An introduction to Physics of Seismic Sources

SP-4: Equivalent source model.

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Forward and inverse source problems

$$u_n(\mathbf{x},t) = \int_T dt \int_{V_o} f_i\, G_{ni} dV' + \int_T dt \int_\Sigma C_{ijkl}\, \Delta u_i\, G_{nk,l} n_j dS' + \int_T dt' \int_\Sigma G_{ni}\, T_i dS'$$

Two different way of using the equation

- Forward problem - calculation of seismogram (displacement field) for assumed fault condition
- Inverse problem - estimation source parameters (fault condition) from observed seismograms. Having estimated $(\Delta u_i,\ T_i)$ over Σ one can examine different physical models of source processes.

Equivalent forces

1. $$u_n(\mathbf{x},t) = \int_T dt \int_{V_o} F_i\, G_{ni} dV'$$

2. $$u_n(\mathbf{x},t) = \int_T dt \int_\Sigma C_{ijkl}\, \Delta u_i\, G_{nk,l} n_j dS'$$

3. $$u_n(\mathbf{x},t) = \int_T dt' \int_\Sigma G_{ni}\, T_i dS'$$

Point source approximation

$$u_n(\mathbf{x}, t) = \int_T dt \int_{V_o} F_i \, G_{ni} \, dV'$$

where F_i are designed to reproduce the observed seismograms.

Let assume that V_o is small:

$$V_o \sim L^3 \qquad (L^2 \times dh)$$

$$L << r_o \qquad (L << \lambda)$$

Point source approximation

$$F_i(t) = \lim_{V_o \to 0} \int_{V_o} F_i(\mathbf{x}', t) dV'$$

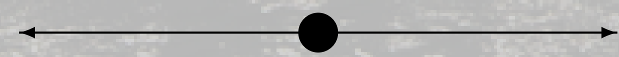
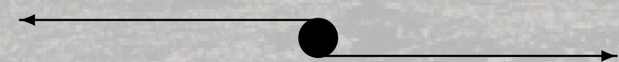
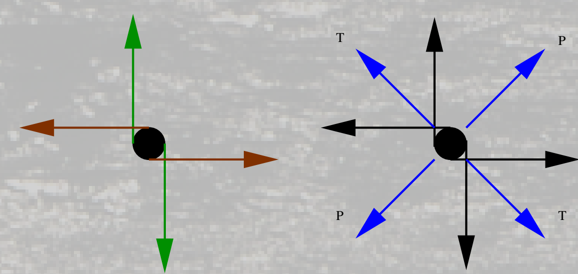
$$u_i(\mathbf{x}, t) = \int_{-\infty}^{\infty} G_{ik}(\mathbf{x}_s - \mathbf{x}, t - t') \, F_k(t') \, dt'$$

$F_i(t)$ - equivalent (virtual) single force

Equivalent forces: basic facts

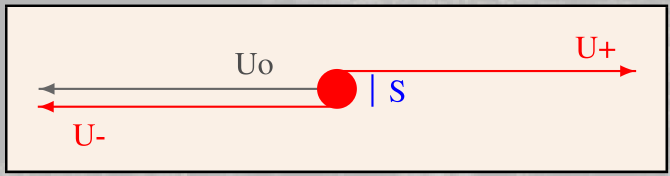
- total forces vanish (no fault displacement)
- total force momentum - vanish (no fault rotation)
- finite duration
- natural earthquakes: fault shearing process
- mining induced events - change of source volume
- fault branching

Force systems

single force	SF	
linear dipol	LD	
single couple	SC	
double couple (shear fracture)	DC	
compensated LD	CLVD	

⋮

Displacement due to a force couple



$$u_i^o(\mathbf{x},t) = \int_{-\infty}^{\infty} G_{ik}(\mathbf{x}_s - \mathbf{x}, t-t')\; F_k(t')\,dt'$$

$$u_i^+ = u_i^o + \frac{s}{2}\frac{\partial u_i^o}{\partial \mathbf{x}_2} \qquad u_i^- = -u_i^o + \frac{s}{2}\frac{\partial u_i^o}{\partial \mathbf{x}_2}$$

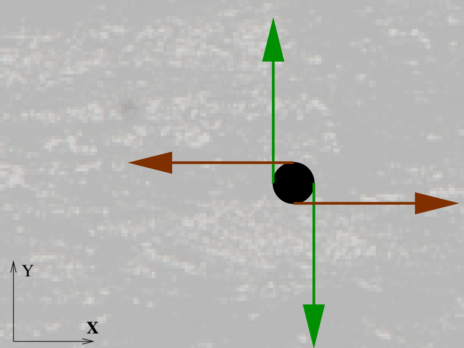
$$u_i^{SC} = u_i^+ + u_i^- \;=\; s\,\frac{\partial u_i^o}{\partial \mathbf{x}_2}$$

