Probabilistic Inverse Theory

Lecture 1

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Title:	**Probabilistic Inverse Theory**
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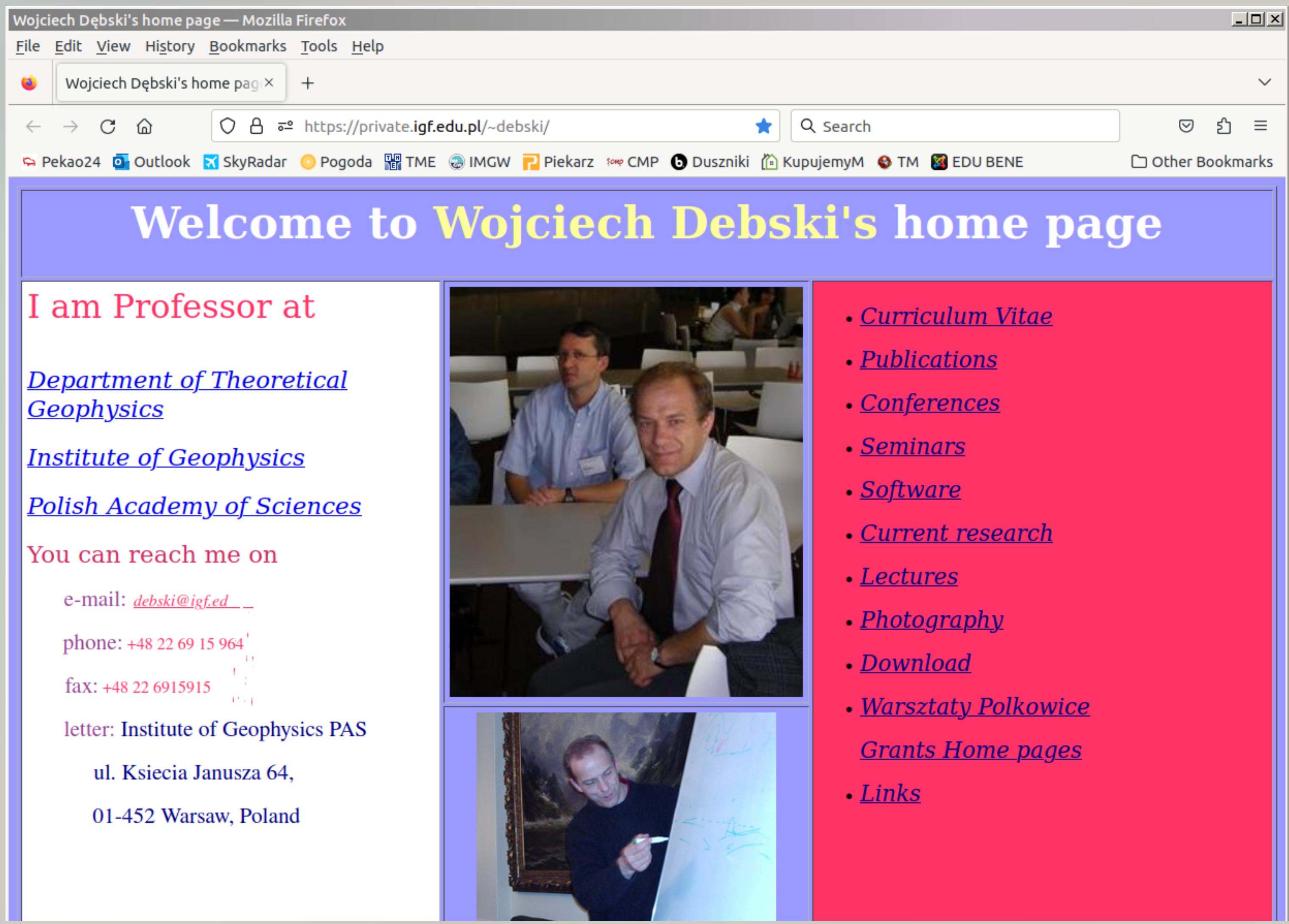
Exam:	individual (preferable solution of simple inverse task)
Duration :	20-24 hours
ETC points :	3

