An introduction to Physics of Seismic Sources

SP-1: Introduction

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Tentative plan

1. Basic observational facts

   - Earthquakes - basic characteristics, description, evidence - etc.
   - Seismic data processing (elementary)

2. Theory of point sources

   - Wave equation and representation theory
   - Point source representation
   - Seismic moment tensor
   - Seismic spectra and scaling relations
   - Madariaga and Bruno models
   - Extended sources - seismic radius

3. Earthquake and fracture mechanics

- Dynamics of fracturing
- Grifith -Irwin’s theory
- State and rate models
- friction and fracture
- sub- and super-shear fracturing

4. Advanced issues

- Stochasticity of rupturing: Fiber Bundle Model
- Electromagnetic effects accompanying rupture process
- Earthquake interaction

Recomended textbooks:

- A. Udias, R. Madariaga, E. Buforn;
  Source Mechanisms of Earthquakes (Cambridge Univ. Press 2014)

- F.H. Cornet,
  Elements of Crustal Geomechanics,(Cambridge Univ. Press 2015)

- S.J. Gibowicz, A. Kijko;
  An itroduction to mining seismology,(Akademic Press, 1994)

+ research papers mentioning during the course

Seismological inference

Seismology deals with two broad classes o problems:

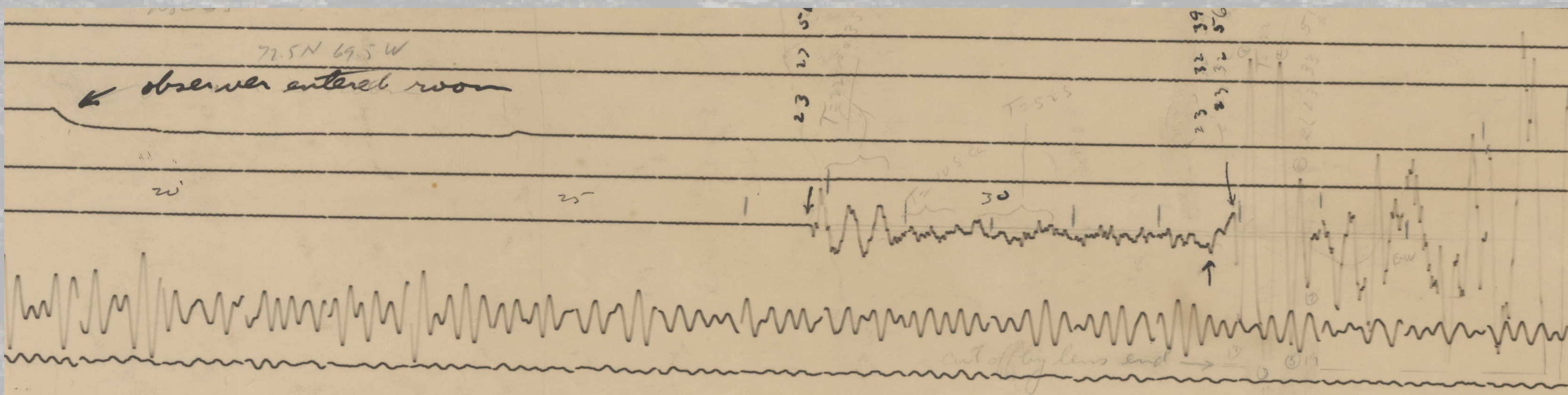
- Structure and composition of the Earth in various scales
  global, regional ($\sim$ 2000 km), local ($\sim$ 100 km), etc.

