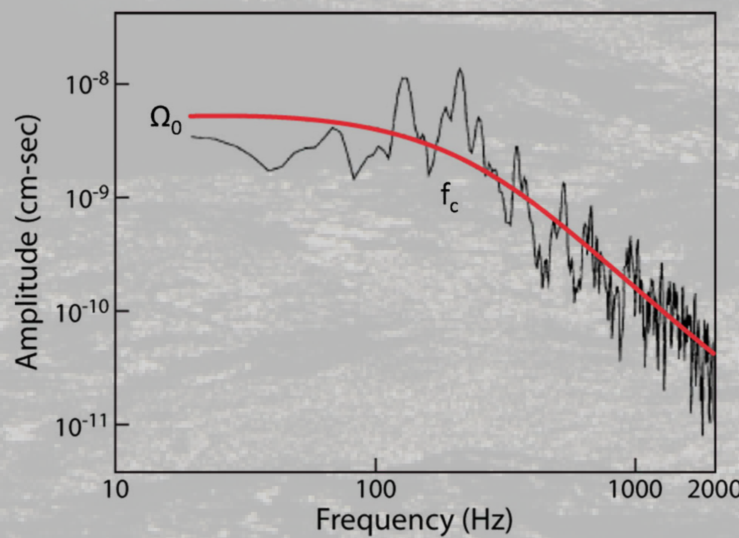
An introduction to Physics of Seismic Sources

SP-9: Kinematics of extended sources

W. Dębski, IGF PAN

debski@igf.edu.pl

Corner frequency



$$\mathbf{u}^P(\omega) = \frac{\dot{M}_o(\omega) R_P}{4\pi\alpha^3\rho r_o}\,\frac{\sin(X)}{X} \exp\left[-i\left(\frac{\omega r_o}{\alpha} - X - \frac{\pi}{2}\right)\right]$$

$$X = -\frac{\omega L}{2\alpha}\left(\frac{\alpha}{v_r} - \cos(\theta)\right)$$

Corner frequency ($\theta = \pi/2$)

$$\mathbf{u}^P(\omega) \sim \dot{M}_o(\omega) \frac{\sin(X)}{X}\, e^{-i\left(\frac{\omega r_o}{\alpha}\right)}\, e^{i\left(\frac{\omega L}{2v_r} + \frac{\pi}{2}\right)}$$

Let us define ω_c as ω for which phase shift equels π

$$\frac{\omega_c L}{2v_r} + \frac{\pi}{2} = \pi$$

$$\boxed{\omega_c = \frac{\pi v_r}{L}}$$

Directivity

Finite size of source causes that amplitudes of observed waves now depends on the location of station with respect to the direction of source rupturing

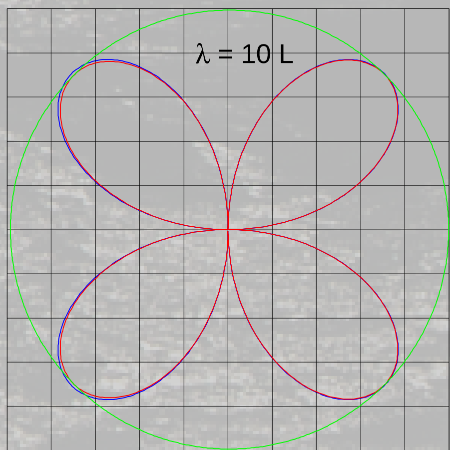
$$\mathbf{u}(\omega) \sim R_p(\theta)\frac{\sin(X)}{X}$$

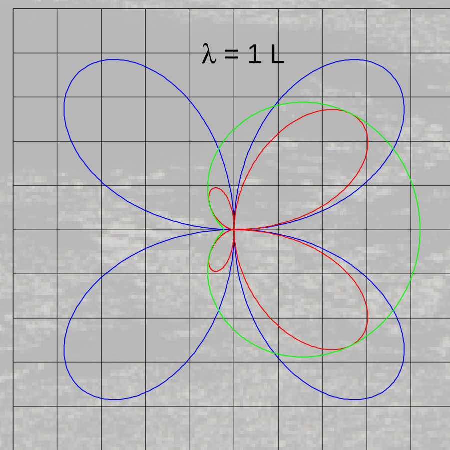
$$X = -\frac{\omega L}{2\alpha}\left(\frac{\alpha}{v_r} - \cos(\theta)\right) = \frac{L}{2\lambda}\left(\frac{\alpha}{v_r} - \cos(\theta)\right)$$