Transparencies



Recomended reading

- A. Tarantola, **Inverse Problem Theory and Methods for Model Parameter Estimation**, SIAM,2005 (on-line available)
- W. Menke, **Geophysical Data Analysis: Discrete Inverse Theory**, 1989
- M. Sen and P. L. Stoffa **Global Optimization methods in Geophysical Inversion**, 1995
- A. Tarantola, **Inverse Problem Theory: Methods for Data Fitting and Model Parameter Estimation**, Elsevier, 1987
- W. Dębski, **Application of Monte Carlo techniques for solving selected seismological inverse problems**, Publ. Inst. Geophys., 2004
- W. Dębski, **Probabilistic Inverse Theory**, Advances in Geophys., 2010

Main tasks

1. Forward and inverse problems
   - basic notions
   - different points of view

2. Inverse problems - introduction
   - indirect measurements
   - parameter fitting
   - inference

3. Algebraic inversion
   - linear inverse problems
   - nonuniquness of inverse problems
   - regularization
   - examples and problems
   - basic features - summary

4. Optimization approach

- ✦ nonlinear inverse problems
  - ★ nonlinear inverse problems - examples
  - ★ weak nonlinearity - linearization
  - ★ strong nonlinearity - optimization approach
- ✦ optimization techniques
  - ★ Gradient methods (overview)
  - ★ Simulated Annealing
  - ★ Genetic algorithm (?)
  - ★ Particle Swarm Optimization (?)
- ✦ examples and problems
- ✦ basic features - summary

5. Probabilistic (Bayesian) approach

- ✦ probabilistic approach - why?
  - ★ (non-) uniqness
  - ★ uncertainties (generalized measurements)
  - ★ inference and “information calculus”
- ✦ probability calculus

- ★ probability and its interpretation
- ★ Bayes theorem
- ★ more rigorous approach
- ★ selected probability distributions
- ★ norms in R^N space and induced probabilities

✦ *a priori* and *a posteriori* probability
- ★ errors ...
- ★ construction of *a posteriori* distribution
- ★ uniqness of solution
- ★ examples

✦ mathematics of inference space (?)

✦ sampling probability distribution
- ★ enumerated sampling and dimensionality problem
- ★ Monte Carlo technique
- ★ Markov Chain Monte Carlo
- ★ Metropolis-Hasting's algorithm
- ★

✦ examples and problems

6. Discrete vs. Continuous inverse problems

- from discrete to continuous
  - naive discretization
  - spectral discretization
  - time reversal method
  - data assimilation
- example - velocity/attenuation tomography
- time reversal method
- data assimilation
- Radon transform

7. Summary and general comments

8. Any particular problem raised by students

Introduction

- Introduction
  - Forward problems
  - Inverse problems
- Inverse problem - different point of views
  - parameter estimation
  - direct and indirect measurements
  - inference

Forward and Inverse problems - a general point of view

Actually, all problems encountered in our scientific activity can be divided into few categories:

- forward problems
- inverse problems
- administrations
- ...

Forward problems

This is a class of problems when one can try to understand qualitatively some observed phenomena or to predict new ones. The ultimate goal of solving such problems is an ability to predict (calculate) behavior of a system in hand.

Question: why...

Two examples:

- building new theories
- modelling physical processes

1. Building a new theory ED $\rightarrow$ QED

Classical electrodynamics:

$$
\begin{aligned}
\nabla \times \vec{E} - \frac{\partial \vec{B}}{\partial t} &= 0 \\
\nabla \times \vec{H} &= \vec{j} + \frac{\partial \vec{D}}{\partial t} \\
\nabla \cdot \vec{D} &= 4\pi\rho + \nabla \cdot \vec{j} \\
\nabla \cdot \vec{B} &= 0
\end{aligned}
$$

