An introduction to Physics of Seismic Sources

SP-6: Spectral properties, Energy

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Source Time Function (STF)

Source time function represents time dependence of seismic moment (slip)

$$u_i^P = \frac{1}{4\pi\rho r}\left[\frac{2}{\alpha^3}\gamma_i(\gamma_j l_j)(\gamma_k n_k)\,\dot{M}_o(t-r/\alpha)\right]$$
$$u_i^S = \frac{1}{4\pi\rho r}\left[\frac{-1}{\beta^3}\Big((2\gamma_i(\gamma_j l_j)(\gamma_k n_k) - n_i(\gamma_j l_j) - (n_k\gamma_k)l_i)\Big)\,\dot{M}_o(t-r/\beta)\right]$$

$$M_o(t) = M_o\bar{S}(t) \qquad \dot{M}_o(t) = M_o S(t)$$

Spectral properties of far field

Udias, Madariaga, Buforn:
*Some of the most useful and universal properties of source time functions are obtained in the **spectral** domain*

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \qquad *$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega \qquad **$$

Basic fact $\lim_{\omega \to o} \dot{M}_o(\omega) = M_o$ obtained taking $f(t) = \dot{M}_o(t)$ in (*)

Seismic energy density

$$E_{tot} = E_K + E_P = \frac{\rho}{2}\dot{u}_i\dot{u}_i + \frac{1}{2}\tau_{ij}\epsilon_{ij}$$

In far field

$$E_{tot} = \rho\dot{u}_i\dot{u}_i$$

Energy:

$$E = \rho\int\limits_S dS\int\limits_{-\infty}^{\infty}\left(\alpha\left|\dot{u}_{iP}\right|^2\beta\left|\dot{u}_{iS}\right|^2\right)dt$$

for point dislocation source

$$\dot{u}_i = \frac{-1}{4\pi\rho\alpha^3 r}R_P\ddot{M}_o(t-r/\alpha) \qquad \frac{-1}{4\pi\rho\beta^3 r}R_S\ddot{M}_o(t-r/\beta)$$

Seismic energy

$$E = \frac{1}{16\pi^2\rho^2 r^2} \int_S \left(\frac{1}{\alpha^5} R_P^2 + \frac{1}{\beta^5} R_S^2 \right) dS \int_{-\infty}^{\infty} |\ddot{M}_o|^2 dt$$

$$\bar{R_P^2} = \frac{1}{4\pi} \int R_P^2(\theta,\phi) \sin\theta d\theta d\phi$$

$$E = \frac{1}{4\pi\rho} \left(\frac{1}{\alpha^5} \bar{R_P^2} + \frac{1}{\beta^5} \bar{R_S^2} \right) \int_{-\infty}^{\infty} |\ddot{M}_o|^2 dt$$

Finite energy $\Longleftrightarrow$ $|\ddot{M}_o|^2$ is finite and has finite support

Seismic energy

Parseval's theorem

$$f(t) \geq 0 \Longrightarrow \int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{\pi}\int_{0}^{\infty} |F(\omega)|^2 d\omega$$

$$\dot{f}(t) = \frac{i}{2\pi}\int \omega F(\omega) e^{i\omega t} d\omega$$

$$\boxed{E = \frac{1}{4\pi\rho}\left(\frac{1}{\alpha^5}\bar{R}_P^2 + \frac{1}{\beta^5}\bar{R}_S^2\right)\int_{0}^{\infty} \omega^2 |\dot{M}_o(\omega)|^2 d\omega}$$

Seismic energy

Important: finite radiation energy requires fast decrease of $M_o(t)$:

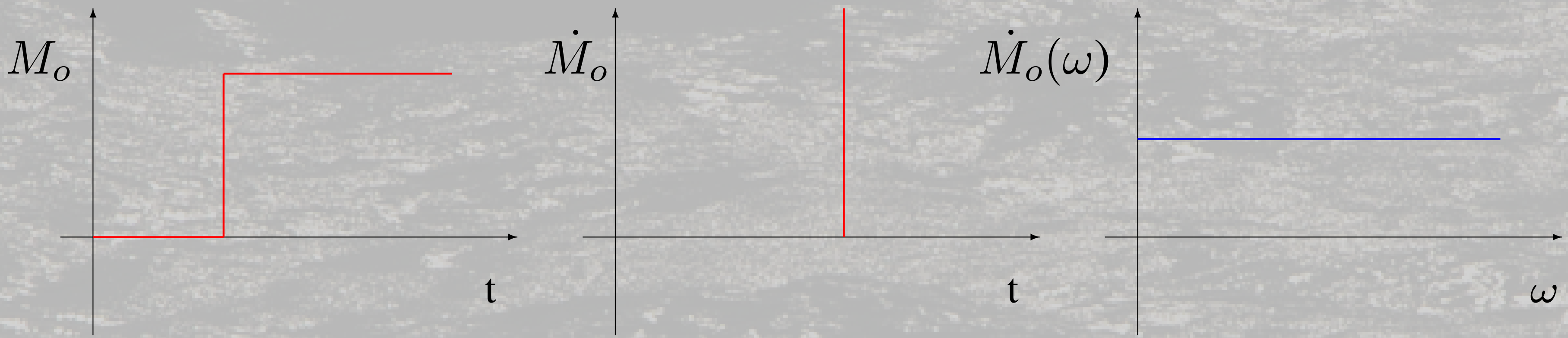
$$|\dot{M}_o(\omega)| \sim \frac{1}{\omega^2}$$

For “minimal” sources: $\dot{M}_o(\omega) = 1/(1 + (\omega/\omega_c)^2)$

$$E = \frac{1}{16\pi\rho}\left(\frac{1}{\alpha^5}\bar{R}_P^2 + \frac{1}{\beta^5}\bar{R}_S^2\right) M_o^2\ \omega_c^3$$

Here ω_c describes the finite rise time of a point source !