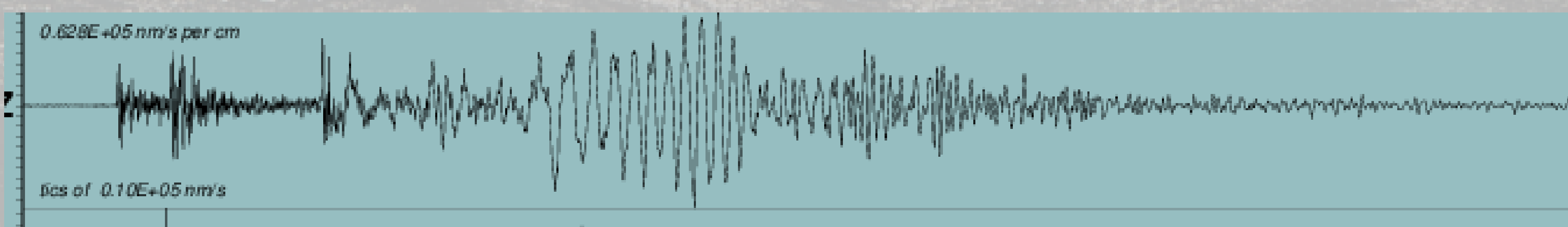
- Earthquakes: their origins, physics, effects and (possible) prediction

Inference is always based on records made on surface (space)
There is no possibility of direct measurements of thought quantities

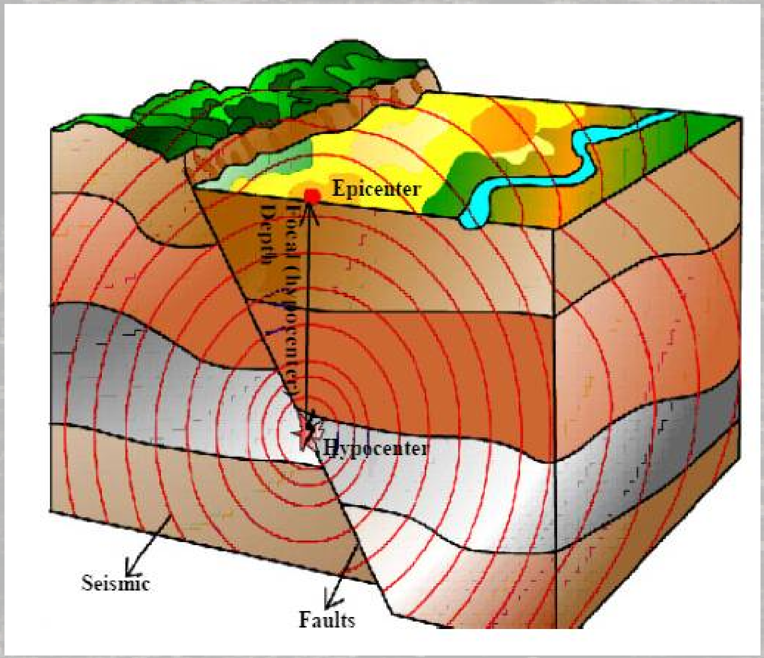
Seismic waves - primary source of information