Classical ED: Lorentz force

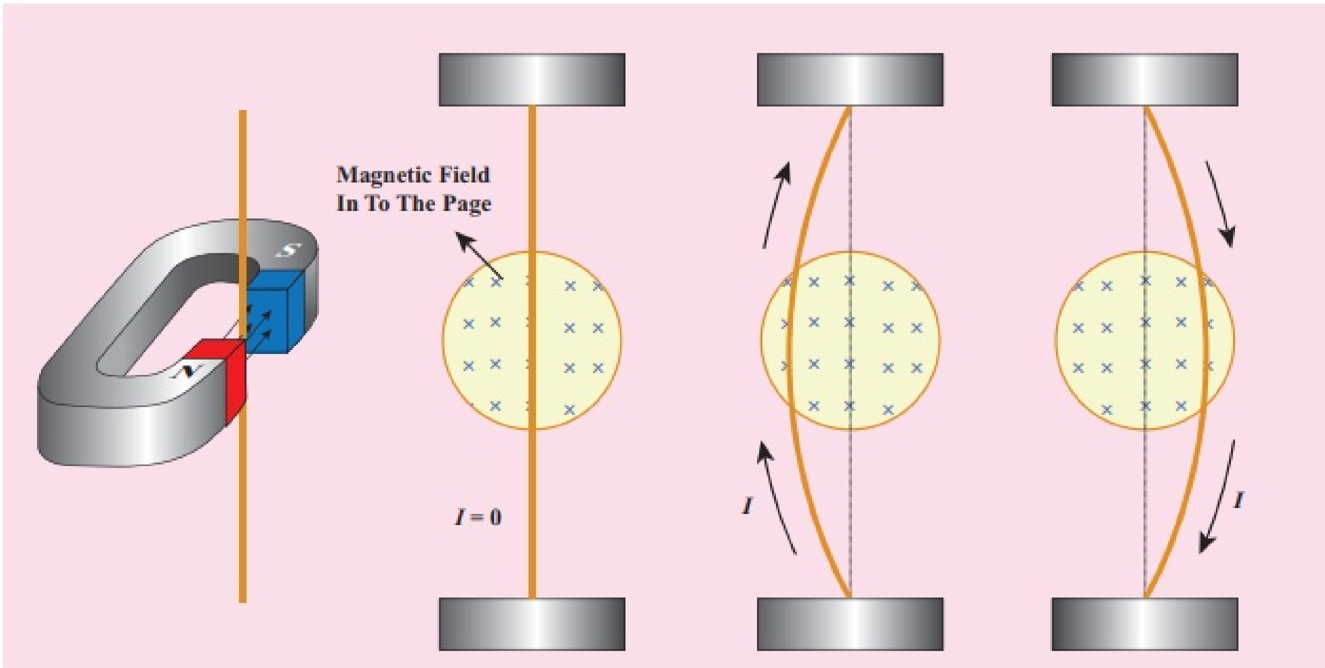