Displacement due to a force couple (SC, DC)

$$u_i^{SC}(\mathbf{x}, t) = \int_{-\infty}^{\infty} \frac{\partial}{\partial \mathbf{x}_2} G_{i1}(\mathbf{x}_s - \mathbf{x}, t - t') \underbrace{s\, F_1(t')}_{M(t')} \; dt'$$

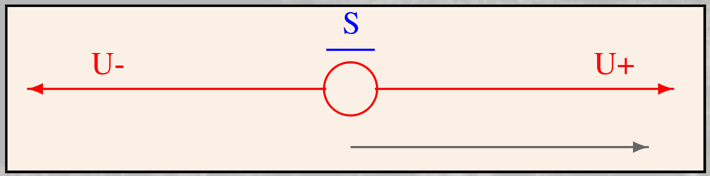
where $M(t)$ is time dependent force moment [N m]

Similar for double-couple (DC)



$$u_i^{DC}(\mathbf{x}, t) = \int_{-\infty}^{\infty} M(t')(e_k e_j + e_j e_k) \frac{\partial G_{ik}(t - t')}{\partial x_j^s} dt'$$

Displacement due to a linear dipole (LD)

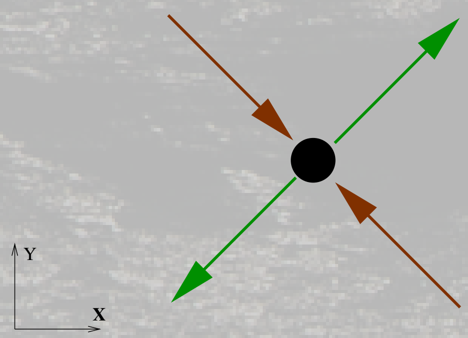


$$u_i^o(\mathbf{x},t) = \int_{-\infty}^{\infty} G_{ik}(\mathbf{x}_s - \mathbf{x}, t - t')\ F_k(t')\, dt'$$

$$u_i^+ = u_i^o + \frac{s}{2}\frac{\partial u_i^o}{\partial \mathbf{x}_1} \qquad u_i^- = -u_i^o + \frac{s}{2}\frac{\partial u_i^o}{\partial \mathbf{x}_1}$$

$$u_i^{LD} = u_i^+ + u_i^- \; = \; s\,\frac{\partial u_i^o}{\partial \mathbf{x}_1}$$

Displacement due to a 2 perpendicular linear dipoles



P - pressure direction

T - Tension axis

$$u_i^{TP}(\mathbf{x},t) = \int_{-\infty}^{\infty} M(t')(T_k T_j + P_j P_k)\frac{\partial G_{ik}(t-t')}{\partial x_j}dt'$$

$$\mathbf{P} = \frac{1}{\sqrt{2}}(\mathbf{X} - \mathbf{Y})$$

$$\mathbf{T} = \frac{1}{\sqrt{2}}(\mathbf{X} + \mathbf{Y})$$

$$\mathbf{Z} = \mathbf{X} \times \mathbf{Y} = \mathbf{P} \times \mathbf{T}$$

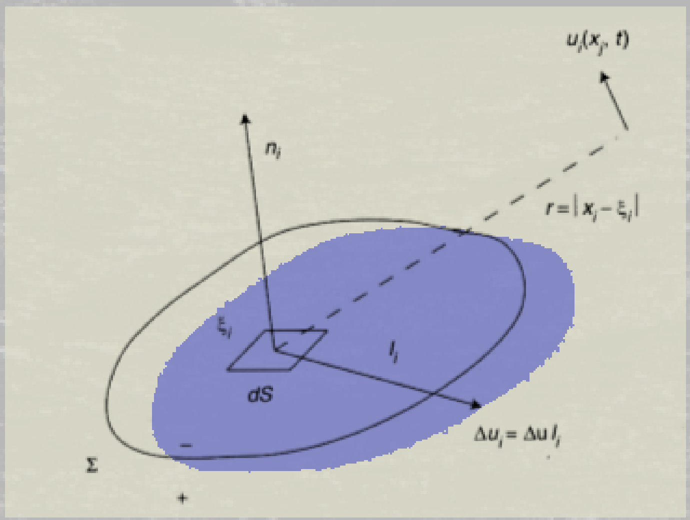
Dislocation source model

1. $$u_n(\mathbf{x},t) = \int_T dt \int_{V_o} F_i\, G_{ni} dV'$$

2. $$u_n(\mathbf{x},t) = \int_T dt \int_{\Sigma} C_{ijkl}\, \Delta u_i\, G_{nk,l}\, n_j dS'$$