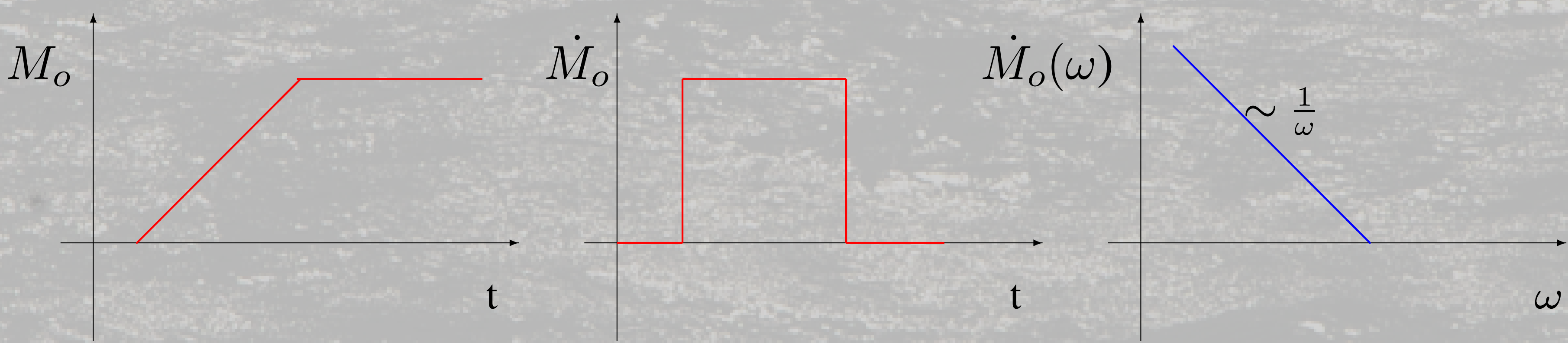
STF - instantaneous fault slip



- unrealistic model
- unacceptable far field
- emitted infinite energy

STF - linear increasing fault slip

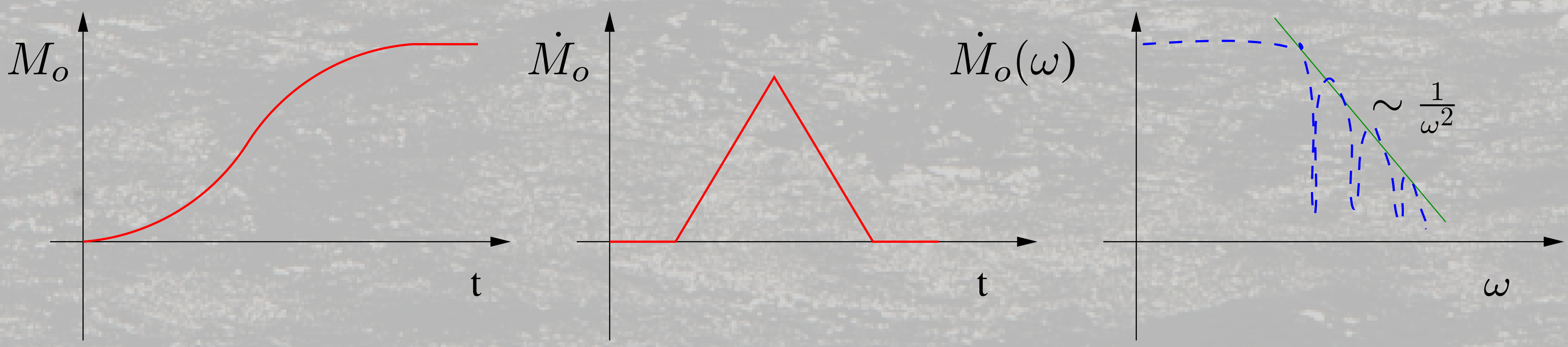
$$M_o(t) = M_o \left[\frac{t}{t_r} H(t) - \frac{t - t_r}{t_r} - H(t - t_r) \right]$$



✦ emitted infinite energy

STF - boxcar stf

$$\dot{M}_o(t) = \begin{cases} M_o \, \frac{4t}{t_r^2} & 0 < t < t_r/2 \\ M_o \, \frac{4(t_r - t)}{t_r^2} & t_r/2 < t < t_r \end{cases}$$

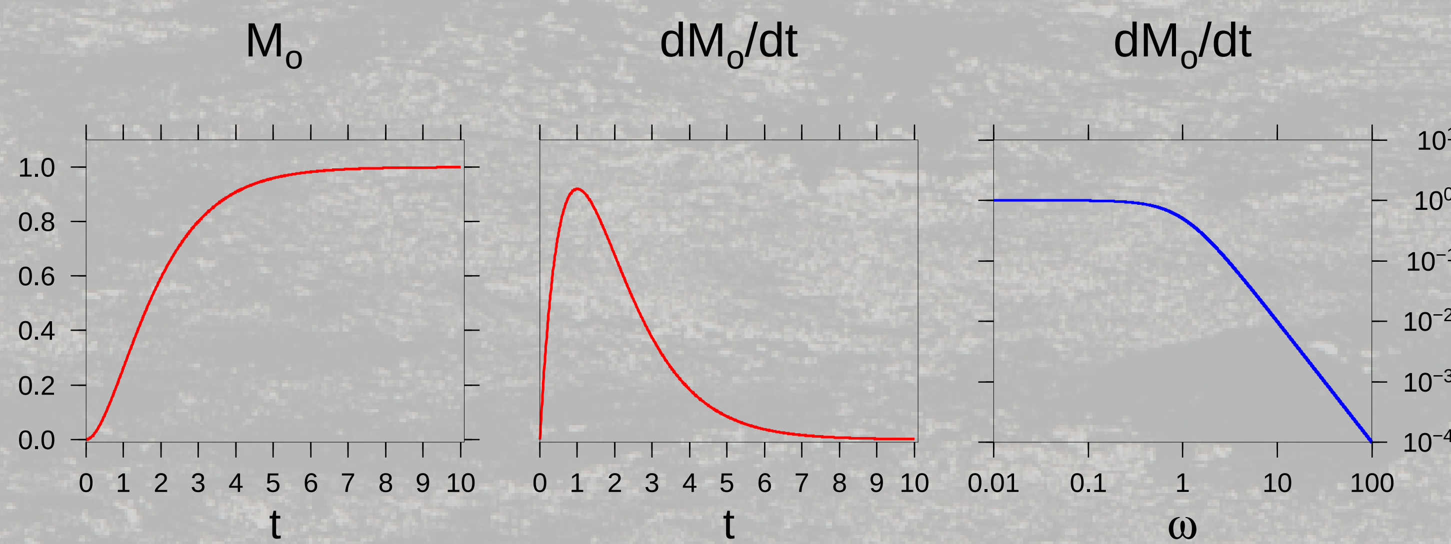


STF - Brune's model

$$M_o(t) = M_o\left[1 - (1 + t/t_r)e^{(-t/t_r)}\right] H(t)$$

$$\dot{M}_o(t) = M_o \frac{t}{t_r^2} e^{(-t/t_r} H(t)$$

$$\dot{M}_o(\omega) = \frac{M_o}{1+(\omega/\omega_c)^2}$$



Moment tensor - generalisation DC

Asumptions:

- source - a non-elastic process in a finite volume
- net sum of forces and moments in source is null
- absence of body forces ($F_i = 0$)

Total stress σ_{ij}

$$\sigma_{ij} = \tau_{ij} + m_{ij}$$

Moment tensor - generalisation

Wave equation

$$\rho \ddot{u}_i = \partial_j \tau_{ij}$$

Generalise (postulate)

$$\rho \ddot{u}_i = \partial_j \sigma_{ij}$$

$$\rho \ddot{u}_i = \partial_j \tau_{ij} - \underbrace{\partial_j m_{ij}}_{F_i}$$

Moment tensor represents the stresses responsible for non-elastic processes in the source volume

Moment tensor - generalisation

Moment tensor m_{ij} represents the general point-like source

$$u_i = \int\limits_{-\infty}^{\infty} \mathrm{d}t' \int\limits_{V_o} F_i(\xi, t') G_{ni}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

Taylor expansion:

$$G_{ik}(\xi_n) = G_{ik}(0) + \xi_j \frac{\partial G_{ik}}{\xi_j} + O(\xi^2)$$

Moment tensor - generalisation

$$u_i = \int_{-\infty}^{\infty} \mathrm{d}t' \int_{V_o} \xi_j F_k G_{ik,j}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

null net forces:

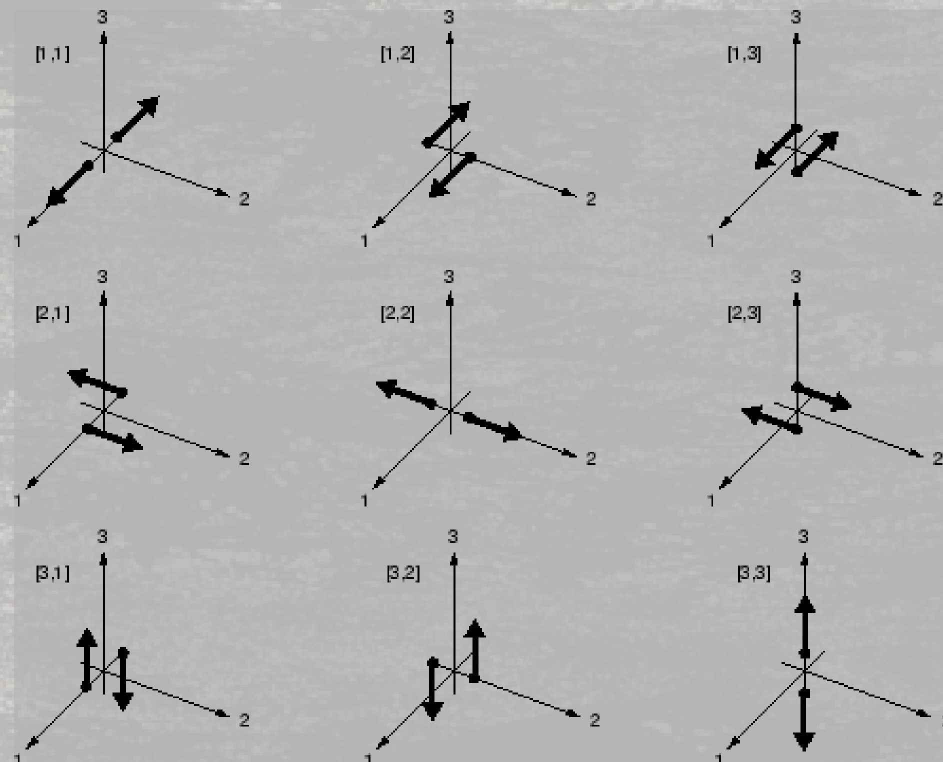
$$\int_{V_o} F_k(\xi) G_{ik}(0, t'; x_s, t) \mathrm{d}V_\xi = 0$$