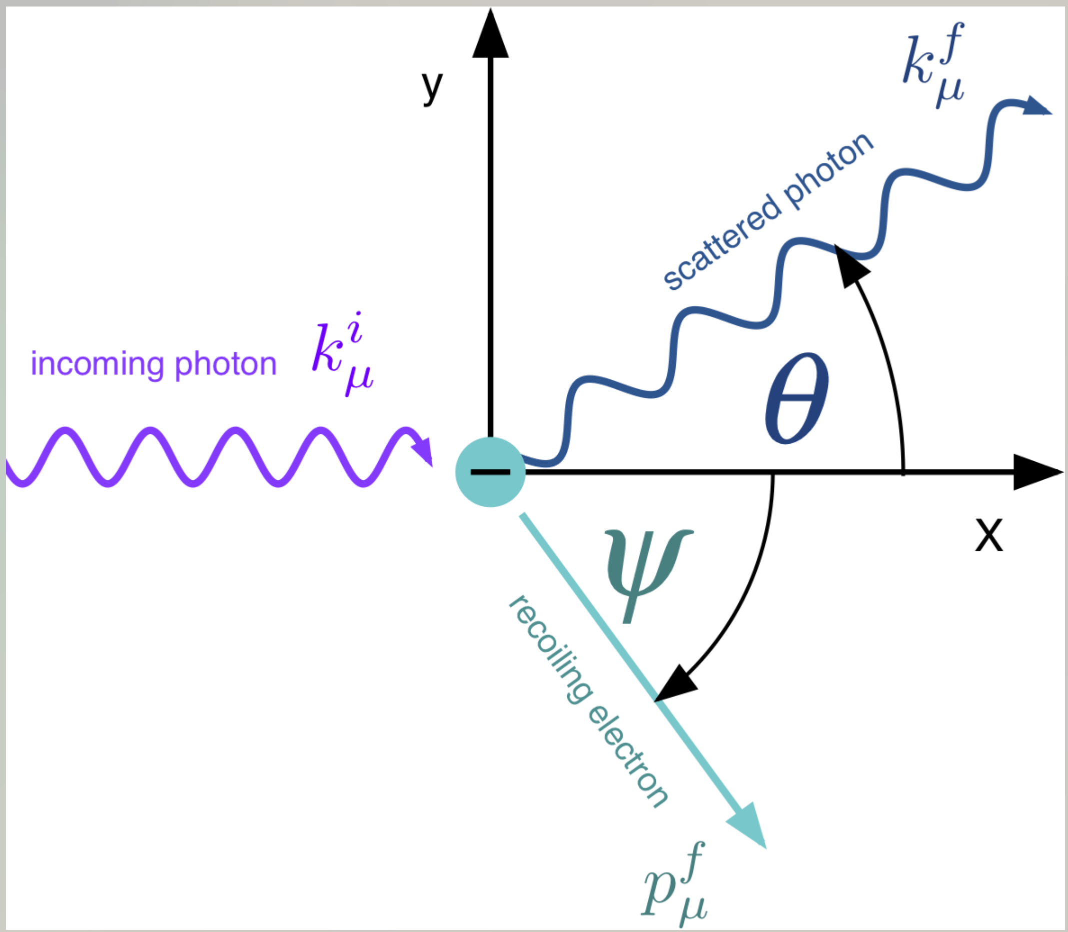
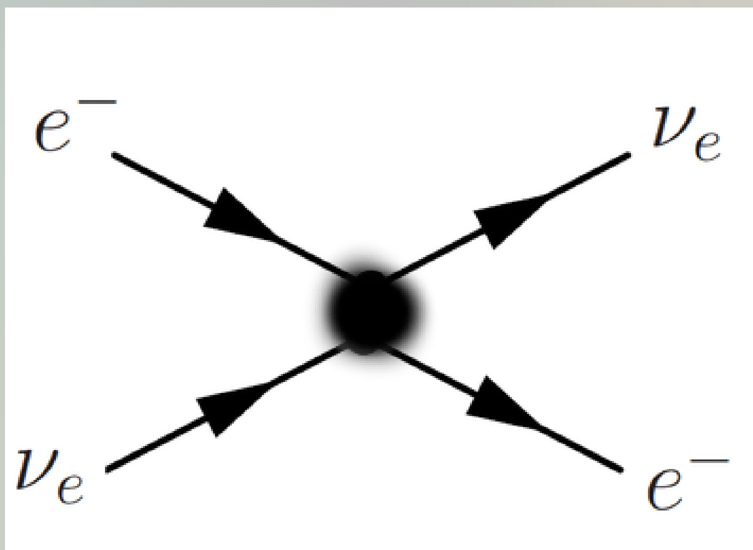