3. $$u_n(\mathbf{x},t) = \int_T dt' \int_{\Sigma} G_{ni}\, T_i dS'$$

Dislocation source



$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\mathbf{\Delta u} \;=\; \Delta u\,\mathbf{l}$$

$$\mathbf{n} \perp \Sigma$$

$$u_n(\mathbf{x}, t) = \int_T dt' S \Delta u \; [\lambda \mathbf{l} \cdot \mathbf{n} \delta_{ij} + \mu(l_i n_j + l_j n_i) G_{ni,j})]$$

Shear fracture dislocation

If $\mathbf{l} \cdot \mathbf{n} = 0$ - pure shearing

$$u_n(\mathbf{x}, t) = \int_T dt' \underbrace{\mu S \Delta u(t')}_{M_o(t')} (l_i n_j + l_j n_i) G_{ni,j}(t - t')$$

otherwise the process with change of source volume (contribution from terms $i = j$)

Shear fracture vs. double couple (DC)

$$u_i^{\Delta}(\mathbf{x},t) = \int\limits_{-\infty}^{\infty} M_o(t')(l_k n_j + l_j n_k) G_{ik,j}(t-t') dt'$$

$$u_i^{DC}(\mathbf{x},t) = \int\limits_{-\infty}^{\infty} M(t')(X_k Y_j + X_j Y_k) G_{ik,j}(t-t') dt'$$

$$u_i^{\Delta}(\mathbf{x},t) \equiv u_i^{DC}(\mathbf{x},t)$$

provided

$$M(t) = sF(t) = M_o(t) \quad \mathbf{l} = \mathbf{X}, \quad \mathbf{n} = \mathbf{Y}$$

Shear fracture source in an infinite medium

Reminder: the Green function:

$$G_{ij} = \frac{1}{4\pi\rho\, r^3}(3\gamma_i\gamma_j - \delta_{ij}) \int_{r/\alpha}^{r/\beta} \tau\delta(t-\tau)d\tau$$

$$+ \frac{1}{\alpha^2 r}\gamma_i\gamma_j\delta(t - r/\alpha)$$

$$- \frac{1}{\beta^2 r}(\gamma_i\gamma_j - \delta_{ij})\delta(t - r/\beta)$$

Notation: $r = ||\mathbf{x}|| = \sqrt{\sum_i x_i^2}$

$$\gamma_i = \frac{\partial r}{\partial x_i} = \frac{x_i}{r} \quad \frac{\partial \gamma_i}{\partial x_j} = -\frac{1}{r}(\gamma_i\gamma_j - \delta_{ij}) \quad \frac{\partial}{\partial x_j}\left(\frac{1}{r}\right) = -\frac{\gamma_i}{r^2}$$

Shear fracture source in an infinite medium

$$G_{ij,k} = \frac{1}{4\pi\rho}\Bigg\{\frac{1}{r^4}\Big(15\gamma_i\gamma_j\gamma_k - 3[\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + \gamma_k\delta_{ij}]\Big)\int_{r/\alpha}^{r/\beta}\tau\delta(t-\tau)d\tau$$

$$+ \frac{1}{r^2\alpha^2}[6\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - \gamma_k\delta_{ij}]\,\delta(t - r/\alpha)$$

$$- \frac{1}{r^2\beta^2}[6\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - \gamma_k\delta_{ij}]\,\delta(t - r/\beta)$$

$$+ \frac{1}{r}\left[\frac{1}{\alpha^3}\gamma_i\gamma_j\gamma_k\,\dot{\delta}(t - r/\alpha) - \frac{1}{\beta^3}(\gamma_i\gamma_j - \delta_{ij})\gamma_k\,\dot{\delta}(t - r/\beta)\right]\Bigg\}$$

for shearing process

$$\mathbf{n}\cdot\mathbf{l} = n_i l_k \delta_{ik} = n_i l_i = 0$$

Displacement due to DC force

$$u_i^{DC}(\mathbf{x},t) = \int_{-\infty}^{\infty} M(t')(n_k l_j + n_j l_k) G_{ik,j}(t-t')dt'$$