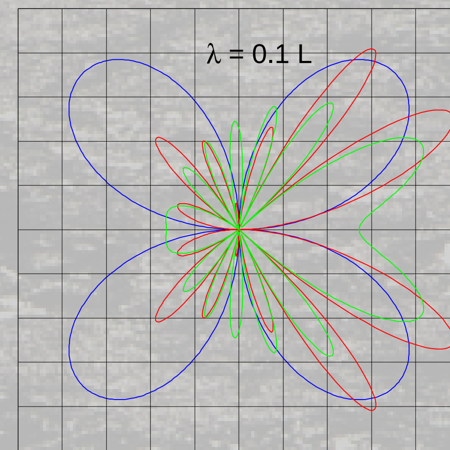
If $\lambda >> L \Rightarrow \sin(X)/X \approx 1$

$$\mathbf{u}^{Finite}(\omega) \sim \mathbf{u}^{Point}(\omega)$$

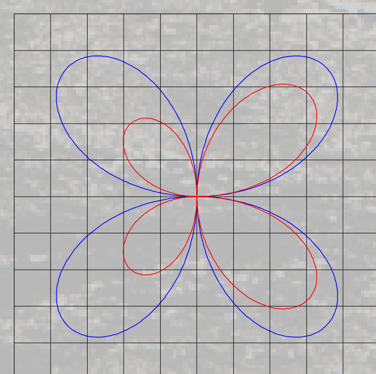
Directivity P-wave ($\alpha/v_r = 2$)





