An introduction to Physics of Seismic Sources

SP-7: From point-like to extended source

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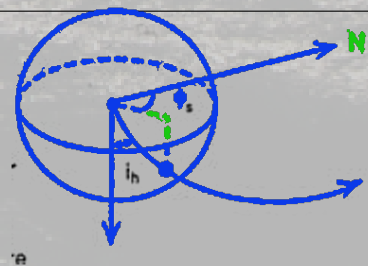
Moment tensor inversion - P and S waves



$$U^P = A\Big[\sin^2(\xi)\big(\cos^2(\psi)m_{11} + \sin^2(\psi)m_{22} + \sin(2\psi)m_{12}\big) + \cos^2(\xi)m_{33} + \sin(2\xi)(cos(\psi)m_{13} + \sin(\psi)m_{23})\Big]$$

$$U^{SV} = B\Big[\frac{1}{2}\sin(2\xi)\big(\cos^2(\psi)m_{11} + \sin^2(\psi)m_{22} - m_{33} + \sin(2\psi)m_{12}\big) + cos(2\xi)\big(\cos(\psi)m_{13} + \sin(\psi)m_{23}\big)\Big]$$

$$U^{SH} = B\Big[\sin(\xi)\Big(\frac{1}{2}\sin(2\xi) - \frac{1}{2}\sin(2\psi)m_{11} + \cos(2\psi)m_{12}\Big) cos(\xi)(\cos(\psi)m_{23} - \sin(\psi)m_{13})\Big]$$

Moment tensor inversion

$$\mathbf{M} = (m_{11}, m_{12}, m_{13}, m_{22}, m_{23}, m_{33})$$

$$U_i^P = G^{ij} M_j$$

- Direct inversion

$$\mathbf{M} = (G^T G + \lambda C^T C)^{-1} \mathbf{U^P_{obs}}$$

- Waveform fitting

$$||U^P_{obs}(t) - U^P_{synth}(M)|| = min \Longrightarrow M^{est}$$

- Other (surface waves, etc)

Source inversion - synchronous source

Source inversion can comprise

➡	Nm=4	*Hypocenter location*	x, y, z, t_o
➡	Nm=6	*Moment tensor*	$m_{11}, m_{12}, \ldots$
➡	Nm=1	*Fault plane solution*	$\gamma_{ij}^{DC} = e_i^s e_j^n + e_j^s e_j^n$
➡	Nm=(∞)	*Source time function*	$S(t) = \Delta \dot{u} / u_o$
➡	Nm=1	*Source rise time*	T_r

$$u_n(x_r, t) = \int_{-\infty}^{\infty} m_{ij}(t')\, G_{ni,j}(x_s, t'; x_r, t)\mathrm{d}t'$$

Centroid moment tensors

Point source - where is it?

- hypocenter - locate by e.g. arrival times - the point of rupture initiating
- centroid solution - the centroid of temporal and spatial moment tensor distribution

For idealized, shear dislocation

$$
\begin{aligned}
x_j^C &= \frac{1}{M_o}\int_S x_j \mu \Delta \dot{u} dS = \frac{1}{M_o}\int_S x_j \dot{m} dS \\
t^C &= \frac{1}{M_o}\int_0^t d\tau \int_S \mu \tau \Delta \dot{u} dS
\end{aligned}
$$

Centroid solution - geometric "gravity center" of fault surface

Higher order moment tensors

$$u_i = \int_{-\infty}^{\infty} \mathrm{d}t' \int_{V_o} F_i(\xi, t') G_{ni}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

Taylor expansion for "point-like source" ($\lambda >> L^2/r$)

$$G_{ik}(\xi_n) = G_{ik}(0) + \xi_j \frac{\partial G_{ik}}{\xi_j} + \frac{1}{2}\xi_j \xi_n G_{ik,jn} + O(\xi^3)$$

$$u_i = \int_{-\infty}^{\infty} \mathrm{d}t' \int_{V_o} \left(\xi_j F_k G_{ik,j} + \frac{1}{2} \xi_k \xi_l F_j G_{ij,kl} \right) \mathrm{d}V_\xi$$

Higher order moment tensors

$$M_{ij} = \xi_i F_j; \qquad M_{ij,k} \sim \xi_k M_{ij}$$

$$u_i = \frac{\partial G_{ij}}{\partial \xi_k} \star M_{jk} + \frac{\partial^2 G_{ij}}{\partial \xi_l \partial \xi_k} \star \frac{\partial M_{jk}}{\partial \xi_l}$$

Assuming synchroneous source

$$m_{ij}(t, \mathbf{x}) = \gamma_{ij} M(t, \mathbf{x})$$

Applying Taylor expansion for time variable (Doormbos 1982, Dahm and Kruger, 1999)

Higher order moment tensors

$$u_i = \Big[G_{ij,k} - \Delta\tau\dot{G}_{ij,k} + \Delta\xi_l\dot{G}_{ij,kl} + \tfrac{1}{2}\Delta(\tau^2)\ddot{G}_{ij,k} - \Delta(\tau\xi_l)\ddot{G}_{ij,kl} + \tfrac{1}{2}\Delta(\xi_l\xi_m)\ddot{G}_{ij,klm}\Big]M_{ij}$$

$$\Delta\tau = \langle \tau - \tau_o \rangle \qquad \Delta(\tau^2) = \langle (\tau - \tau_o)(\tau - \tau_o) \rangle, \quad \tau_o - \text{origin time}$$

For centroid location

$$u_i = \Big[G_{ij,k} - \frac{1}{2}\Delta(\tau^2)\ddot{G}_{ij,k} - \Delta(\tau\xi_l)\ddot{G}_{ij,kl} + \frac{1}{2}\Delta(\xi_l\xi_m)\ddot{G}_{ij,klm}\Big]M_{ij}$$

END