$$\begin{aligned} u_i \; = \; & \frac{1}{4\pi\rho} \times \\ & \Big\{ \frac{1}{r^4} \Big(30\gamma_i\gamma_j\gamma_k l_j n_k - 6[\gamma_j l_j n_i + \gamma_k n_k l_i] \Big) \int_{r/\alpha}^{r/\beta} \tau \mathbf{M}_o(t-\tau) d\tau \\ & + \frac{1}{r^2\alpha^2} \Big[12\gamma_i\gamma_j\gamma_k l_j n_k - 2(\gamma_j l_j n_i - \gamma_k n_k l_i) \Big] M_o(t - r/\alpha) \\ & - \frac{1}{r^2\beta^2} \Big[12\gamma_i\gamma_j\gamma_k l_j n_k - 2(\gamma_j l_j n_i - \gamma_k n_k l_i) \Big] M_o(t - r/\beta) \\ & + \frac{1}{r\alpha^3} \, 2\gamma_i\gamma_j\gamma_k l_j n_k \, \dot{M}_o(t - r/\alpha) \\ & - \frac{1}{r\beta^3} \, (2\gamma_i\gamma_j\gamma_k l_j n_k - n_i\gamma_j l_j - n_k\gamma_k l_i) \, \dot{M}_o(t - r/\beta) \Big\} \end{aligned}$$

DC force - far field

$$u_i^P = \frac{1}{4\pi\rho r}\left[\frac{2}{\alpha^3}\gamma_i(\gamma_j l_j)(\gamma_k n_k)\,\dot{M}_o(t-r/\alpha)\right]$$

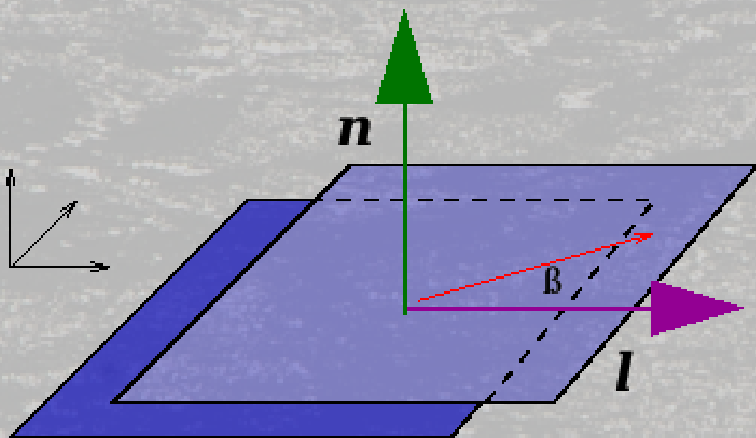
$$u_i^S = \frac{1}{4\pi\rho r}\left[\frac{-1}{\beta^3}\Big((2\gamma_i(\gamma_j l_j)(\gamma_k n_k) - n_i(\gamma_j l_j) - (n_k\gamma_k)l_i\Big)\,\dot{M}_o(t-r/\beta)\right]$$

or

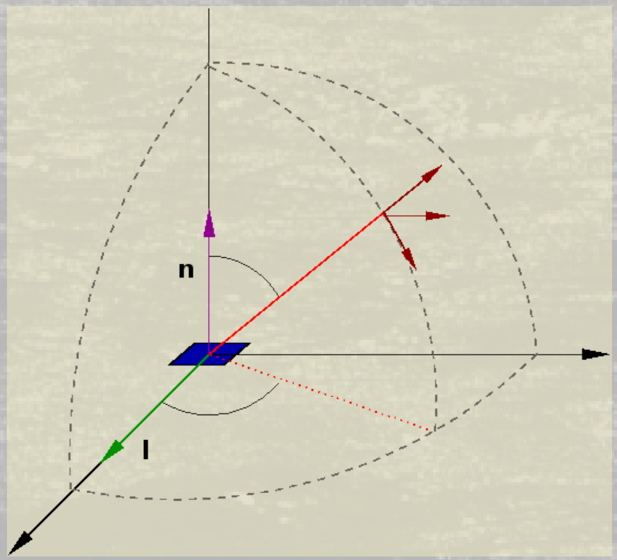
$$u_i^P = \frac{1}{4\pi\rho\alpha^3 r}(T_j T_k - P_j P_k)\,\gamma_i\gamma_j\gamma_k\,\dot{M}_o(t-r/\alpha)$$
$$u_i^S = \frac{1}{4\pi\rho\beta^3 r}(T_j T_k - P_j P_k)\gamma_k(\delta_{ij} - \gamma_i\gamma_j)\,\dot{M}_o(t-r/\beta)$$