$$u_i = \int_{-\infty}^{\infty} \mathrm{d}t' \int_{V_o} \xi_j F_k G_{ik,j}(\xi, t'; x_s, t) \mathrm{d}V_\xi$$

$$\boxed{m_{jk} = \xi_j F_k}$$

Moment tensor - physical interpretation

$$m_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$



Moment tensor - principle stresses

m_{ij} real, symmetric - can be diagonalized

$$(m_{ij} - \delta_{ij}\sigma) \cdot \vec{\mathbf{v}}_j = 0$$

$\vec{\mathbf{v}}_i$ - principle axes of inelastic stress in source volume

$$m_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \qquad \sigma_1 > \sigma_2 > \sigma_3$$

Moment tensor - isotropic part

Isotropic part - inelastic dilatation - change of source volume

$$m_{ij}^{I} = \frac{1}{3} Tr(m_{ij})\delta_{ij}$$

$$m_{ij}^{I} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Deviatoric part - processes with no volume change, e.g. shearing

$$m_{ij}^{D} = m_{ij} - m_{ij}^{I}$$

Moment tensor - deviatoric part

$$m_{ij}^{D} = m_{ij}^{DC} + m_{ij}^{CL}$$

where

$$\det(m_{ij}^{DC}) = 0$$

Double Couple part (pure shearing)

$$m_{ij}^{DC} = m_{ij}^{D} - m_{ij}^{CL}$$

Compensated Linear Vector Dipol (e.g. tensil crack)

$$\mathbf{m} = \mathbf{m}^{ISO} + \mathbf{m}^{DC} + \mathbf{m}^{CL}$$

DC - CLVD partitionig

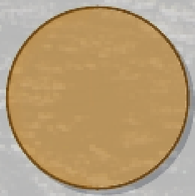
$$\mathbf{m}^{D} \qquad = \qquad \mathbf{m}^{DC} \qquad + \qquad \mathbf{m}^{CL}$$

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\sigma_1 - \sigma_3) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(\sigma_1 - \sigma_3) \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}\sigma_2 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & -\frac{1}{2}\sigma_2 \end{pmatrix}$$

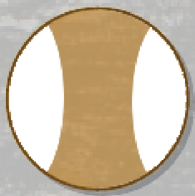
- DC - shearing process
- CLVD - change in μ perpendicular to rupture plane (crack opening, etc.)

Moment tensor - examples

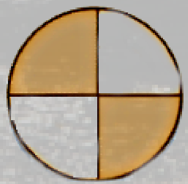
Explosive (ISO) $$m_{ij} = (\lambda + \frac{2}{3}\mu)\Delta u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Tensil fracture (CLVD) $$m_{ij} = A(\lambda, \mu)\Delta u \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Shear fracture (DC) $$m_{ij} = \mu\Delta u \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Moment tensor inversion

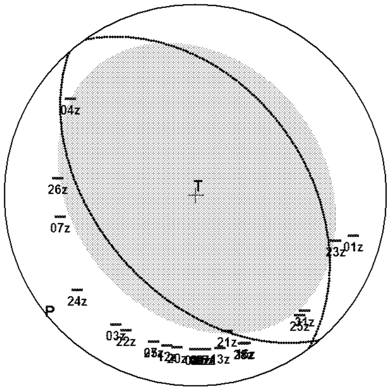
$$m_{ij}(t) \; = \; \gamma_{ij} M_o S(t)$$

Moment tensor inversion should comprise

➡	Nm=6	*“Fault plane solution”*	$\gamma_{ij}^{DC} = e_i^s e_j^n + e_j^s e_j^n$
➡	Nm=1	Seismic scalar moment	$M_o = \mu S \Delta u_o$
➡	Nm=(∞)	Source time function	$S(t) = \Delta \dot{u} / u_o$

$$u_n(x_r, t) = \int_{-\infty}^{\infty} m_{ij}(t') \, G_{ni,j}(x_s, t'; x_r, t) \mathrm{d}t'$$

