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Application of Global Optimization Methods to Operational Control of Multireservoir Systems

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Abstract

The paper considers global optimization algorithms. The focus is on heuristic methods, which do not guarantee the optimum solution, but rather provide with the reasonable solution in a reasonable time. This is due to the fact that traditional optimization methods often fail for complex, non-convex problems. Because of growing complexity of systems taken into consideration and possibilities of modern computers, we can observe increasing interest in the development of the global algorithms.

The paper consists of two parts. First, the features of selected optimization methods - Controlled Random Search, Simulated Annealing, Genetic Algorithms and Evolutionary Strategies are summarized. The comparison of them based on the numerical experiments is presented. Next, the application of considered global algorithms designed to search the global minimum (maximum) of performance function in the case of complex control systems is discussed.

The case study considered in this paper is operational control in multireservoir systems working in flood conditions. Taking into account the complexity of the problem a two-level control structure with periodic coordination is proposed for real-time flood operation. This control structure incorporates two decision levels each: the upper level with the control center and the local level formed by the operators of the reservoirs. It is based on the use of the repetitive optimization of the outflow trajectories, using predicted inflows. Within this structure the central dispatcher performs an analysis of possible future scenarios of the flood and determines the optimal vector of parameters influencing local operators' decisions about the releases from the reservoirs. Because of the problem nonlinearity, cumbersome calculations (e.g. numerical simulation), it seems reasonable to apply the nongradient global optimization methods and parallel supercomputers to solve it. The global algorithms were applied to solve this problem. The paper describes how the optimization algorithms were chosen and how their efficiency was improved. Results of numerical simulations performed for the real-world water system of the Upper Vistula river basin are presented and discussed.

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1 Introduction

Operational control of water storage reservoirs during flood is very complicated task. Even in the case of a single reservoir system the solution of the control problem is difficult because of limited knowledge regarding future inflows. In the case of a multiple-reservoir system the control problem becomes much more complicated. The dimensionality of the overall problem, the nonlinearity of the flow transformation model, the nonlinear and state-dependent constraints on control variables (water releases) make it already quite difficult. The next important factor involved in the control of such a complex system is connected with the multiple decision units (reservoir's operators) and, at the some time, many different individual objectives, i.e., minimization of a local damages caused by high outflow from the reservoir. These local goals are often in contradiction with the global objective, i.e., and the minimization of flood damages in the whole river basin. For example, it may be useful from the global point of view to desynchronize the peak flows on various rivers by accelerating the wave on one river and retarding the waves on the others. However, this might cause greater local damage than in the case of independent local operation of reservoirs. Taking into account these features it is necessary to introduce control mechanisms capable of satisfying the global objectives. Such mechanism for real-time flood operation was developed and investigated. The hierarchical control structure with periodic coordination was proposed [3], [7]. It is based on the use of repetitive optimization taking into account predicted inflows. The considered optimization problem consists in determining for all reservoirs some parameters, through which a coordinator influences releases from the reservoirs. The problem is non-convex. Two heuristic optimization algorithms were applied to solve it. The results of calculations based on local and global optimization methods are presented and discussed in the final part of the article.

2 Considered global optimization methods

Two global optimization methods: Controlled Random Search (two versions: CRS2 and CRS3) and Evolutionary Strategies were considered in the experiments. These algorithms have certain advantages over many other global optimization procedures. They can be used for many classes of functions.