Far field radiation pattern

Radiation pattern: dependence of amplitude of displacement field on observational direction for fixed source-receiver distance.



$$\mathbf{n} = (0,\ 0,\ 1)$$
$$\mathbf{l} = (1,\ 0,\ 0)$$



P wave far field radiation pattern

$$u_x = K \sin(2\theta)\sin(\theta)\cos^2(\phi)$$

$$u_y = K/2 \sin(2\theta)\sin(\theta)\sin(2\phi)$$

$$u_z = K \sin(2\theta)\cos(\theta)\cos(\phi)$$

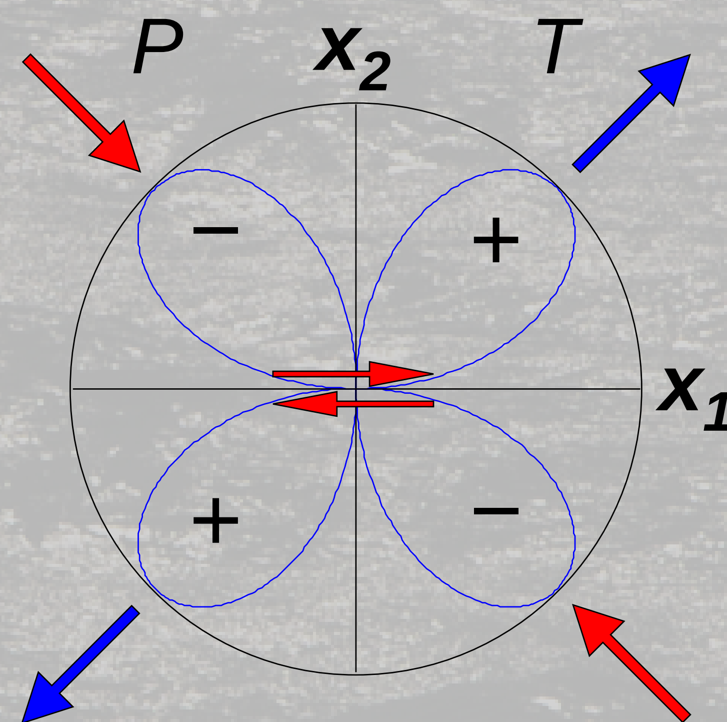
$$u_r = K \sin(2\theta)\cos(\phi)$$

$$u_\theta = 0$$

$$u_\phi = 0$$

where $K = \frac{\dot{M}_o(t-r/\alpha)}{4\pi\rho\alpha^3 r}$

$u_r(\theta)_{|\phi=0}$

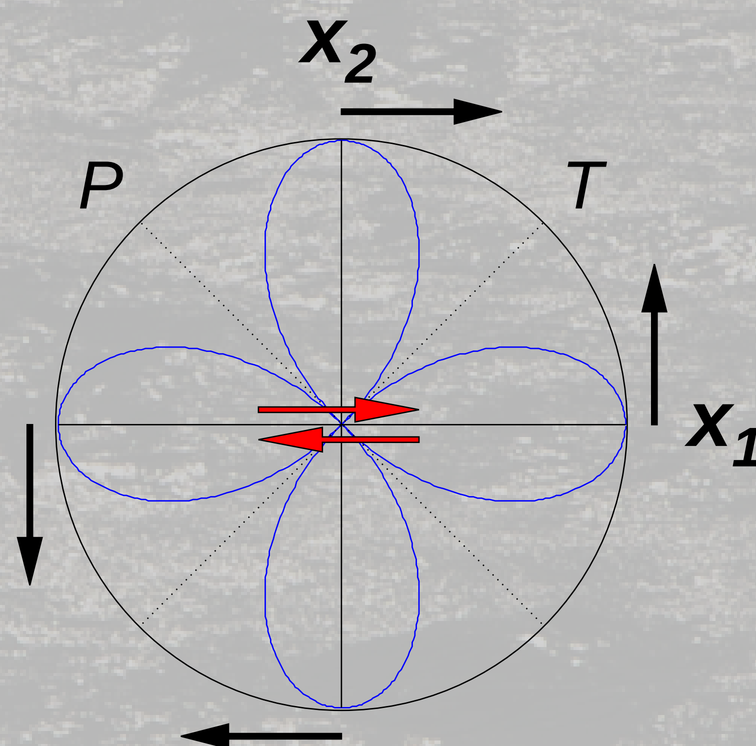


S wave far field radiation pattern

$$u_x = -K' \cos(\theta) \left[\cos(2\theta) - \sin^2(\theta)\sin^2(\phi)\right]$$

