An introduction to Physics of Seismic Sources

SP-13: Dynamic models - fracture mechanics

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Static crack

Dynamic source models:

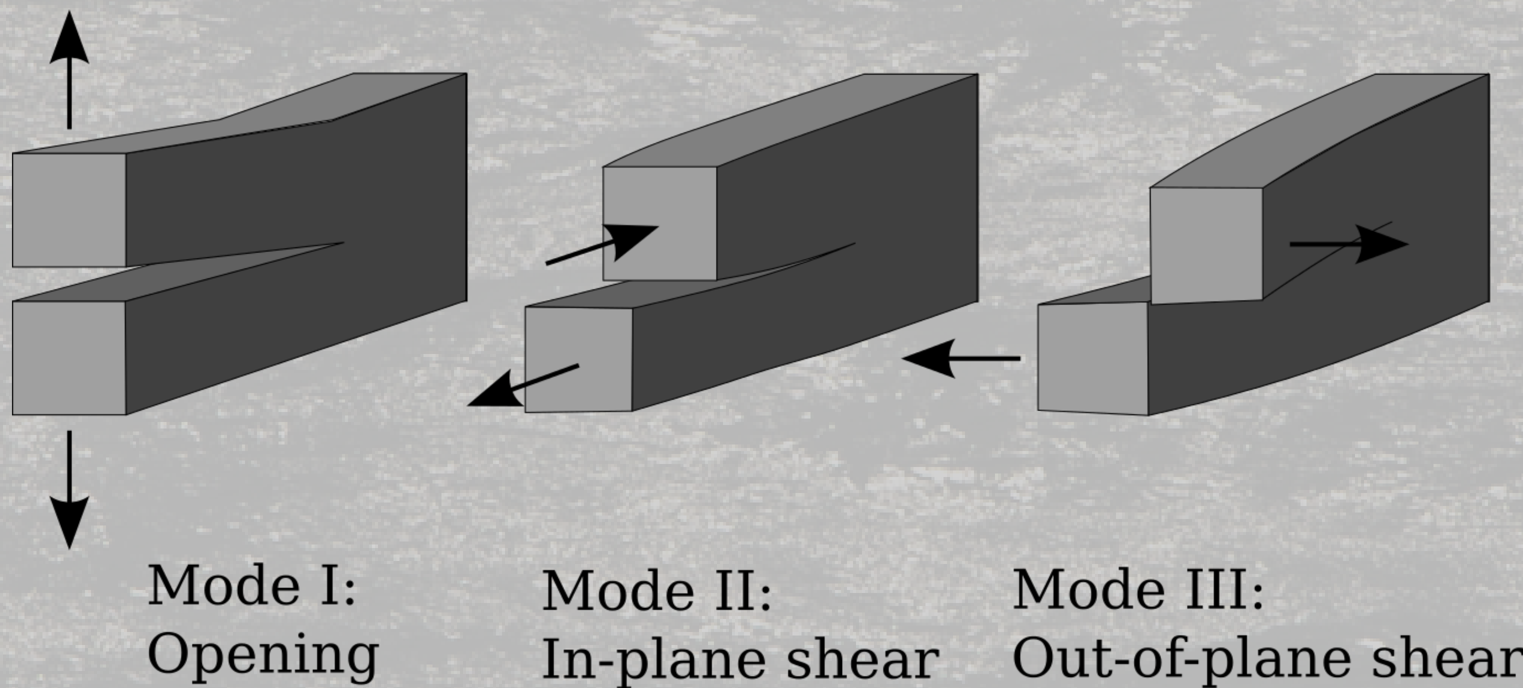
stress condition $\Longrightarrow$ slip on the fault

Brune model - the first “dynamic” source model had been build on *ad hoch* assumptions in a similar as kinematic models was proposed. Systematic approach to construct dynamic models has been based on results of the **(linear) fracture mechanics**

Basic objects of this theory are **fractures** (cracks). Foundations of the theory has been layed out by Griffith and Irvin, in early 30’th (XX).