2.1 Controlled Random Search methods

In principle, Controlled Random Search methods (CRS) [9] were designed as a combination of a local optimization algorithm with the global search procedure. Two versions were proposed: CRS2 and CRS3. The CRS2 algorithm starts from the creation of the set of points (much more than $n+1$ points in n-dimensional space), selected randomly from the domain. Let us denote it as *P*. Then the best x_L (i.e., that of the minimal value of the performance index) and the worst x_H (i.e., that of the maximal value of the performance index) points are determined and a simplex in n-space is formed with the best point x_L and n points ($x_2,...,x_{n+1}$) randomly chosen from *P*. Afterwards, the centroid x_G of points x_L, x_2 ,…, x_n is determined. The next trial point x_Q is calculated, $x_Q = 2x_G - x_{n+1}$. Then the last, if the point x_Q is admissible and better (i.e., $f(x_Q) \le f(x_H)$),

replaces the worst point x_H in the set *P*. Otherwise, a new simplex is formed randomly and so on.
The CRS3 algorithm is a combination of the CRS2 procedure with the local optimization procedure based on Nelder-Mead simplex method [1]. The local algorithm is switched when a newly generated point in CRS2 fell within the bottom one-tenth of the ordered array *P*. After completing the local search the global search is continued. The CRS3 method tends to speed the convergence of the algorithm with respect to CRS2. The local optimization procedure operates only on the small part of set *P* and thus has a minimal effect on the global search performance of the CRS2 phase. The local procedure can operate at any stage of CRS3. It is triggered automatic but it can be modified to permit the user to switch the local procedure in or out according to his decision.

2.2 Evolutionary Strategies

Evolutionary Strategies (ESs) [5] emulate biological evolutionary theories to solve optimization problems. The ESs comprise a set of individual elements (the population *P*) and a set of biologically inspired operators defined over the population. According to evolutionary theories the most suited elements in a population are likely to survive and reproduce. In computing terms, evolutionary strategies map a problem onto a set of real-value vectors $x \in \Re^n$ (chromosomes), each vector representing a potential solution. The ESs manipulate the most promising individuals in its search for improved solution. A general Evolutionary Strategy algorithm operates through a simple cycle of stages:

Initialization - An initial population *P* of potential solutions is generated at random.
Evaluation - The performance (fitness) of each individual is evaluated with respect to the constraints imposed by the problem.
Selection - The population for genetic manipulation is chosen from *P* based on each individual's fitness.
Recombination - Recombination exchanges a population's genetic material. It takes two chromosomes and swaps part of their genetic information to produce new chromosomes. There are several types of recombination operator.
Mutation - The mutation operator introduces new genetic structures in the population by randomly modifying of individuals. Each coordinate x_i of a vector $x \in \Re^n$ representing an individual is mutated by adding an individual normally distributed random number, $(0, \sigma_i)$. The σ_i are also subject to mutation and recombination.

Several schemes of Evolutionary Strategies can be selected [5]. In this paper the multimember (μ+λ)-ES variant was considered. In the case of this scheme λ offspring individuals are created from μ parents by means of recombination and mutation. The best μ individuals out of parents and offspring are selected to form the next population.

2.3 Comparative performance of CRS and ES algorithms

The CRS and ES algorithms were applied to search the global minimum of some standard non-convex functions: Branin, Camel-Back, Goldstein and Price, Griewank, Rosenbrock, Shubert, presented in literature [10]. The stop criterion was defined in terms of the convergence to the global minimum with the assumed accuracy. When the global optimum was reached with the accuracy 0.1, the algorithms stopped. The Table 1 shows the average numbers of function evaluations needed to find the global minimum with assumed accuracy over a series of 100 trials. The CRS2 and ES procedures were compared with other, well known global optimization methods, such as: genetic algorithm (GA) [2], Meewella-Mayne algorithm [4] of linear subapproximations and simulated annealing algorithm (SA) taken from [8]. In the tests the methods ES and CRS2 were able to find the global minimum more quickly than the other tested methods.

Test Function	*Optimization Algorithm*				
	M-M	*SA*	*GA*	*ES*	*CRS2*
Branin	1195	623	643	80	143
Camel-Back	495	246	200	68	53
Camel-Back 6	225	364	350	68	88
Goldstein Price	13555	424	7500	40	263
Griewank	420	810	1650	122	192
Shubert	-	605	3200	422	1434

('-' denotes that the algorithm did not converge)

Table 1: Speed tests of the global optimization algorithms

3 Case study: Flood control in multireservoir system

The ES and CRS methods were chosen as the basic optimization algorithms in the control structure developed for the optimal flood operation in a multireservoir system.