Seismograms: records of seismic waves



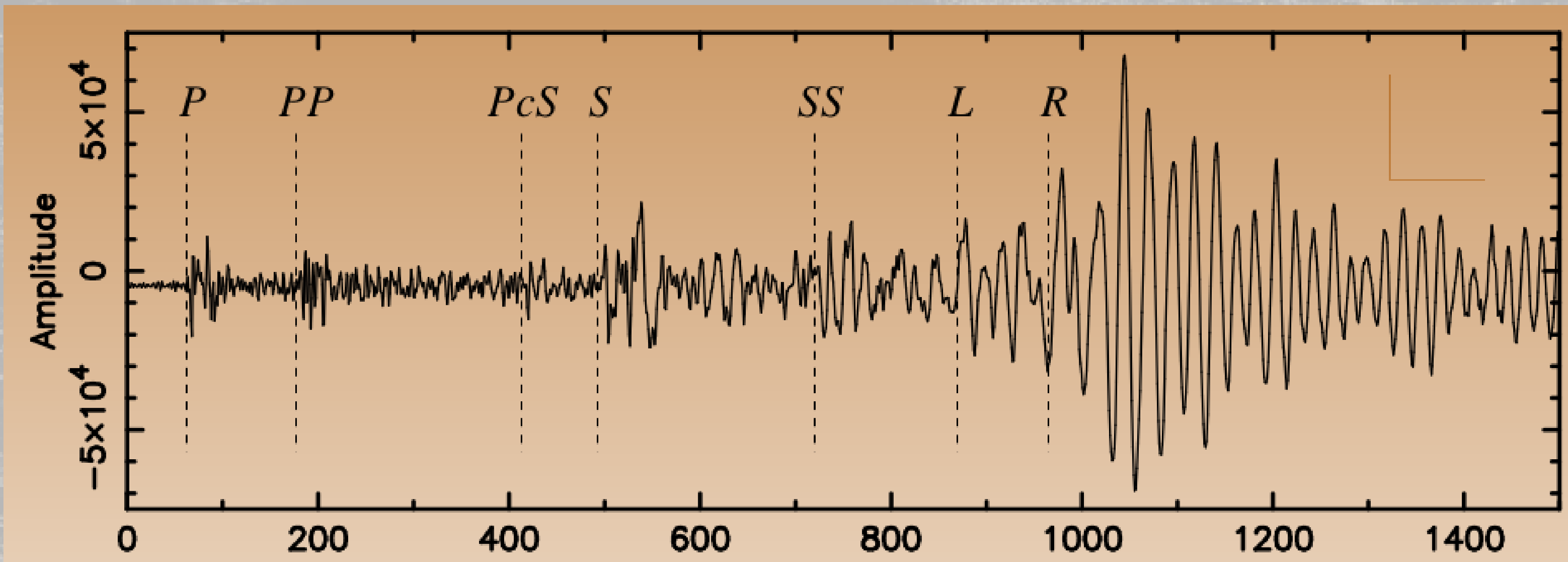


Seismic waves - information contents

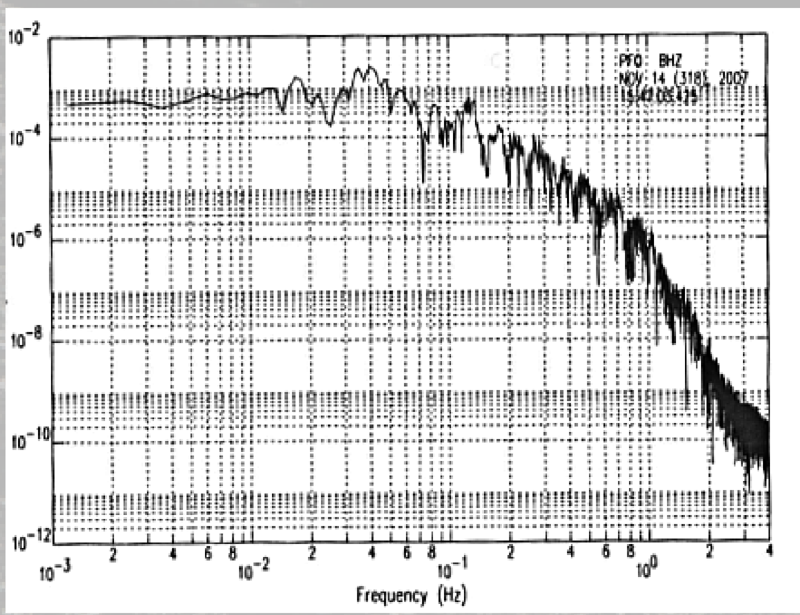


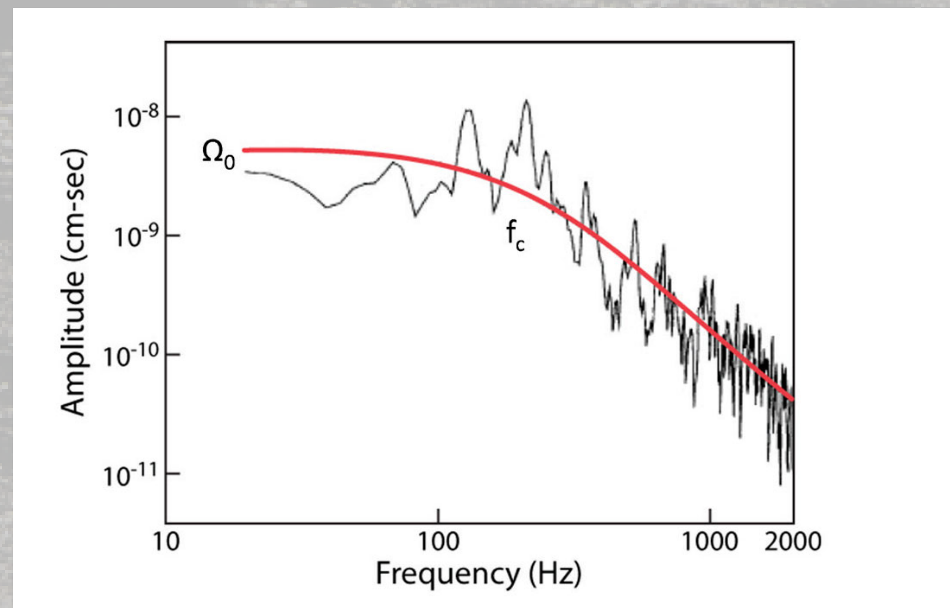
$$\mathbf{u}(\mathbf{r},t) \;=\; \int\limits_{R',T'} G(\mathbf{r},\mathbf{r}',t-t')\;\; S(\mathbf{r}',t')\,\mathrm{d}\mathbf{r}'\mathrm{d}t'$$