Moment tensor inversion - examples



Full solution

M_{11} = **-1,12E+13** [Nm] M_{12} = **-7,81E+12** [Nm] M_{13} = **+4,04E+12** [Nm]
M_{21} = **-7,81E+12** [Nm] M_{22} = **-1,50E+13** [Nm] M_{23} = **+1,37E+12** [Nm]
M_{31} = **+4,04E+12** [Nm] M_{32} = **+1,37E+12** [Nm] M_{33} = **+4,54E+13** [Nm]

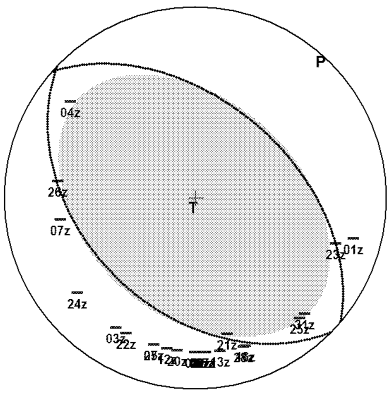
Rupture time: 50,7[ms]
Seismic moment: 3,59E+13[Nm]
Total seismic moment: 4,06E+13[Nm]
Error: 6,77E+11[Nm]
Seismic moment magnitude: 3,0

Decomposition:
EXPL: 18,8 CLVD: 33,9 DBCP: 47,4

Nodal planes:
1: 139,0/48,1 2: 324,2/42,0

Axes (trend/plunge):
P: 231/3 T: 11/86 B: 141/3

Quality factor: 68
Classification: Reverse fault



Trace-null solution

M_{11} = **-1,41E+13** [Nm] M_{12} = **-1,00E+13** [Nm] M_{13} = **-3,05E+12** [Nm]
M_{21} = **-1,00E+13** [Nm] M_{22} = **-1,24E+13** [Nm] M_{23} = **-1,63E+12** [Nm]
M_{31} = **-3,05E+12** [Nm] M_{32} = **-1,63E+12** [Nm] M_{33} = **+2,65E+13** [Nm]

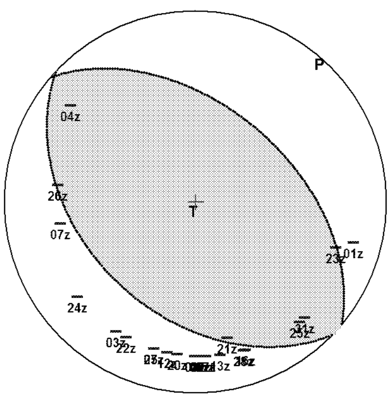
Rupture time: 50,7[ms]
Seismic moment: 2,35E+13[Nm]
Total seismic moment: 2,53E+13[Nm]
Error: 7,07E+11[Nm]
Seismic moment magnitude: 2,9

Decomposition:
CLVD: 13,5 DBCP: 86,5

Nodal planes:
1: 310,9/48,9 2: 134,2/41,2

Axes (trend/plunge):
P: 42/4 T: 199/86 B: 312/2

Quality factor: 72
Classification: Reverse fault



Double couple solution

M_{11} = **-1,29E+13** [Nm] M_{12} = **-1,16E+13** [Nm] M_{13} = **-2,78E+12** [Nm]
M_{21} = **-1,16E+13** [Nm] M_{22} = **-1,04E+13** [Nm] M_{23} = **-1,59E+12** [Nm]
M_{31} = **-2,78E+12** [Nm] M_{32} = **-1,59E+12** [Nm] M_{33} = **+2,33E+13** [Nm]

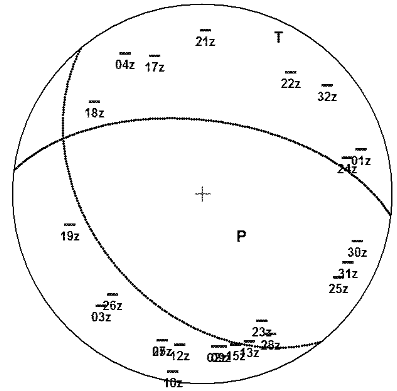
Rupture time: 50,7[ms]
Seismic moment: 2,35E+13[Nm]
Total seismic moment: 2,35E+13[Nm]
Error: 7,50E+11[Nm]
Seismic moment magnitude: 2,9