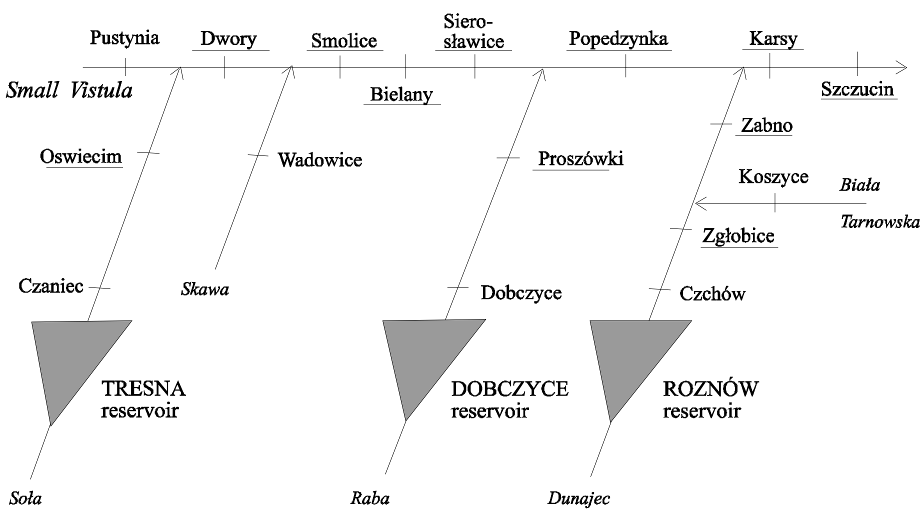

Figure 1: Layout of the considered river system

3.1 The water system description

A multireservoir water system, where several reservoirs are located on tributaries to the main river, was considered (see Fig. 1). The overall objective was to minimize damages created by a flood wave passing through the river basin, related to the peak flows at the important cross-sections. The mathematical expression for the considered performance index is:

$$J(Q(\cdot)) = \sum_{k=1}^{K} v_k \cdot (Q_k^{cul} - Q_k^{mcul})_+ \qquad \text{where} \qquad (x)_+ = \max(0,x) \tag{1}$$

$$Q_k^{cul} = \max_{t\in[t_c^{'},t_{cf}^{'}]} Q_k(t) \qquad k = 1,...,K$$

where $[t_c^{'}, t_{cf}^{'}]$ denotes control horizon, v_k weight related to the flow at the *k*-th cross-section (different points have different importance), $Q_k(t)$ flow at the *k*-th cross-section at time *t*, Q_k^{mcul} maximal flow at the *k*-th cross-section which does not induce damages, *K* the number of cross-sections, $J(Q(\cdot))$ performance (loss) function associated with flow levels trajectories.

Each reservoir is described by the dynamics of a simple tank, with one input $d_i(t)$ (the forecasted value of this inflow rate is $\overline{d}_i(t)$, *i=1,...,m*; where *m* - the number of reservoirs) and one controlled output $u_i(t)$, i.e.

$$\dot{w}_i(t) = \overline{d}_i(t) - u_i(t) \tag{2}$$

where - $w_i(t)$ storage of the *i*-th reservoir at time *t* and $u_i(t)$ - rate of outflow from the *i*-th reservoir at time *t.*

The entire river system is divided into several sections and each section is represented by a cascade of *Z* non-linear elements ("reservoirs") described by identical equations:

$$s_j = l \cdot Q_{out_j}^{r} \qquad \frac{ds_j}{dt} = Q_{in_j}(t) - Q_{out_j}(t) \qquad j = 1,...,Z \tag{3}$$

where $Q_{in_j}(t)$ denotes inflow to the *j*-th element at time t, $Q_{out_j}(t)$ outflow from the *j*-th element at time *t*, $s_j(t)$ storage of the *j*-th element at time *t* and *l*, *r* denote parameters connected with given river reach (together with the number of elements *Z* they are experimentally determined). Obviously, $Q_{in_j}(t) = Q_{out_{j-1}}(t), \forall t$.