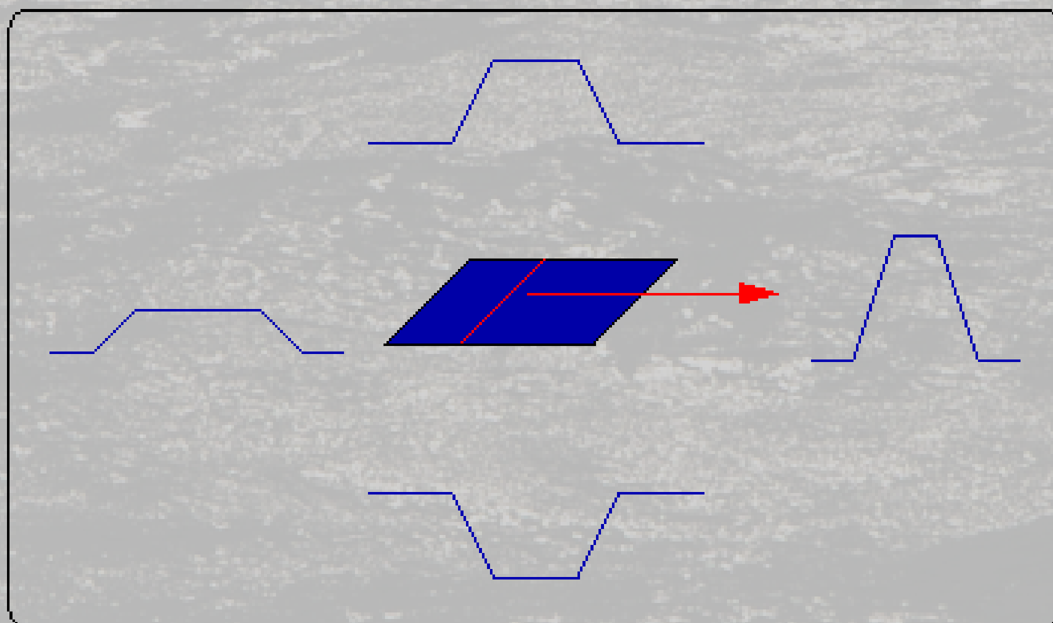


Radiation pattern average over frequency with $\lambda^{base} \sim 5L$



Directivity: amplitude and duration time

$$T_d = \frac{L}{v_r} - \frac{L}{\alpha}\cos(\theta)$$



Nucleation and arresting

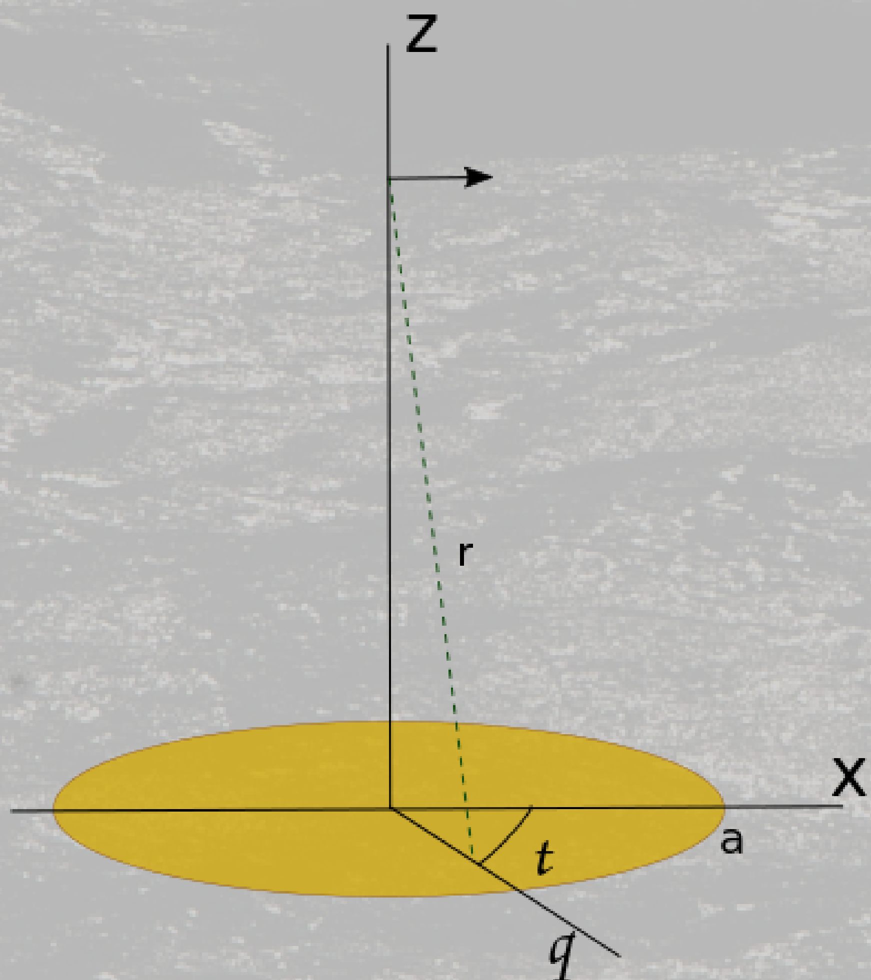
Haskell's model assume instantaneous starting (nucleation) of the rupture process and its instantaneous arresting over the whole fault. Both are highly unphysical since predict delta-like pulses in far field and are not acceptable from mecanics point of view (e.g., infinite energy) Look in depth on the problem

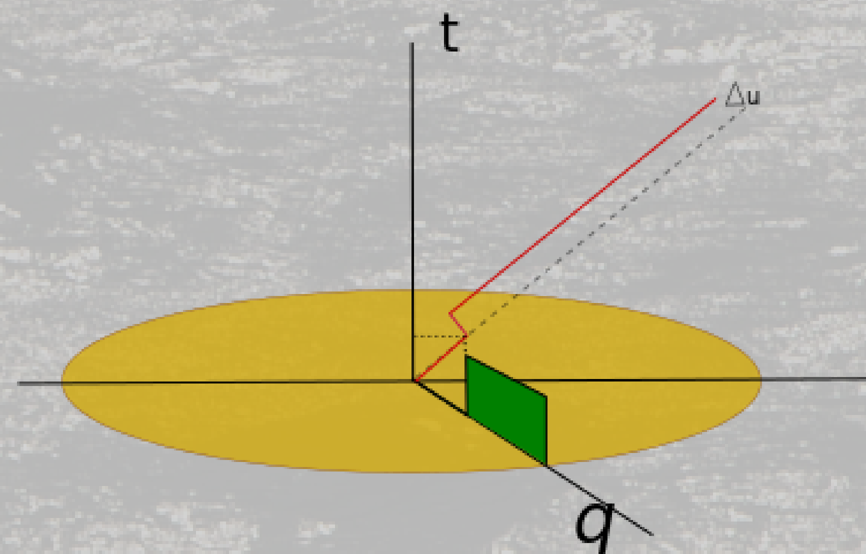
Simplified Savage model

- circular fault of radius a
- slip constant for all points
- rupture begins at the center
- propagates radially with $v_r = const.$
- rupture stops at circular edge
- slip depends only on ρ

$$\Delta u(\rho, t) = \Delta u H(t - \rho/v_r)[1 - H(\rho - a)]$$

Savage model





Savage model

$$u(z,t) = \int_0^{2\pi}\int_0^{a} \Delta\dot{u}\left(\rho, t - \frac{r(\rho,\phi)}{\alpha}\right)\rho d\rho d\phi$$