Seismic info - seismic phases

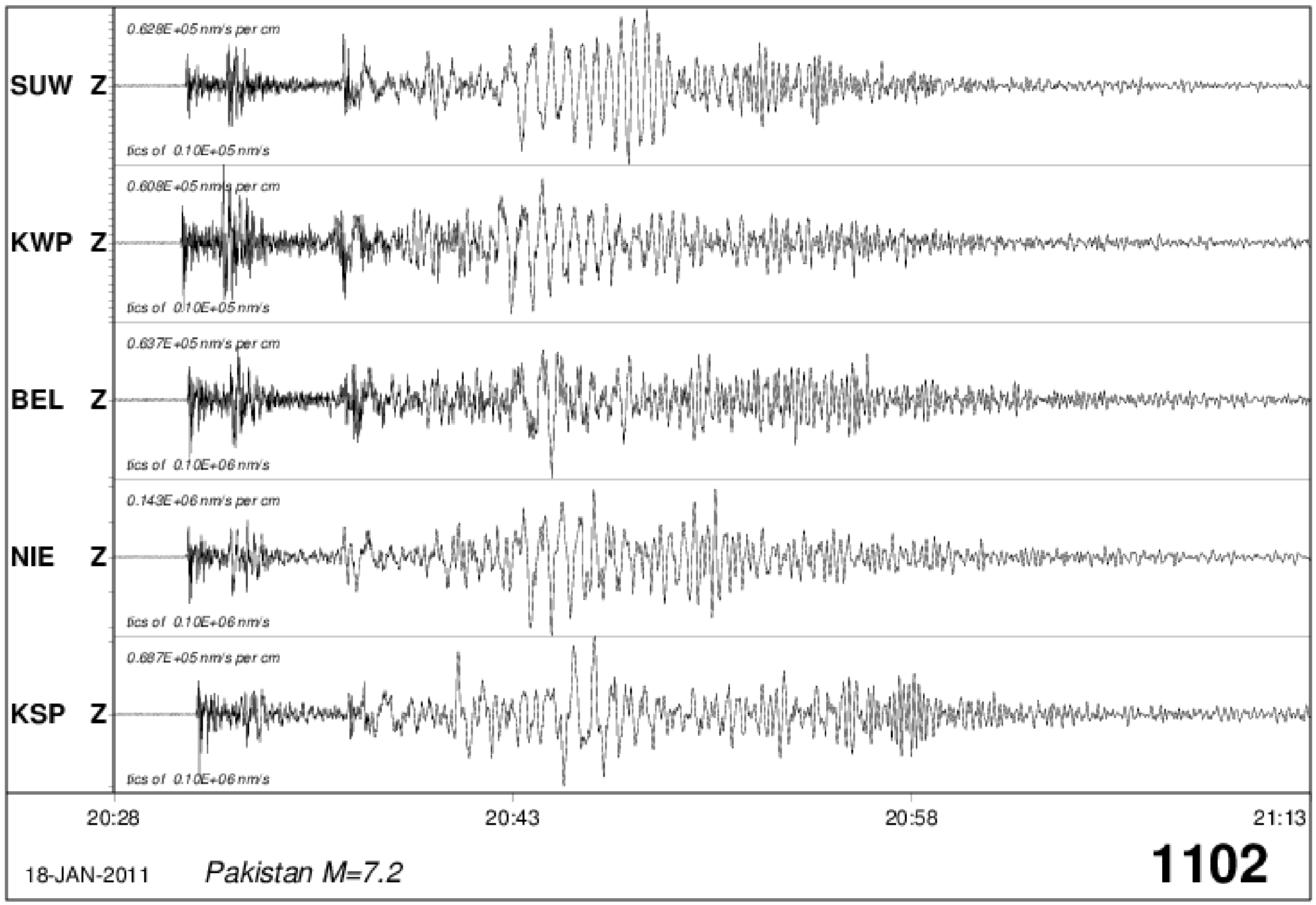


Seismic info - spectrum

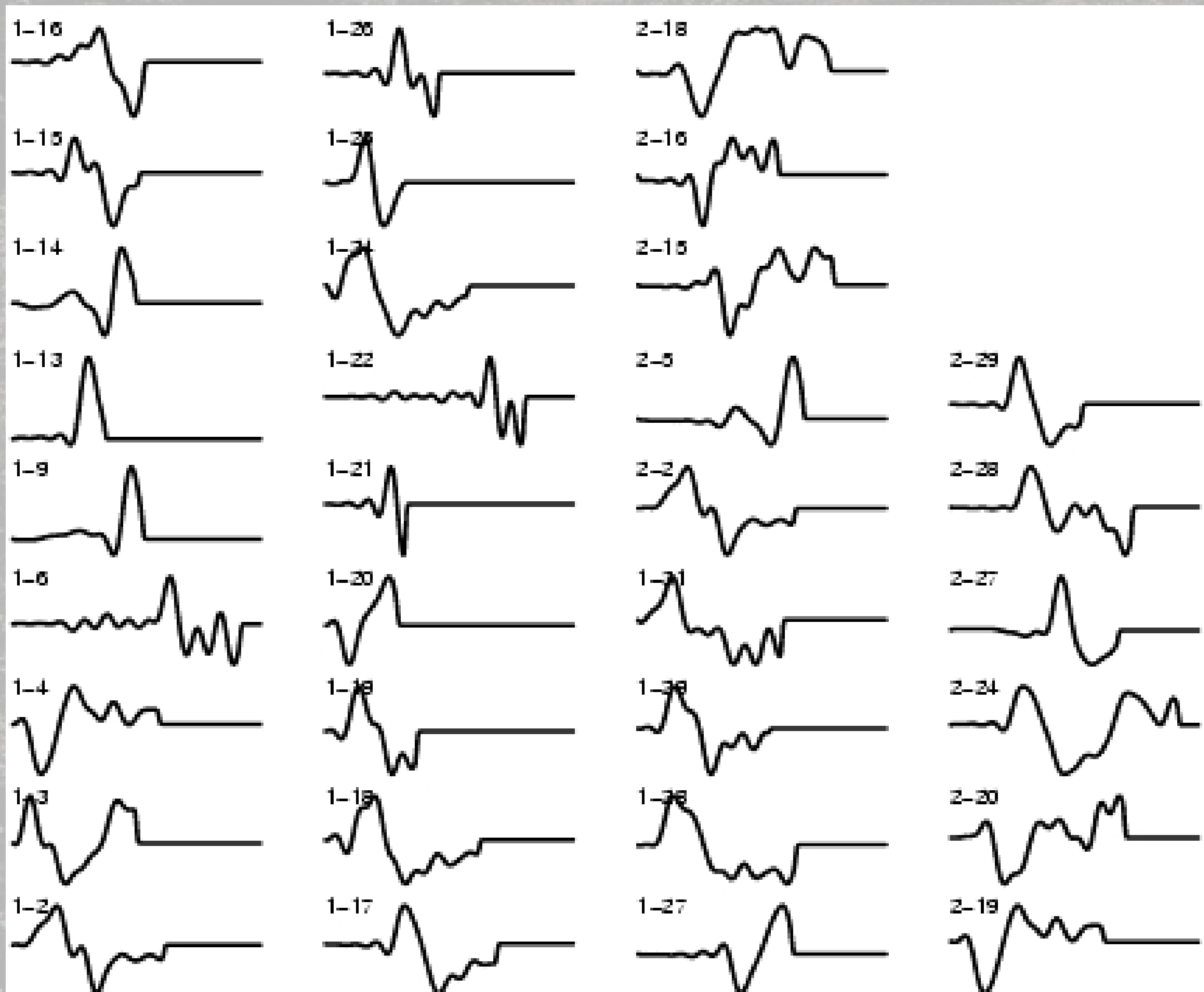




Seismic info - spatial variation (heterogeneities)



Seismic info - spatial variation (source physics)



Seismic source inference

Having recorded seismic data

- can we describe processes in earthquake foci?
- what can we say about energy budget of earthquake?
- can we understand rupture dynamics ?