The objective, as considered by the central operator at time $t_c^{'}$, is to determine such outflows from the reservoirs, that the performance function (1) is minimized. The following constraints on the reservoir storage and releases are taken into account:

$$w_i(t_c^{'}) = w_{i_0} \tag{4}$$

$$w_i(t_{cf}^{'}) = w_{i_{\max}} \tag{5}$$

$$w_{i_{\min}} \le w_i(t) \le w_{i_{\max}} \qquad t \in [t_c^{'}, t_{cf}^{'}] \tag{6}$$

$$u_{i_{\min}} \le u_i(t) \le u_{i_{\max}} \qquad t \in [t_c^{'}, t_{cf}^{'}] \tag{7}$$

The hierarchical control structure with periodic coordination was proposed for controlling this system [7]. This structure incorporates two decision levels: the upper level with the control center (coordinator) and the local level formed by several reservoir operators (see Fig. 2).

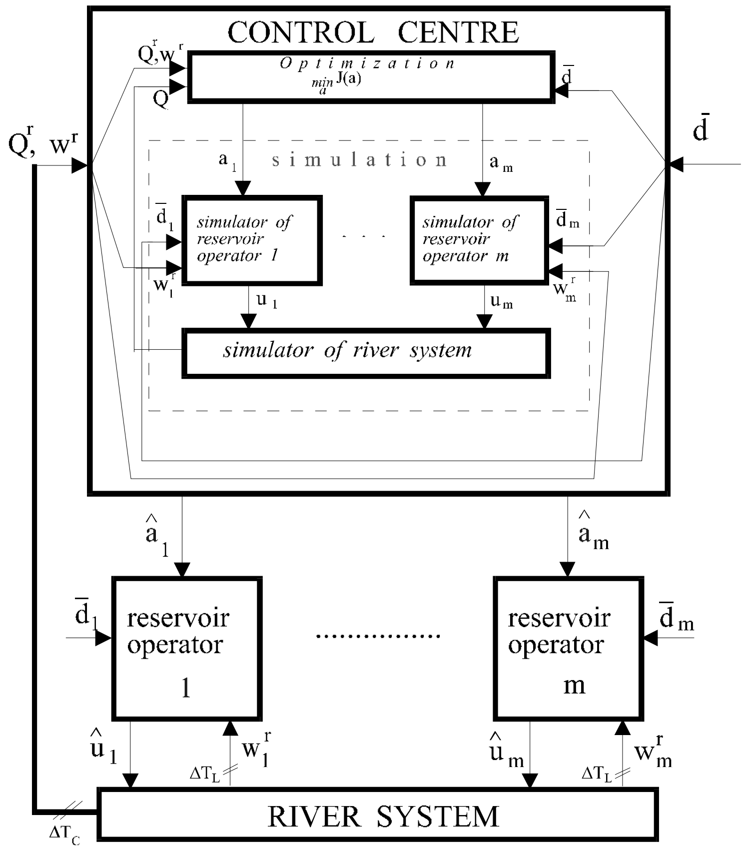

Figure 2: Two-level structure for flood control

3.2 Optimization problem formulation

The upper level mechanism of parametric coordination is based on the assumption that each reservoir operator rule (local control law) is periodically modified using the vector a_i of parameters set by the coordinator, i.e. by the control center (see Fig. 2).