If $z >> a$

$$u(z,t) = 2\pi \int_0^{a} \Delta\dot{u}\left(\rho, t - \frac{z}{\alpha}\right)\rho d\rho$$

Savage model

$$u(z,t) = 2\pi \Delta u H\left(t - \frac{z}{\alpha}\right) \int_0^a \delta\left(t - \frac{z}{\alpha} - \frac{\rho}{v_r}\right) [1 - H(\rho - a)] \rho d\rho$$

$$\int f(x)\delta(ax-b)dx = \frac{1}{a} f(\left(\frac{b}{a}\right)$$

$$u(z,t) = 2\pi \Delta u v_r^2 \left(t - \frac{z}{\alpha}\right)\ H\left(t - \frac{z}{\alpha}\right) \left[1 - H\left(v_r\left(t - \frac{z}{\alpha}\right) - a\right)\right]$$

Savage model - stoping phase

$$u(z,t) = 0 \quad \text{for} \;\; t \le \frac{z}{\alpha} \quad \text{and} \;\; t \ge \frac{z}{\alpha} + \frac{a}{v_r}$$

otherwise

$$u(z,t) = 2\pi \Delta u v_r^2 \left(t - \frac{z}{\alpha}\right) \quad \text{for} \;\; \frac{z}{\alpha} < t < \frac{z}{\alpha} + \frac{a}{v_r}$$

Displacement begins at $t = z/\alpha$ (nucleation) and continue until $t_s = z/\alpha + a/v_r$ when fracture stops at the border and then drops to zero. At t_s arrives signal produced by stopping rupture - stopping phase. Since displacement drops discontinuously to zero velocity/acceleration will have delta-like shape - high frequency radiation - dominated by stopping phase

Extended Savage models

- Savage model with rise time (rectangular displacement velocity pulse)

$$\Delta u(\rho,t) = \Delta u H(t-\rho/v_r)[1-H(\rho-a)]\left[H(t)\frac{t}{\tau_r} - H(t-\tau_r)\frac{t-\tau_r}{\tau_r}\right]$$

In this model slip ceases afer a time τ_r independent of final arrest at $t_r = a/v_r$

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$$\Delta \dot{u}(\rho,t) = \Delta V\left[H\left(t-\frac{\rho}{v_r}\right) - H\left(t-\frac{a}{v_r}\right)\right]H(a-\rho)$$

Slip persists on everypoint until fracture stops at the eadge. Information on fracture arresting at the eadge reaches instantenously all points

Healing front

- Extension - stoping phase propagates insight the fault with finite velocity (here α). this is so called healing front

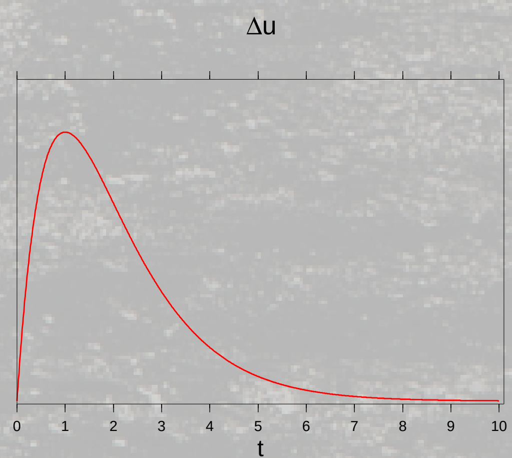
$$\Delta\dot{u}(\rho,t) = \Delta V \left[H\left(t - \frac{\rho}{v_r}\right) - H\left(t - \frac{a}{v_r} - \frac{a-\rho}{\alpha}\right)\right] H(a-\rho)$$