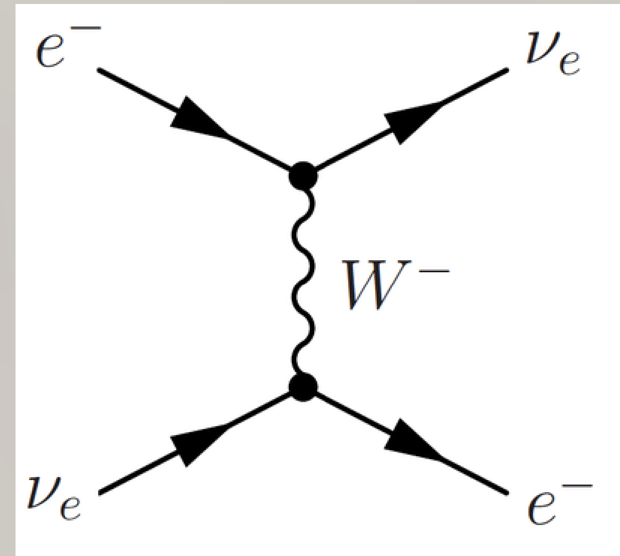
Figure 3.11 Deflection of current carrying wire in magnetic field

Compton scattering - photons



Building a new theory : QED





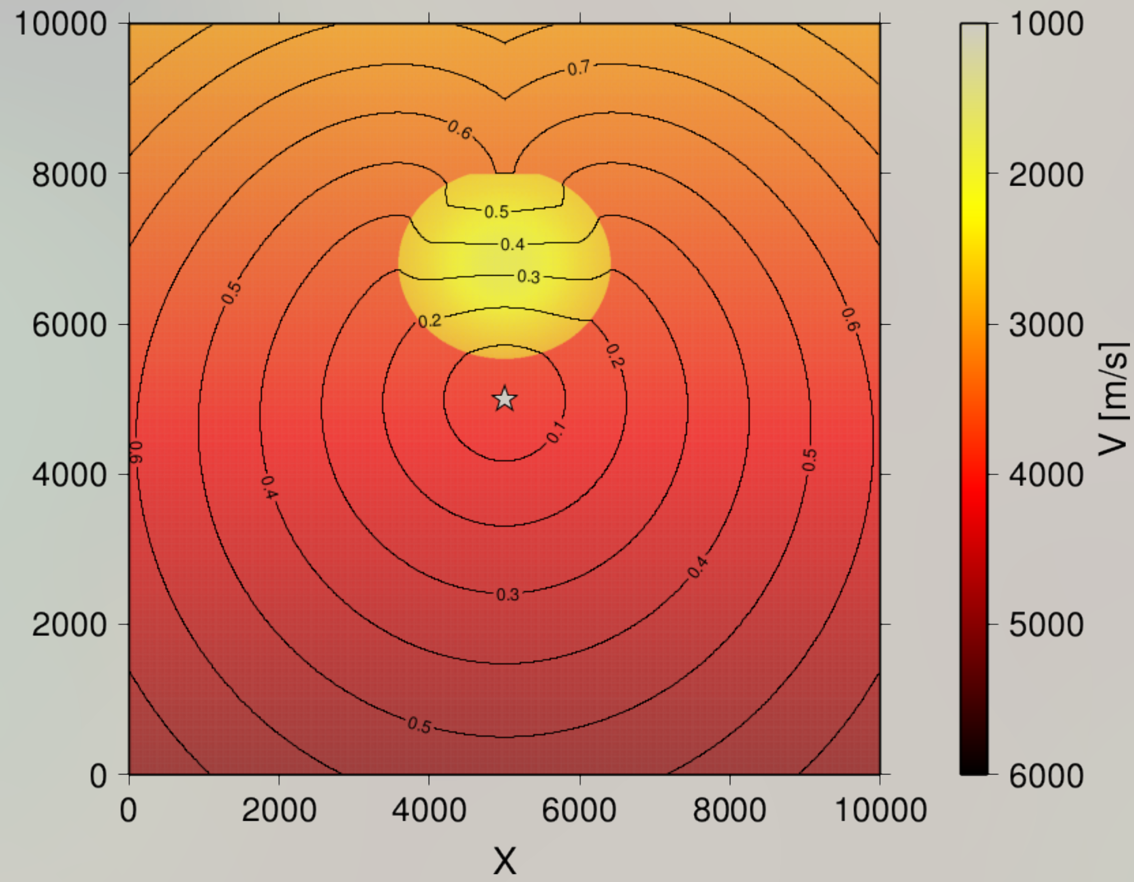
2. Forward modelling (seismic waves)

$$\rho \frac{\partial^2 u^i}{\partial t^2} - \partial_j \sigma^{ij} = S^i(\mathbf{r}, t)$$