$$u_y = K' \cos(\theta)\sin^2(\theta)\sin(2\phi)$$

$$u_z = K' \cos(2\theta)\sin(\theta)\cos(\phi)$$



$$u_r = 0$$

$$u_\theta = K' \cos(2\theta)\cos(\phi)$$

$$u_\phi = -K' \cos(\theta)\sin(\phi)$$

where $K' = \frac{\dot{M}_o(t-r/\beta)}{4\pi\rho\beta^3 r}$

Near field radiation pattern

the $1/r^4$ term

$$\frac{1}{4\pi\rho r^4}\Big[30\gamma_i\gamma_j\gamma_k l_j n_k - 6(\gamma_i\delta_{jk}+\gamma_j\delta_{ik}+\gamma_k\delta_{ij})\Big]\int_{r/\alpha}^{r/\beta}\tau\mathbf{M}_o(t-\tau)d\tau$$

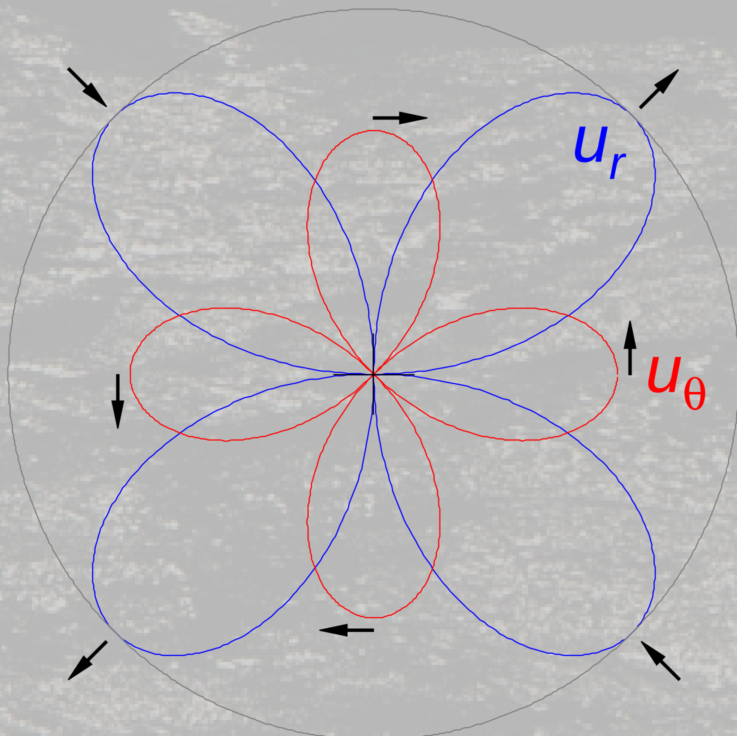
for the same fault geometry ($\mathbf{l}\cdot\mathbf{n}=0, \quad \mathbf{l}=(1,0,0)$)

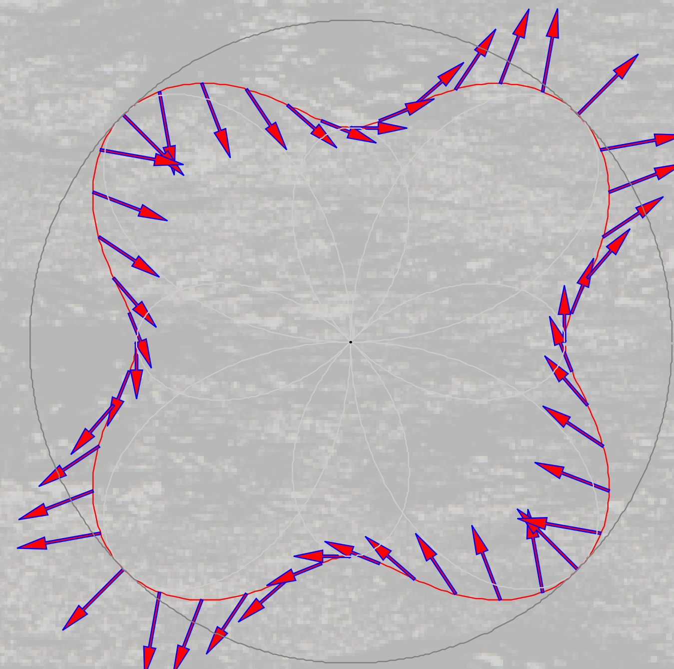
$$u_x = 6A\cos(\theta)\left(5\sin^2(\theta)\cos^2(\phi)-1\right) \qquad \textit{for}\ \ \phi = 0$$

$$u_y = 15/2A\sin(\theta)\sin(2\theta)\sin(2\phi) \qquad u_r = 9A\sin(2\theta)$$

$$u_z = 6A\sin(\theta)\cos(\phi)\left(5\cos^2(\theta)-1\right) \qquad u_\theta = 6A\cos(2\theta)$$

Near field radiation pattern





No separation into fast (P-like) and slow (S-like) phases.