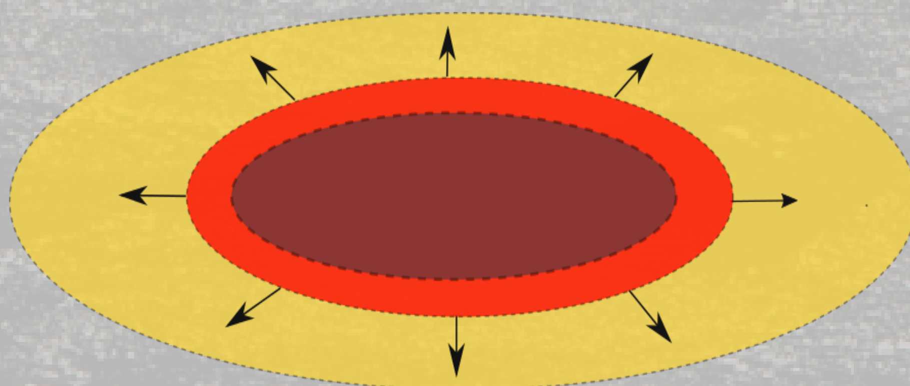
- Molnar's model
  Circular fault ruptures from the center to the border and next contracts back to the center at the same speed

$$\Delta\dot{u}(\rho,t) = \Delta V \left[H\left(t - \frac{a}{v_r} + \frac{a-\rho}{\alpha}\right) - H\left(t - \frac{a}{v_r} - \frac{a-\rho}{\alpha}\right)\right] H(a-\rho)$$

Heaton model

- Heaton model (Heaton 1990)
  Slip velocity starts at $t = \rho/v_r$, lineary increases to its maximum value and next decreases to zero after τ_r. for $\tau_r < a/v_r$ this is propagating pulse from the center to the border of time width τ_r. This is so called self-healing pulse because motion ceases separatelly at each point of the fault after τ_r time. No need of healing pulse. Quite realistic model observed for some strike-slip faults.





Beyond kinematical models

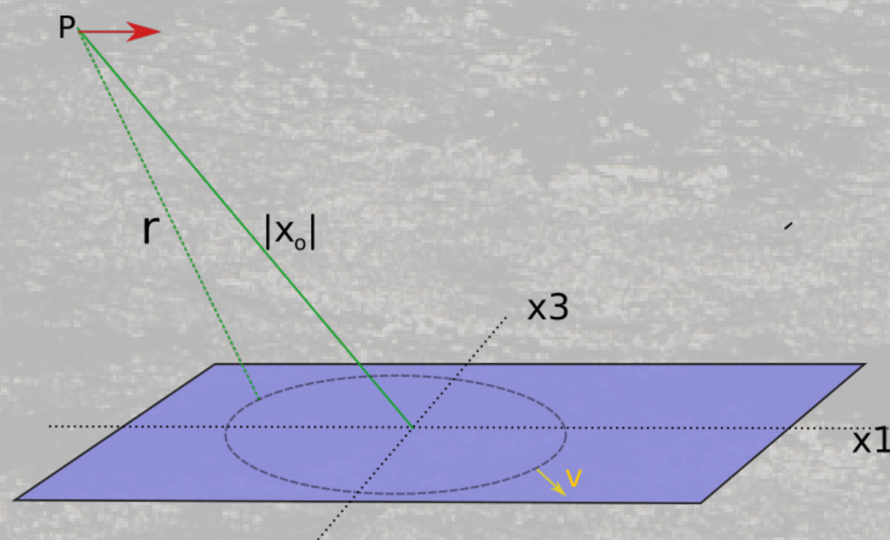
The discussed kinematic source models were very successful in describing basic source characteristics and established the theoretical base for for source physics inference based on seismic data. However, all of them have more or less serious problems with displacement continuity at edges of fault, instantaneous rupturing, or stopping, etc. From the point of view continuous mechanics are simply not acceptable. Dealing with this issue requires considering dynamical rupture models. The first step in this direction was the model of Sato and Hirasawa (1973).