$$\epsilon_{kl} = 1/2(\partial_k u_l + \partial_l u_k)$$

$$\sigma^{ij} = c^{ijkl} \epsilon_{kl}$$

2. Solution: wave propagation fora given velocity model

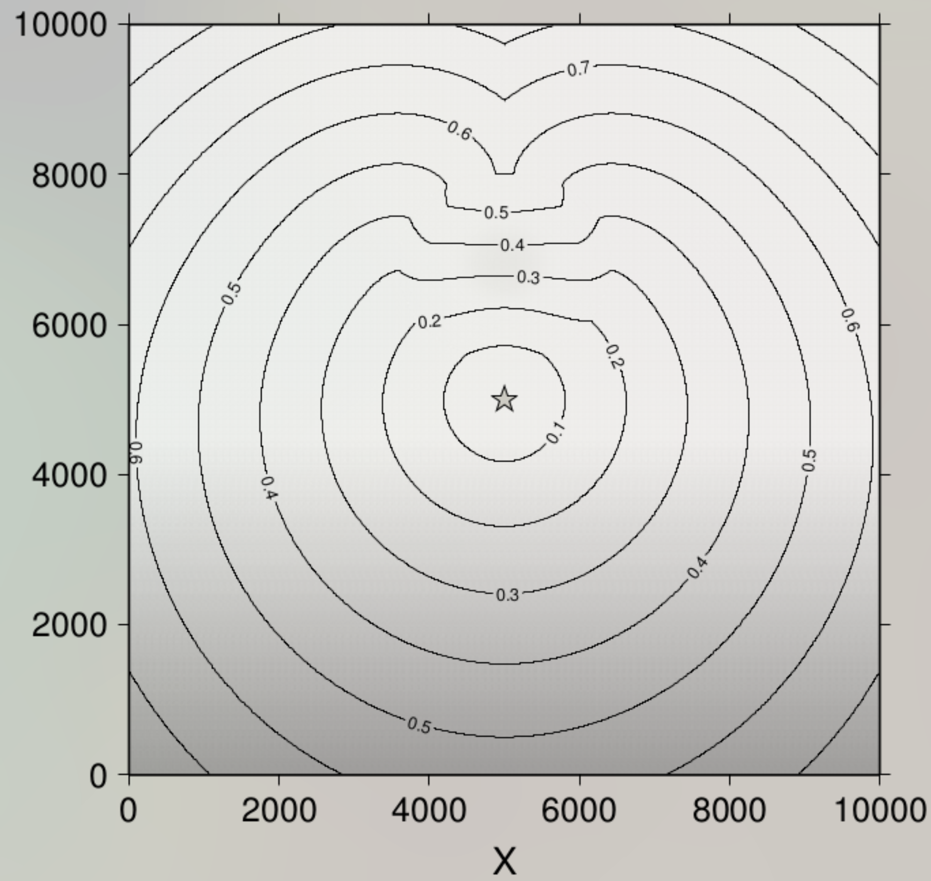


Inverse problems

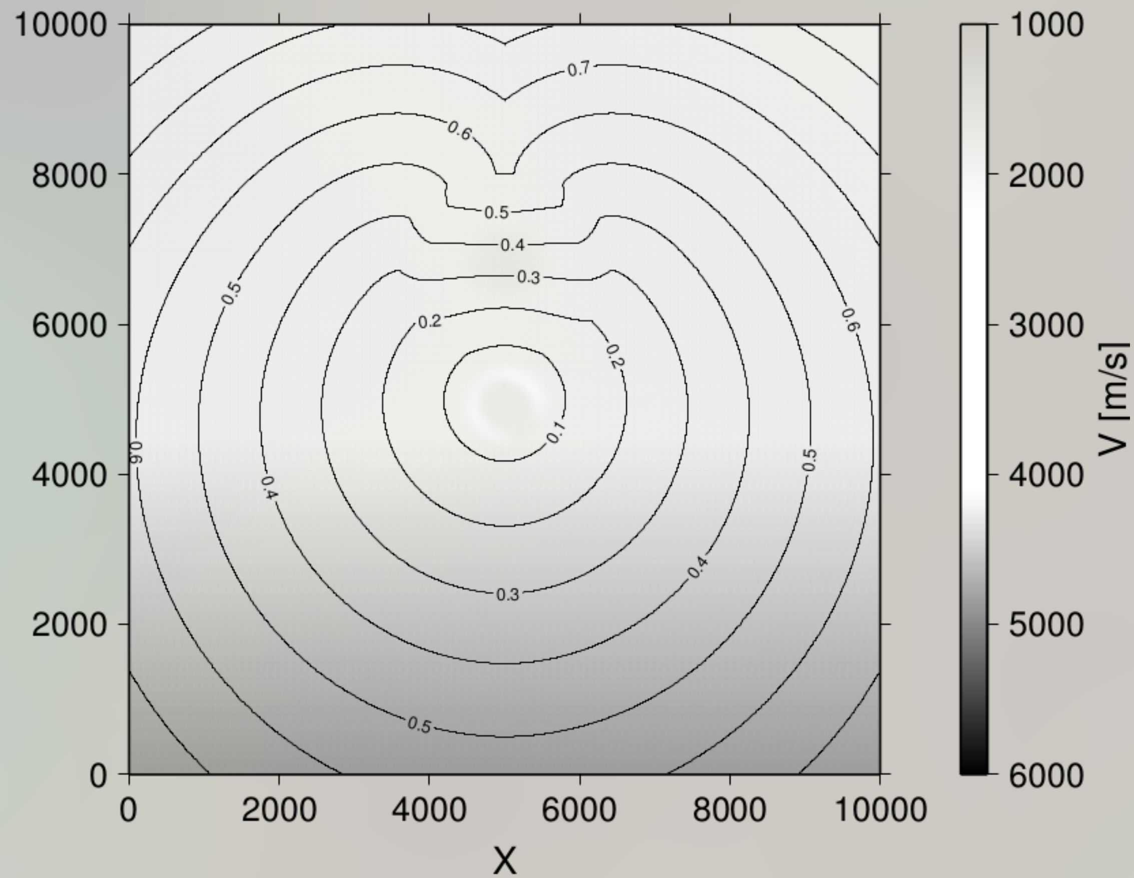
Inverse problems are tasks when one can try to grasp quantitative description of given system in hand or observed processes. The goal is not to provide a general description how the system behaves but to infer information on it allowing its realistic description.