Intermediate field radiation pattern

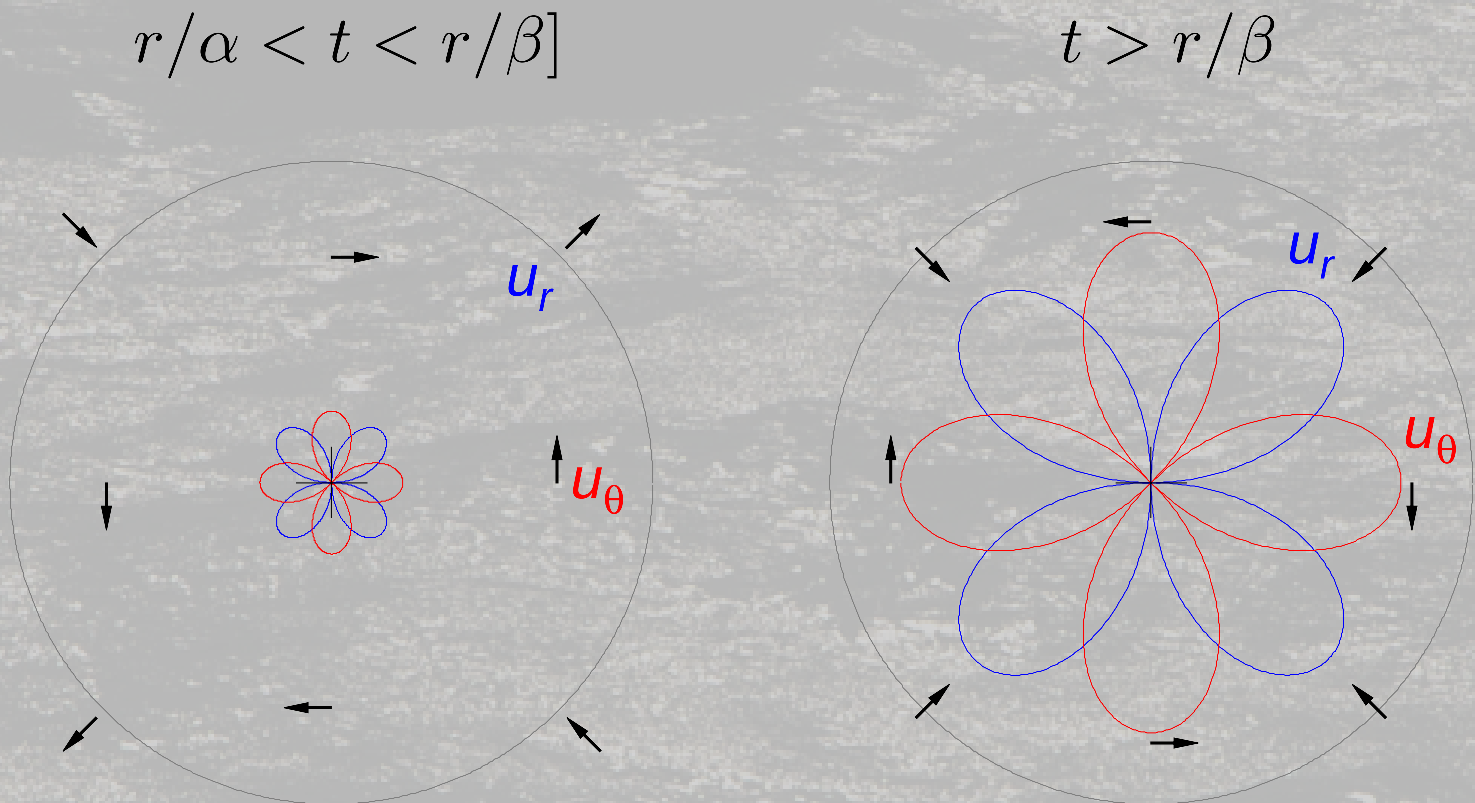
the $1/r^2$ term

$$\frac{1}{r^2\alpha^2}\Big[12\gamma_i\gamma_j\gamma_k l_j n_k - 2(\gamma_j l_j n_i - \gamma_k n_k l_i)\Big] M_o(t - r/\alpha)$$

$$-\frac{1}{r^2\beta^2}\Big[12\gamma_i\gamma_j\gamma_k l_j n_k - 2(\gamma_j l_j n_i - \gamma_k n_k l_i)\Big] M_o(t - r/\beta)$$