Their model describes a circular fault rupturing with constant velocity from the center outwards (like Savage's one), but now the slip is not constant but is a function of the stress drop over the fault

Variable slip models

Until now we have considered models with slip (slip rate) constant over the whole fault. Let generalize it to spatially variable slip. First Haskell's like model

$$u^P(x,t) = A \int_0^L \int_0^W \frac{1}{r} R_P(n_k, l_k(\xi_i)\gamma_k(x,\xi_i)\Delta\dot{u}\left(\xi_1, \xi_3, t - \frac{r}{\alpha}\right) d\xi_1 d\xi_3$$



Variable slip models

Again for Fraunhofer approximation and assuming shearing process ($n_k l_k = 0$), and constant slip direction ($l_i = const$) for rupture starting at $t = 0$ and $\xi_i = 0$

$$u^P(x,t) = A' R_P(n_k, l_k, \gamma_k) \int\limits_0^L \int\limits_0^W \Delta\dot{u}\left(\xi_1, \xi_3, t - \frac{r_o}{\alpha} + \frac{\xi_i \gamma_i}{\alpha}\right) d\xi_1 d\xi_3$$

Let assume that rupture speed can vary and $t'(\xi_i)$ is rupture front arrival at given point.

$$\Delta\dot{u}\left(\xi_1, \xi_3, t - \frac{r_o}{\alpha} - \frac{\xi_i \gamma_i}{\alpha}\right) = \Delta\dot{u}\left(\xi_1, \xi_3, t - t'(\xi_i) + \frac{\xi_i \gamma_i}{\alpha}\right)$$

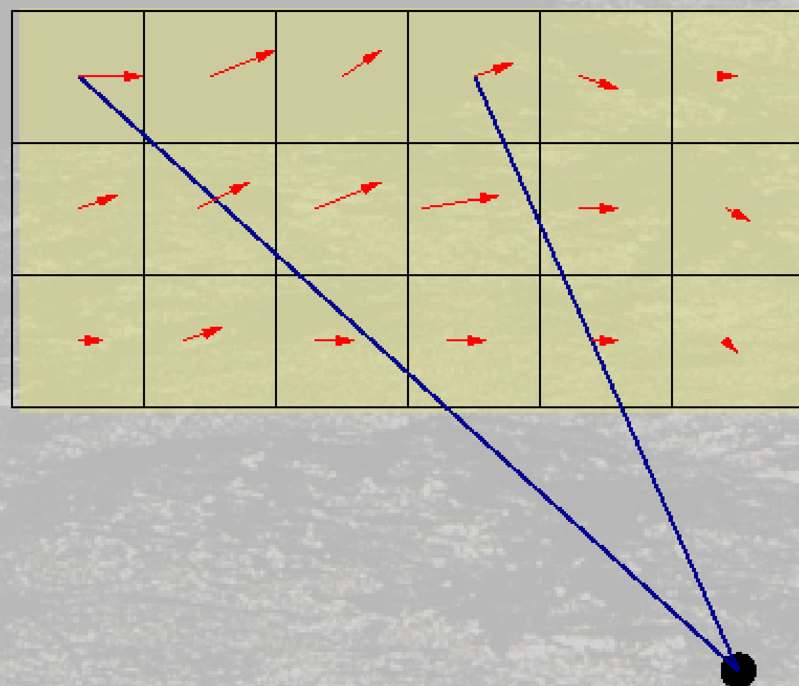
Kinematic models - practical issue

$$u^P(x,t) = A' R_P(n_k, l_k, \gamma_k) \int_0^L \int_0^W \Delta \dot{u} \left(\xi_1, \xi_3, t - \frac{r_o}{\alpha} - t'(\xi) + \frac{\xi_i \gamma_i}{\alpha} \right) d\xi_1 d\xi_3$$

It can be used to calculate synthetic seismograms for as general source slip and source time functions. We need assume fault plane size (L, W), its position in space (strike, dip) and slip distribution (dependence of $\Delta \dot{u}$ over fault plane) and source time function (dependence of $\Delta \dot{u}$ on time) This practically can be done only numerically.