Question: What is ....

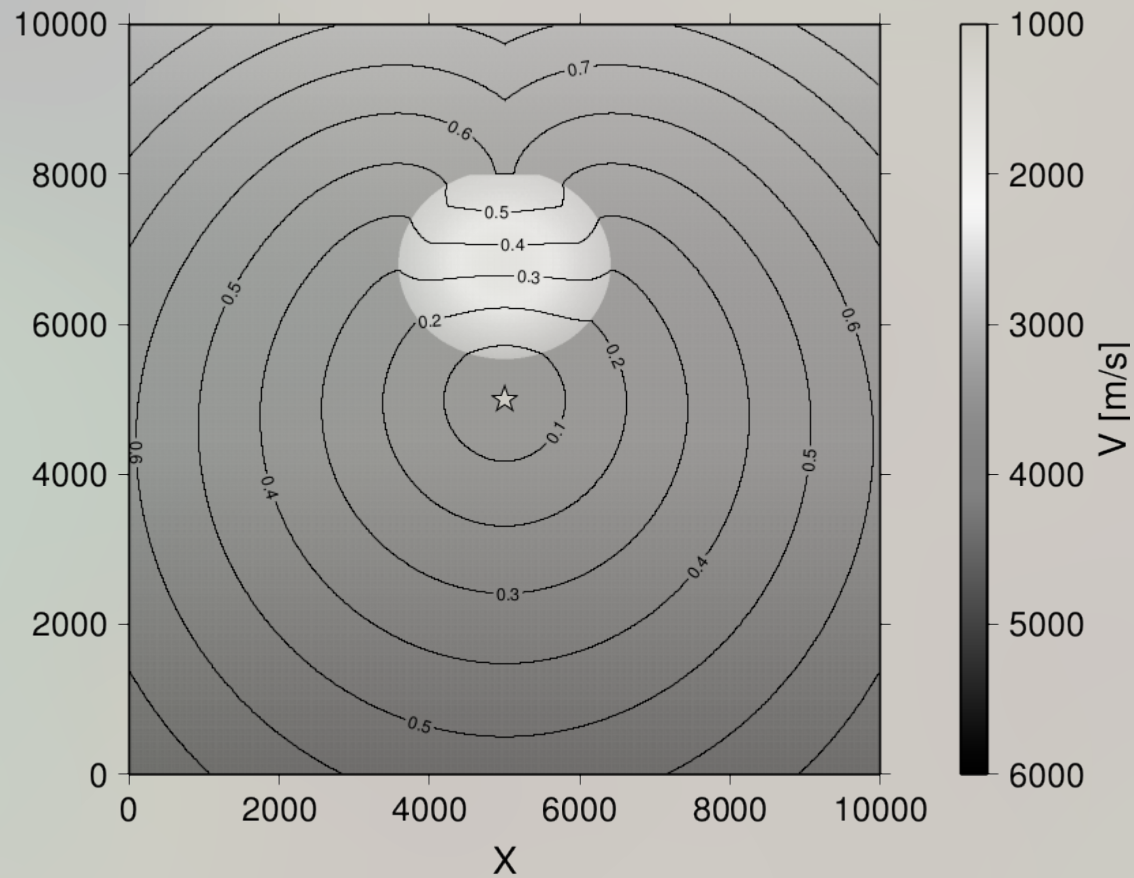
Inv. problem 1: velocity distribution for which we observe ...



Inv. problem 2: location of source



Inv. problem 3: size of an opening



Inverse problems - comments

- to solve inverse problems we HAVE TO to be able to solve a corresponding forward problem
- for given forward problem there can be many different inverse problems - we can pose different questions
- solution of inverse problems - some characteristics of studied object/process

Inverse problems - cd

- forward modelling problem is always unique - for given process/object predictions are always unique
- uniqueness of inverse problems may be problematic
- solving inverse problems usually requires some observational information ....
- solution of inverse problem - interpretation of available data

See you next week ...