- what about heterogeneities in source area?
- classical or quantum ....

Earthquake origin - preliminary

Origin:

- natural earthquakes
- induced earthquakes
- triggered earthquakes

Depth:

- shallow (crustal) earthquakes
- deep earthquakes

Earthquakes and faults

Fault - a rupture (discontinuity) in Earth's crust with relative displacement of its two size

Properties:

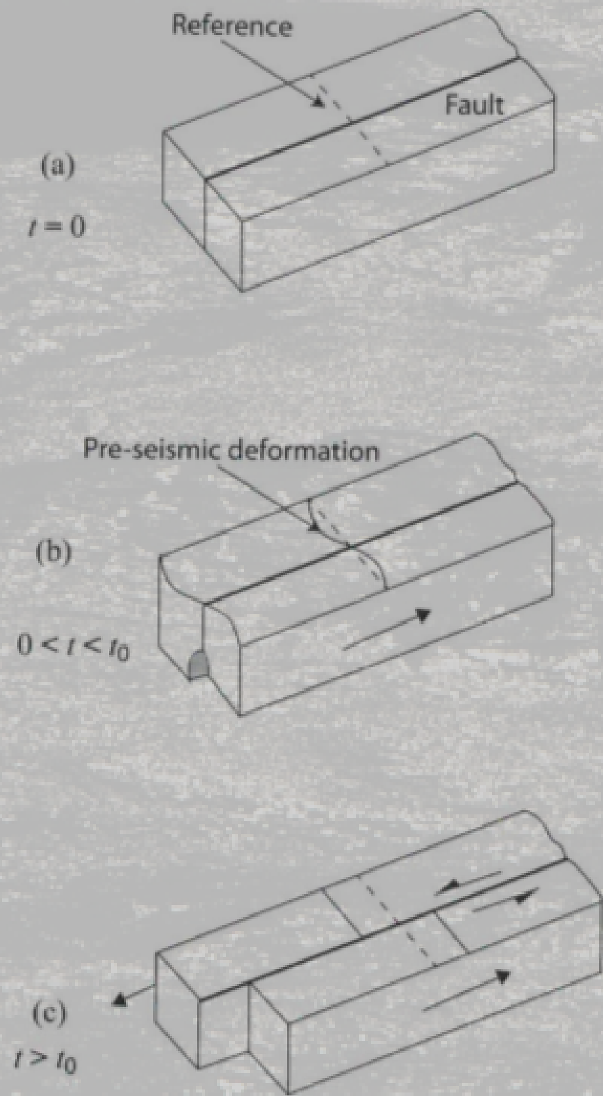
- fault lengths from 1m to 1000 km
- fault width for shallow earthquakes ¡ 40 km
- fault slip - from millimeters to tens of meters
- complexity: non-planarity, branching, bifurcation, etc
- sense of motion: strike-, and dip-slip faults

Elastic rebound theory - natural earthquakes

The background model of Earth's seismicity has been proposed by Reid [1910] after studying the catastrophic San Francisco (1906) earthquake.

