# *An introduction to Physics of Seismic Sources*

## *SP-12: Brune model (cd)*

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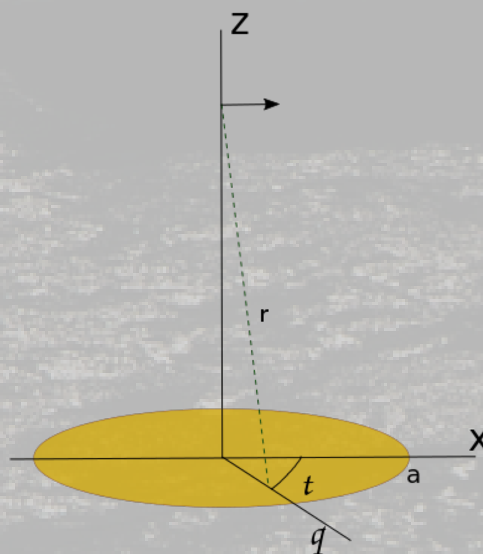
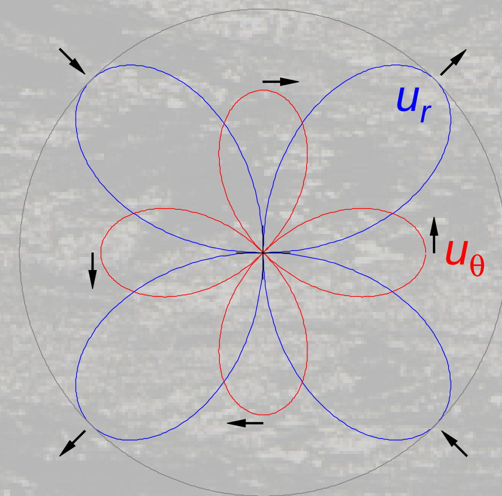
# Brune model - assumptions

- circular model
- finite size source
- sudden shear stress pulse
- stress instanteneous over the whole area
- no fracture propagation (infinite fracture speed)

# Brune model (simplified)




$$\Delta\sigma(z,t) = \Delta\sigma H\left(t - \frac{z}{\beta}\right)$$

This stress drop generates the waves which on the z-axis are pure SH wave propagating perpendicular to the fault

# Brune model - near field

using

$$\sigma_{ij} \sim 1/2(u_{i,j} + u_{j,i}) \Longrightarrow \Delta\sigma = \mu \partial u / \partial z$$

$$u\left(t - \frac{z}{\beta}\right) = \frac{\Delta\sigma}{\mu} \int H\left(t - \frac{z}{\beta}\right) dz = \frac{\Delta\sigma}{\mu} \beta t H\left(t - \frac{z}{\beta}\right)$$

On the fault $(z = 0)$

$$u(t) = \frac{\Delta\sigma}{\mu} \beta\, \mathbf{t}\, H(t)$$

On the fault displacement increases linearly with time until reachs the fault edge

# Brune model - near field

$$[tH(t-z/\beta)]^{(fft)} = i\pi\frac{d}{d\omega}\delta(\omega) - \frac{\beta\Delta\sigma}{\mu\omega^2}$$

$$Re[u(t)] = \frac{\beta\Delta\sigma}{\mu\omega^2}$$

However, the source has a finite dimension ($R = a$) which is ignored.
The finite size effect will be visible only for $t > \tau = a/\beta$ Thus, Brune has proposed make an extension

$$u(t) = \frac{\Delta\sigma}{\mu}\beta\tau\left(1 - e^{-t/\tau}\right)$$

# Brune model - near field spectrum

$$u(t) = \frac{\Delta\sigma}{\mu}\beta \begin{cases} t & \text{for } t < \tau \\ 1 & \text{for } t >> \tau \end{cases}$$

Spectrum

$$|u(\omega)| = \frac{\Delta\sigma}{\mu}\beta \frac{1}{\omega\sqrt{(\omega^2 + \tau^{-2})}}$$

# Brune model - far field

Using the representation integral

$$u^Z(z,t) = A \iint_\Sigma \frac{1}{r} R_S \left(n_k, l_k(\xi_i)\gamma_k(z,x_i)\right) \Delta\dot{u} \left(x_1, x_3, t - \frac{r}{\alpha}\right) dx_1 dx_3$$

$$\dot{u}(t) = \frac{\Delta\sigma}{\mu} \beta e^{-t/\tau}$$

We arrive in

$$u(t) = A' R_S \frac{a}{r} \frac{\Delta\sigma}{\mu} \beta \left(t - \frac{r}{\beta}\right) \exp\left[-\omega_c \left(t - \frac{r}{\beta}\right)\right]$$

# Brune model - far field spectrum

$$|u(\omega)| = A' R_S \frac{a}{r} \frac{\Delta\sigma}{\mu} \beta \frac{1}{\omega^2 + \omega_c^2}$$

Problem: what A and $\omega_c$ are?

Brune has fixed them to obtain low and high frequency energy radiation consistent with kinematic (instanteneous) circular shear crack.

$$A' = \frac{4}{7\pi} \left( \frac{\omega_c a}{\beta} \right)^2$$

$$|u(0)| = \frac{4}{7\pi\beta} \frac{1}{r} R_s \frac{\Delta\sigma}{\mu} a^3 \sim \frac{M_o}{\beta^3}$$

# Brune model - far field spectrum

Next Brune has observed that $A' \approx 1$ so he assumed

$$\omega_c = \sqrt{\frac{7\pi}{2}}\,\frac{\beta}{a} \approx 2.34\frac{\beta}{a}$$

than

$$|u(\omega)| = \frac{1}{4\pi\beta^3}\frac{1}{r}R_s M_o \frac{\omega_c^2}{\omega^2 + \omega_c^2}$$

and source radius

$$a = 2.34\beta/\omega_c$$

# Energy balance

Strain energy release due to static slip over the circular crack when the average shear stress on the fault is $\bar{\sigma}$

$$\Delta W = \int_S \bar{\sigma} \Delta \, u \mathrm{d}S$$

disspiates mainly as

$$\Delta F = \int_S \bar{\sigma_f} \Delta u \, \mathrm{d}S \quad \text{frictional energy (heat)}$$

$$\Delta U = \int_S \Delta\sigma \Delta u \, \mathrm{d}S \quad \text{elastic energy (radiation, fracturing)}$$

$$\Delta W = \Delta F + \Delta U$$

# END



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