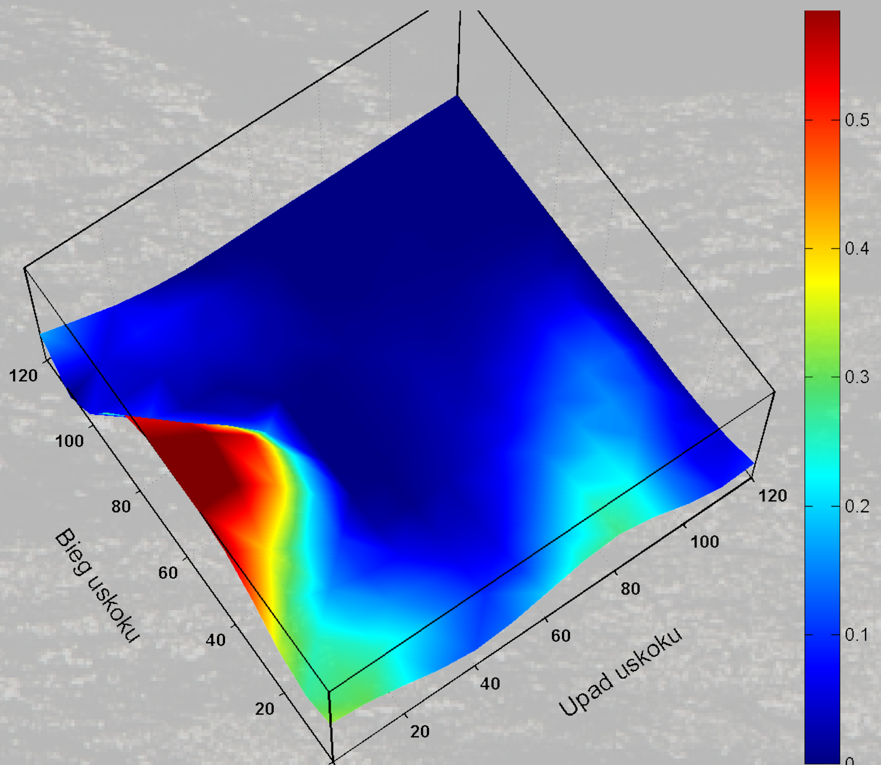
$$u^P(x,t) = \frac{1}{4\pi\rho\alpha^3 r_o} \int_0^L \int_0^W R_P(m_{ij}(\xi), \gamma_i)\, \dot{m} \left(\xi_1, \xi_3, t - \frac{r_o}{\alpha} - t'(\xi) + \frac{\xi_i \gamma_i}{\alpha} \right) d\xi_1 d\xi_3$$

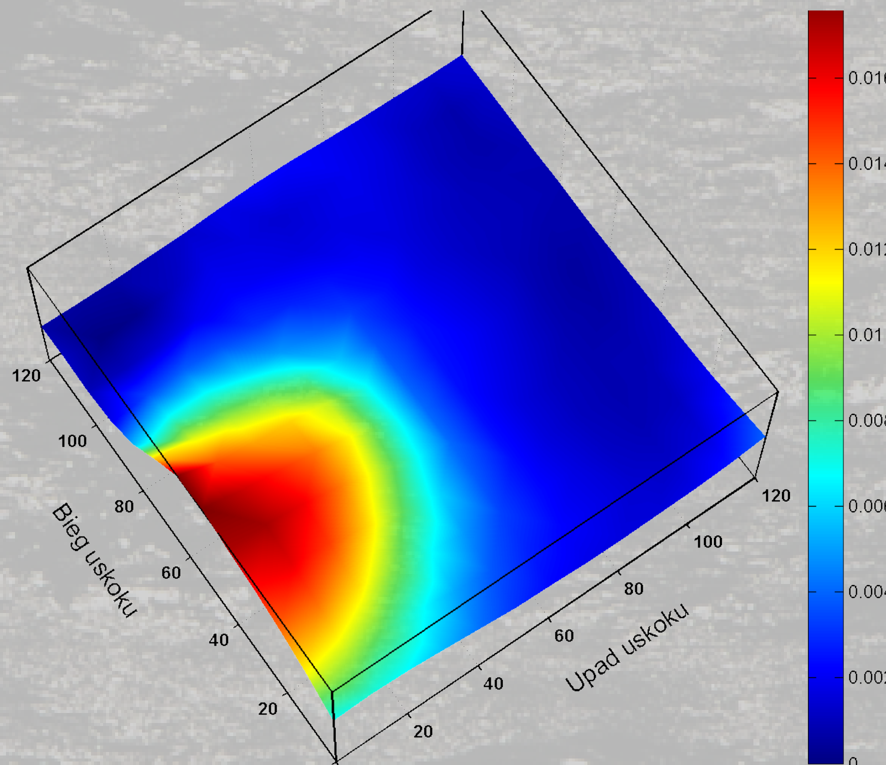
Kinematic source tomography



$$u^P(x,t) = \frac{1}{4\pi\rho\alpha^3 r_o} \sum_k S_k R_{P_k}\, \dot{m}_k \left(t - \frac{r}{\alpha} - t'_k + \phi_k \right)$$

Kinematic source tomography Rudna mine case





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