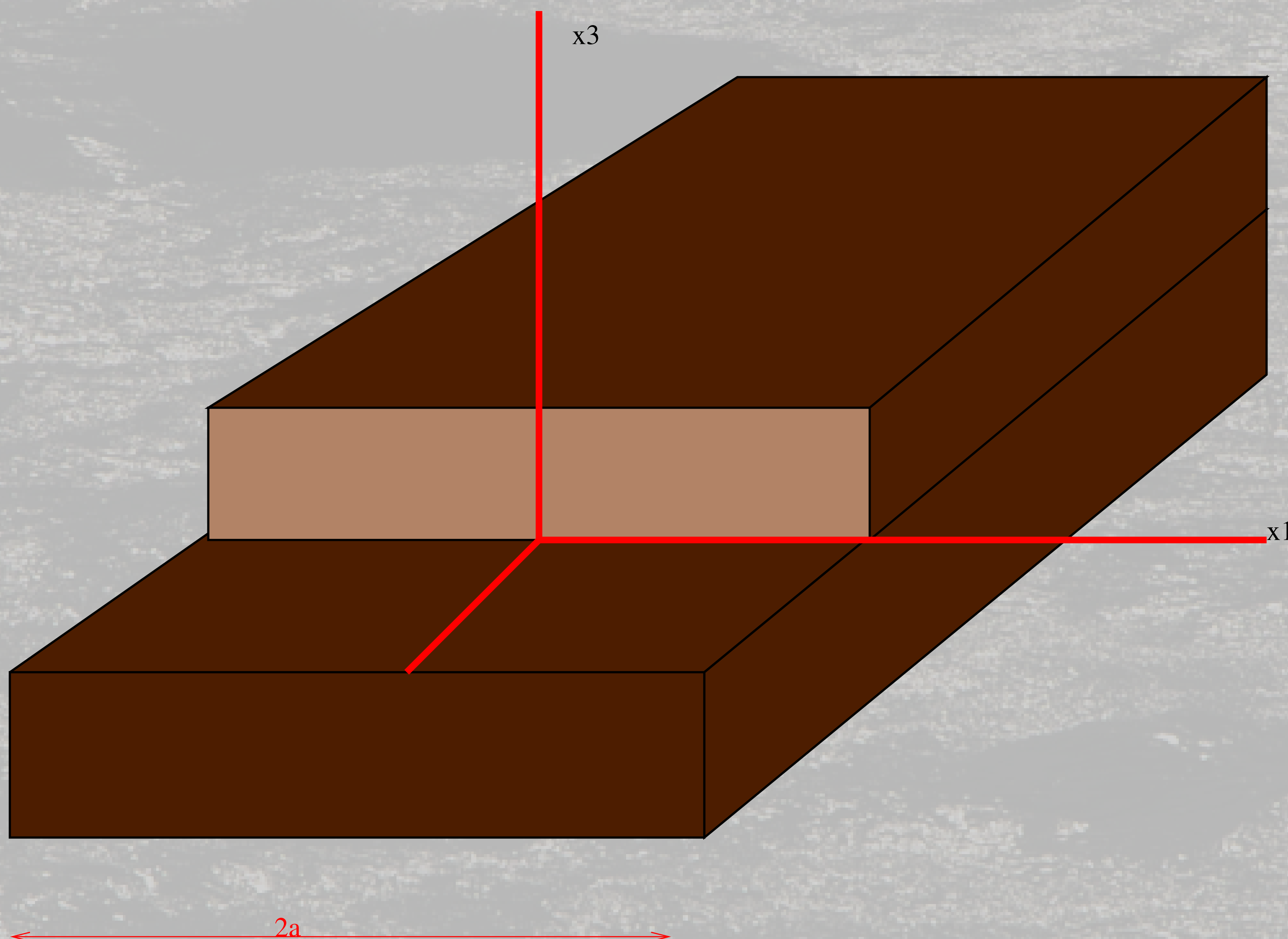
Earthquake $\equiv$ super-crack

Fracture basic types



(after Wikipedia)

Static crack -type II (shear crack)



Static crack - assumptions

- infinite elastic medium
- uniform shear stres in medium $\tau_{32} = (0, \sigma_o, 0)$ befor crack formation
- crack of width $2a$ in $x_3 = 0$ plane with displacement $u = (0, u, 0)$
- stress in the ruptured area $(-a < x_1 < a)$ zero and σ_o outside
- static stress drop inside crack $\Delta\sigma = \sigma_o$, zero outside