$t \in [r/\alpha,\ \ r/\beta]$	$t > r/\beta$
$u_r = 2K\sin(2\theta)$	$-3K'\sin(2\theta)$
$u_\theta = K\cos(2\theta)$	$-3K'\cos(2\theta)$
$K = \frac{M_o(t-r/\alpha)}{2\pi\rho\alpha^2 r^2}$	$K' = \frac{M_o(t-r/\beta)}{2\pi\rho\beta^2 r^2}$

Intermediate field radiation pattern



Both intermediate displacements: traveling with α and β velocities have both radial and azimuthal components

Source time functions

$$u_i(t) \approx \frac{a}{r^4} \int_{r/\alpha}^{r/\beta} \tau \mathbf{M}_o(t-\tau) d\tau + \frac{b}{r^2} \mathbf{M}_o(t - r/v_i) + \frac{c}{r} \dot{M}_o(t - r/v_i)$$

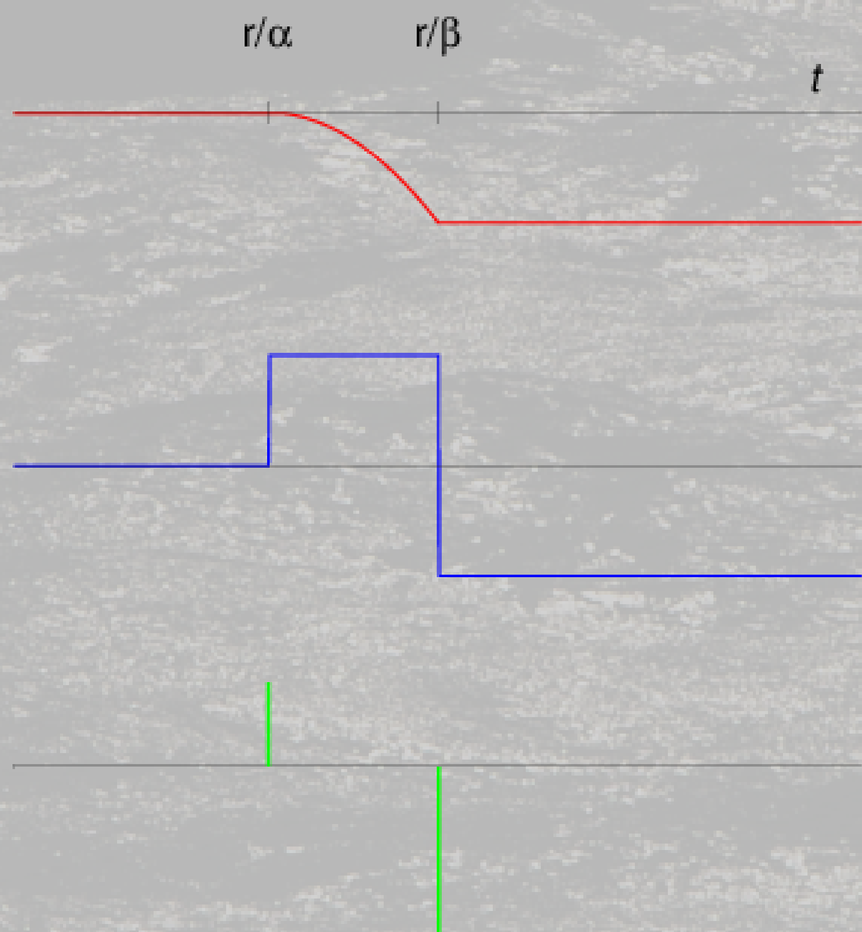
Let us assume $M_o(t) = M_o H(t)$

$$S^N \sim \frac{M_o}{2} \left[\left(t^2 - \frac{r^2}{\beta^2} \right) H(t - r/\beta) - \left(t^2 - \frac{r^2}{\alpha^2} \right) H(t - r/\alpha) \right]$$

$$S^I \sim M_o(t) = M_o \left[\frac{1}{\alpha^2} H(t - r/\alpha) - \frac{1}{\beta^2} H(t - r/\beta) \right]$$

$$S^F \sim M_o \left[\frac{1}{\alpha^3} \delta(t - r/\alpha) - \frac{1}{\beta^3} \delta(t - r/\beta) \right]$$

Source time functions



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