Decomposition:
DBCP: 100,0

Nodal planes:
1: 310,4/48,9 2: 133,7/41,2

Axes (trend/plunge):
P: 42/4 T: 199/86 B: 312/2

Quality factor: 54
Classification: Reverse fault



Full solution

M_{11} = **-1,30E+12** [Nm] M_{12} = **+1,08E+12** [Nm] M_{13} = **+1,95E+12** [Nm]
M_{21} = **+1,08E+12** [Nm] M_{22} = **-2,07E+12** [Nm] M_{23} = **-1,53E+12** [Nm]
M_{31} = **+1,95E+12** [Nm] M_{32} = **-1,53E+12** [Nm] M_{33} = **-6,98E+12** [Nm]

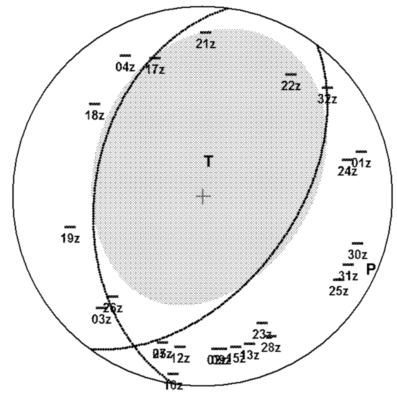
Rupture time: 39,7[ms]
Seismic moment: 5,89E+12[Nm]
Total seismic moment: 7,70E+12[Nm]
Error: 5,14E+11[Nm]
Seismic moment magnitude: 2,5

Decomposition:
EXPL: -30,3 CLVD: -14,4 DBCP: 55,3

Nodal planes:
1: 276,9/58,2 2: 140,7/40,6

Axes (trend/plunge):
P: 137/65 T: 26/9 B: 292/23

Quality factor: 65
Classification: Normal fault



Trace-null solution

M_{11} = **-1,55E+12** [Nm] M_{12} = **+1,52E+12** [Nm] M_{13} = **+2,21E+12** [Nm]
M_{21} = **+1,52E+12** [Nm] M_{22} = **-4,81E+12** [Nm] M_{23} = **+3,07E+10** [Nm]
M_{31} = **+2,21E+12** [Nm] M_{32} = **+3,07E+10** [Nm] M_{33} = **+6,36E+12** [Nm]

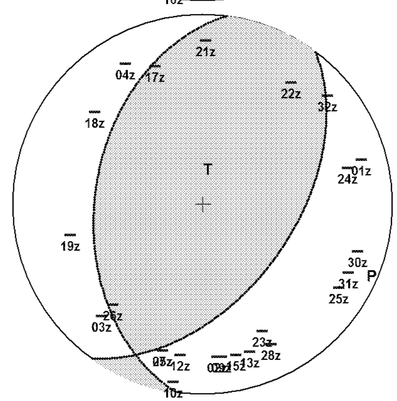
Rupture time: 39,7[ms]
Seismic moment: 5,46E+12[Nm]
Total seismic moment: 6,34E+12[Nm]
Error: 6,09E+11[Nm]
Seismic moment magnitude: 2,5

Decomposition:
CLVD: 27,2 DBCP: 72,8

Nodal planes:
1: 36,7/50,8 2: 188,3/42,8

Axes (trend/plunge):
P: 113/4 T: 8/75 B: 204/14

Quality factor: 67
Classification: Reverse fault



Double couple solution

M_{11} = **-4,55E+11** [Nm] M_{12} = **+1,99E+12** [Nm] M_{13} = **+1,52E+12** [Nm]
M_{21} = **+1,99E+12** [Nm] M_{22} = **-4,61E+12** [Nm] M_{23} = **-1,85E+11** [Nm]
M_{31} = **+1,52E+12** [Nm] M_{32} = **-1,85E+11** [Nm] M_{33} = **+5,07E+12** [Nm]

Rupture time: 39,7[ms]
Seismic moment: 5,46E+12[Nm]
Total seismic moment: 5,46E+12[Nm]
Error: 7,22E+11[Nm]
Seismic moment magnitude: 2,5

Decomposition:
DBCP: 100,0

Nodal planes:
1: 36,1/50,8 2: 187,7/42,8

Axes (trend/plunge):
P: 113/4 T: 7/75 B: 204/14

Quality factor: 49
Classification: Reverse fault

END