Displacement field

What is the displacement field

$$u = u(x_1, x_2, x_3)$$

Equation of motion

$$\partial_t^2 u_i - \partial_j \tau_{ij} = f_i$$

Static crack

$$\frac{\partial \tau_{ij}}{\partial x_j} = 0$$

boundary condition:

$$\tau_{32} = \begin{cases} \sigma_o & \text{for } |x_1| >> a; \\ 0 & \text{for } |x_1| < a \end{cases}$$

Laplace equation

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_3^2} = 0$$

Static crack - solution

$$z = x_1 + ix_2; \qquad f(z) = v(z) + iu(z)$$

$$f(z) = \frac{\sigma_o}{\mu}\sqrt{z^2 - a^2}$$

$$u(z) = \frac{\sigma_o}{\mu} Im \sqrt{z^2 - a^2}$$

$$t_{32} = \mu \frac{\partial u}{\partial x_3} = \sigma_o Re \frac{z}{\sqrt{z^2 - a^2}}$$

Static crack - solution

On the fault ($x_3 = 0$)

$$u(x_1, 0) = \begin{cases} \frac{\sigma_o}{\mu}\sqrt{a^2 - x_1^2} & |x_1| < a \\ 0 & |x_1| > a \end{cases}$$

$$\tau_{32}(x_1, 0) = \begin{cases} 0 & |x_1| < a \\ \frac{\sigma_o x_1}{\sqrt{x_1^2 - a^2}} & |x_1| > a \end{cases}$$

Circular static crack

- circular crack of radius a
- slip only inside the crack: $u(\rho) = 0$ for $\rho > a$
- maximum slip (at center) $u(0) = u_{max}$
- stress far from the fracture (fault) σ_o parallel to x_1
- zero stress inside ($\Delta\sigma = \sigma_o$)

Circular static crack solution

Solution for $\rho < a$ (Poisson ratio 0.25)

$$u(\rho) = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}\sqrt{a^2 - \rho^2} \qquad (\star)$$

static stress drop $\leftrightarrow$ maximum displacement

$$u_{max} = \frac{24}{7\pi}\frac{\Delta\sigma}{\mu}a$$

Circular static crack solution

Stress outside ($\rho > a$) is not axial symmetrical (combinations of crack modes II and III)

$$\sigma_{x_1 x_2}(\rho, \phi) = (K_{II} \cos\phi + K_{III} \sin\phi) \frac{1}{\sqrt{2\pi(\rho - a)}}$$

K_{II}, K_{III} - stress intensity factors

$$K_{II} = \frac{16}{7\sqrt{\pi}} \frac{\Delta\sigma\sqrt{a}}{1-\nu} \qquad K_{III} = \frac{16}{7\sqrt{\pi}} \Delta\sigma\sqrt{a}$$

Circular static crack displacement and M_o

$$\Delta\bar{u} = \frac{1}{S}\int_S \Delta u dS$$

$$\Delta\bar{u} = \frac{48\Delta\sigma}{7\mu S}\int_0^a \rho\sqrt{a^2-\rho^2}d\rho = \frac{16a^3\Delta\sigma}{7\mu S}$$

$$\Delta\bar{u} = \frac{16}{7\pi}\frac{\Delta\sigma}{\mu}a$$

$$M_o = \mu\Delta\bar{u}S = \frac{16}{7}\Delta\sigma a^3$$

Energy balance

Strain energy release due to static slip over the circular crack when the average shear stress on the fault is $\bar{\sigma}$

$$\Delta W = \int_S \bar{\sigma} \Delta u \, \mathrm{d}S$$

disspiates mainly as

$$\Delta F = \int_S \bar{\sigma_f} \Delta u \, \mathrm{d}S \quad \text{frictional energy (heat)}$$

$$\Delta U = \int_S \Delta\sigma \Delta u \, \mathrm{d}S \quad \text{elastic energy (radiation, fracturing)}$$

$$\Delta W = \Delta F + \Delta U$$

Energy balance Brune's model

From $(\star)$

$$\Delta U = \frac{1}{2}\Delta\sigma \times \frac{16\Delta\sigma}{7\pi\mu}a \times \pi a^2 = \frac{8}{7}\frac{\Delta\sigma^2}{\mu}a^3$$

Energy radiated as S-wave to far-field

$$E_s = \frac{1}{16\pi\mu} < R_S^2 > M_o^2 \frac{w_c^3}{\beta^3}$$

$$E_s \approx 0.535\frac{\Delta\sigma^2}{\mu}a^3 = 0.47\Delta U$$

where is remaining energy ???

END

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