- driving force - plate tectonics caused by thermodynamical instability of Earth's interior (hot core - cool crust)
- tectonic stress and and related stress localizations along fault. Localization occurs because at depth 10-20 km crust loosing its brittle character and becomes more visco-plastic. Tectonic driven motion become continues aseismic slip (creep)
- when deformation along upper part of fault is large enough, induced stresses exceed material strength - an earthquake is triggered releasing accumulate stress
- after that earthquake aftershocks occurs due to possible relaxation processes
- fault heals (locks) and stress starts to accumulate again

Elastic rebound model



Seismic energy

Shear stress overcomes the material strength or friction forces	$\Longrightarrow$	Motion on fault and permanent deformation

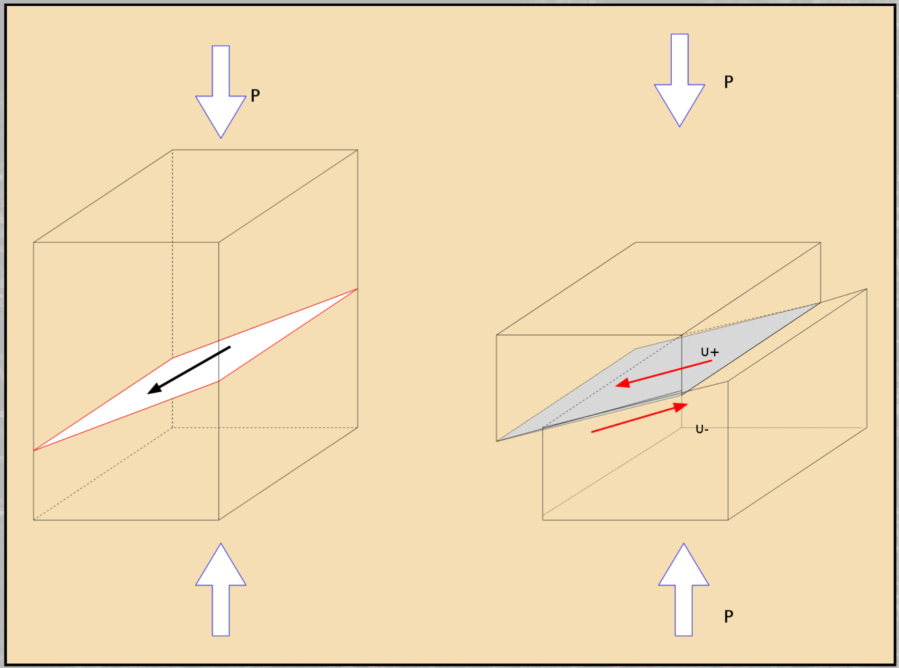
Different material behaviors:

- plastic - (permanent deformation without breaking)
- ductile - (breaking with sides deformations
- brittle - sharp and sudden breaking with sides displacements but no deformations

Elastic rebound theory - natural earthquakes

We shall consider only brittle failure

Simple seismic source model (after UMB)



$$\Delta\sigma = \sigma_0 - \sigma_f$$

$$\Delta u = u^+ - u^-$$

Energy balance

$$E_T = E_F + E_S + E_H$$

E_T – total energy dissipated during earthquake

E_F – fracture energy (creation new surface)

E_S – seismic wave energy

E_H – energy dissipated by friction during sliding (mostly heat)

Energy balance

Seismic energy (only directly measurable component)

$$E_S = \int_{-\infty}^{\infty} v\rho \cdot u^2 \mathrm{dt}$$

can be estimated from far-field measurements (after propagation effects and radiation pattern correction). Finally, it also converts mostly to heat.

E_H, E_F are dissipated during lasting earthquake: $t_o < t < t_f$

Seismic efficiency

$$\eta = \frac{E_S}{E_T}$$

Measure the proportion of total dissipated energy radiated as seismic waves

Seismic [scalar] moment

$$M_o = \mu \Delta \bar{u} S$$

μ – shear (rigity) modulus

$\Delta \bar{u}$ – average final slip: $\Delta \bar{u} = \frac{1}{S} \int_S \Delta u(\xi) \mathrm{d}\xi$

S – fault surface area

M_o is very convenient measure of “size” of earthquake

$\boxed{M_o \text{ units: } \mathrm{N\,m}}$

Average and Static stress drop

Fault starts to slip (breaks) when shear stress exceeds strength of fault material or static friction

