139,0/48,1 2: 324,2/42,0

Axes (trend/plunge):
P: 231/3 T: 11/86 B: 141/3

Quality factor: 68
Classification: Reverse fault



Trace-null solution

M_{11} = **-1,41E+13** [Nm] M_{12} = **-1,00E+13** [Nm] M_{13} = **-3,05E+12** [Nm]
M_{21} = **-1,00E+13** [Nm] M_{22} = **-1,24E+13** [Nm] M_{23} = **-1,63E+12** [Nm]
M_{31} = **-3,05E+12** [Nm] M_{32} = **-1,63E+12** [Nm] M_{33} = **+2,65E+13** [Nm]

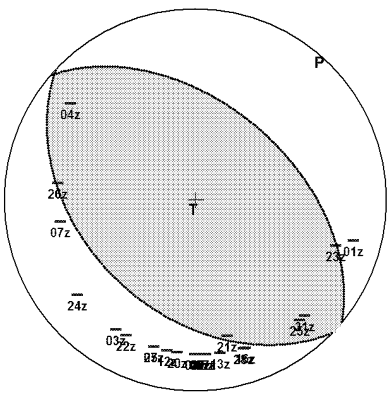
Rupture time: 50,7[ms]
Seismic moment: 2,35E+13[Nm]
Total seismic moment: 2,53E+13[Nm]
Error: 7,07E+11[Nm]
Seismic moment magnitude: 2,9

Decomposition:
CLVD: 13,5 DBCP: 86,5

Nodal planes:
1: 310,9/48,9 2: 134,2/41,2

Axes (trend/plunge):
P: 42/4 T: 199/86 B: 312/2

Quality factor: 72
Classification: Reverse fault



Double couple solution

M_{11} = **-1,29E+13** [Nm] M_{12} = **-1,16E+13** [Nm] M_{13} = **-2,78E+12** [Nm]
M_{21} = **-1,16E+13** [Nm] M_{22} = **-1,04E+13** [Nm] M_{23} = **-1,59E+12** [Nm]
M_{31} = **-2,78E+12** [Nm] M_{32} = **-1,59E+12** [Nm] M_{33} = **+2,33E+13** [Nm]

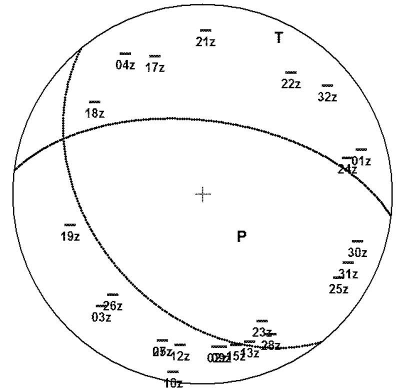
Rupture time: 50,7[ms]
Seismic moment: 2,35E+13[Nm]
Total seismic moment: 2,35E+13[Nm]
Error: 7,50E+11[Nm]
Seismic moment magnitude: 2,9

Decomposition:
DBCP: 100,0

Nodal planes:
1: 310,4/48,9 2: 133,7/41,2

Axes (trend/plunge):
P: 42/4 T: 199/86 B: 312/2

Quality factor: 54
Classification: Reverse fault



Full solution

M_{11} = **-1,30E+13** [Nm] M_{12} = **+1,08E+13** [Nm] M_{13} = **+1,95E+13** [Nm]
M_{21} = **+1,08E+13** [Nm] M_{22} = **-2,07E+13** [Nm] M_{23} = **-1,53E+13** [Nm]
M_{31} = **+1,95E+13** [Nm] M_{32} = **-1,53E+13** [Nm] M_{33} = **-6,98E+12** [Nm]

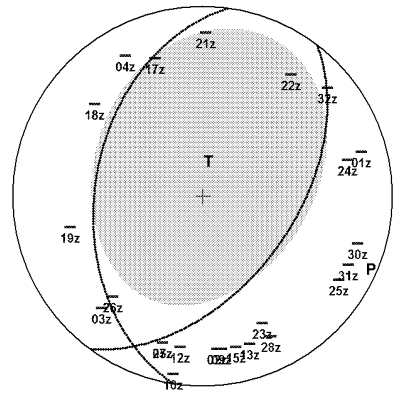
Rupture time: 39,7[ms]
Seismic moment: 5,89E+12[Nm]
Total seismic moment: 7,70E+12[Nm]
Error: 5,14E+11[Nm]
Seismic moment magnitude: 2,5

Decomposition:
EXPL: -30,3 CLVD: -14,4 DBCP: 55,3

Nodal planes:
1: 276,9/58,2 2: 140,7/40,6

Axes (trend/plunge):
P: 137/65 T: 26/9 B: 292/23

Quality factor: 65
Classification: Normal fault



Trace-null solution

M_{11} = **-1,55E+12** [Nm] M_{12} = **+1,52E+12** [Nm] M_{13} = **+2,21E+12** [Nm]
M_{21} = **+1,52E+12** [Nm] M_{22} = **-4,81E+12** [Nm] M_{23} = **+3,07E+10** [Nm]
M_{31} = **+2,21E+12** [Nm] M_{32} = **+3,07E+10** [Nm] M_{33} = **+6,36E+12** [Nm]

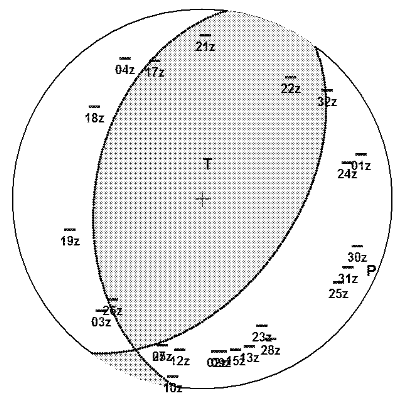
Rupture time: 39,7[ms]
Seismic moment: 5,46E+12[Nm]
Total seismic moment: 6,34E+12[Nm]
Error: 6,09E+11[Nm]
Seismic moment magnitude: 2,5

Decomposition:
CLVD: 27,2 DBCP: 72,8

Nodal planes:
1: 36,7/50,8 2: 188,3/42,8

Axes (trend/plunge):
P: 113/4 T: 8/75 B: 204/14

Quality factor: 67
Classification: Reverse fault



Double couple solution

M_{11} = **-4,55E+11** [Nm] M_{12} = **+1,99E+12** [Nm] M_{13} = **+1,52E+12** [Nm]
M_{21} = **+1,99E+12** [Nm] M_{22} = **-4,61E+12** [Nm] M_{23} = **-1,85E+11** [Nm]
M_{31} = **+1,52E+12** [Nm] M_{32} = **-1,85E+11** [Nm] M_{33} = **+5,07E+12** [Nm]

Rupture time: 39,7[ms]
Seismic moment: 5,46E+12[Nm]
Total seismic moment: 5,46E+12[Nm]
Error: 7,22E+11[Nm]
Seismic moment magnitude: 2,5

Decomposition:
DBCP: 100,0

Nodal planes:
1: 36,1/50,8 2: 187,7/42,8

Axes (trend/plunge):
P: 113/4 T: 7/75 B: 204/14

Quality factor: 49
Classification: Reverse fault

Thank you