Static stress drop

$$\Delta\sigma = \sigma_0 - \sigma_f$$

Average stress

$$\bar{\sigma} = \frac{1}{2}(\sigma_0 + \sigma_f)$$

$\sigma_f = 0 \Longrightarrow \Delta\sigma = 2\bar{\sigma}$

Static stress drop

Since σ_0, σ_f are initial (pre-earthquake) and final (post-earthquake) shear stresses $\Delta\sigma$ **does not** depend on kinematics and dynamics of stress release process

Average stress drop:

$$\Delta\bar{\sigma} = \frac{1}{S} \int\limits_S \Delta\sigma(\xi) \mathrm{d}\xi$$

$\sigma_o, \sigma_f, \Delta\sigma$ units: `Pa, bar`

Energy balance - strain energy

Energy dissipated by earthquake is elastic potential energy ΔW

Simplifying problem

$$E_T = \Delta W = \Delta \bar{u} \, \bar{\sigma} S = \frac{\bar{\sigma}}{\mu} M_o$$

But $E_S = \eta E_T$ one can define **apparent average stress**

$$\bar{\sigma}_a = \eta \bar{\sigma} = \mu \frac{E_S}{M_o}$$

Apparent stress

using definition of M_o $[\mu \bar{u} S]$

$$\boxed{\bar{\sigma}_a = \frac{E_S}{\bar{u} S}}$$

Apparent stress $\bar{\sigma}_a$ express how much energy is radiated from source per unit area and unit fault displacement.

Full distressing ($\sigma_f = 0$)

$\Delta\sigma = 2\bar{\sigma}$

$$E_T = E_o = \frac{\Delta\bar{\sigma}}{2\mu} M_o$$

Seismic efficiency

$$\eta = \frac{2\mu E_S}{\Delta\bar{\sigma} M_o}$$

E_S, M_o, $\Delta\bar{\sigma}$ are parameters which can be estimated from seismograms !

Partial distressing ($\sigma_f \neq 0$)

$$\bar{\sigma} \;=\; \frac{\Delta\bar{\sigma}}{2} + \sigma_f$$

$$E_T \;=\; E_o + \sigma_f \Delta\bar{u} S$$

frictional stress: σ_{fr}

$$E_R = E_F + E_H \;=\; \sigma_{fr}\Delta\bar{u} S$$

$$E_S \;=\; E_T - E_R = \frac{\Delta\bar{\sigma}}{2\mu} M_o + \Delta\bar{u} S(\sigma_f - \sigma_{fr})$$

Orowan's model ($\sigma_f = \sigma_{fr}$)

$$E_S = \frac{1}{2}\Delta\bar{\sigma}\Delta\bar{u}S$$

In this case the the radiated energy brings information ONLY on stress drop and total (final) displacement and not about rupture evolution.

Seismic efficiency: after using definition of M_o

$$\eta = \frac{\Delta\bar{\sigma}}{2\bar{\sigma}}$$

Statistical properties of earthquakes

✦ Gutenberg-Richter law

Logarithm of number N of earthquakes greater than given magnitude M decreases linearly with M

$$log(N) = a - bM$$

l seismic activity in given area
mbers of large-to-small earthquakes

Observing temporal changes of b one can tried to predict earthquakes
- rather unsatisfactory approach

Statistical properties of earthquakes

✦ Omori's - Utsu law

Number of aftershocks following a main shock is proportional to inverse power of time after the main shock

$$N(t) = \frac{A}{(c+t)^{\gamma}}$$

of aftershocks per unit time
t expressing the overall aftershock activity
oefficient (usually ~ 1)

Statistical properties of earthquakes

✦ Båth rule

The largest aftershock has magnitude approximately one unit less than the main event (shallow earthquake) or between 2 and 4 in case of deep earthquake.

Recently, (Maule 2010, $M_w = 8, 8$ and Tohoku 2011, $Mw = 9$ the rule has failed. In the first case the largest aftershock ($M_w = 7.3$) occurred almost two years after the main earthquake. In the second case the largest aftershock of magnitude ($M_w = 7.8$) occurred a few minutes after the main event.

END