The objective of the coordinator during flood, say at time $t_c^{'}$, is then to determine such joint vector of parameters $a = [a_1, ..., a_m]$ (where *m* is the number of reservoirs), that the performance measure related to damages in the whole river basin is minimized. This problem can be formulated as follows:

$$\min_{a=[a_1,..,a_m]\in A} [\Psi(a) = J(Q(\cdot))]$$
$$Q(t) = F(Q^r(t_c^{'}), \hat{u}(\cdot), \bar{d}^{t_c^{'}}(\cdot)) = F(Q^r(t_c^{'}), R(w^r(t_c^{'}), \bar{d}^{t_c^{'}}(\cdot), a), \bar{d}^{t_c^{'}}(\cdot)) \tag{8}$$

where *A* denotes set of admissible values of parameters a, $[t_c^{'}, t_{cf}^{'}]$ control horizon, $Q(t)$ vector of trajectories of flows in the river system at time *t*, $Q^r(t_c^{'})$ vector of real flows measured at time $t_c^{'}$, $\bar{d}^{t_c^{'}}(\cdot)$ vector of trajectories of forecasts of all inflows in period $[t_c^{'}, t_{cf}^{'}]$

calculated at time $t_c^{'}$, $\hat{u}^{t_c}(\cdot)$ vector of releases from the reservoirs in period $[t_c^{'}, t_{cf}^{'}]$, $R(\cdot,\cdot,\cdot)$ the set of local decision rules, $Q(t)$ performance (loss) function associated with flow levels $J(Q(\cdot))$ in period $[t_c^{'}, t_{cf}^{'}]$.

While formulating local optimization problems we should take into account that the natural objective of a single retention reservoir management during flood is the minimization of damages created by high water levels just downstream the reservoir - in an adjacent river reach. This is equivalent to minimizing the peak release from the reservoir. Hence, we can formulate the decision problem of the *i*-th local reservoir operator at time $t_l^{"}$ as follows:

$$\min_{u_i}\left[q_i(u_i(\cdot)) = \max_{t\in[t_l^{"}, t_{lf}^{"}]} u_i(t) \right] \tag{9}$$

under the constrains (4) - (7).

The instruments of coordination a_i, $i = 1,...m$ provided by the control center, can be used to modify local operators' performance measures. In particular, the i-th reservoir operator control problem can be defined as follows:

$$\min_{u_i}\left[q_{i\,\mathrm{mod}}(u_i(\cdot), a_i) = \max_{t\in[t_l^{"}, t_{lf}^{"}]} (u_i(t)\cdot \alpha(t)) \right] \tag{10}$$

where $[t_l^{"}, t_{lf}^{"}]$ optimization horizon $t_l^{"} \geq t_c^{'}$, $q_{i\,\mathrm{mod}}(\cdot,\cdot)$ modified local operator cost function, a_i parameters specified by the control center.

In the above problem, the vector a_i of coordinating parameters for the *i*-th reservoir is related to the weighting function . This function is defined as follows:

$$\alpha_i(t) = 1 + (c_i - 1)1(t - T_i^*) = \begin{cases} 1 & t \in [t_l^{"}, T_i^*) \\ c_i & t \in [T_i^*, t_{lf}^{"}] \end{cases} \tag{11}$$

where $1()$ denotes the Heaviside's step function.

Finally the vector is given as $a_i = [c_i, T_i^*]$. The objective of these parameters is to shift the peak of the flow downstream of the *i*-th reservoir. It can be accelerated to occur before time T_i^* in the case when $c_i > 1$ or delayed when $c_i < 1$. For $c_i = 1$, the performance index (11) reduces to the one given by (10), and so local, independent control policy is realized.

In the calculation process performed by the central operator two phases can be distinguish (see Fig. 2): optimization and simulation. Taking into account the description of the entire problem (nonlinear model of flow transformation, internal optimization of the outflows from the reservoirs) we can not lead the analytical form of the performance function describing flood damages in the whole river basin. Every optimization step value of the performance index is the result of the simulation process. The optimization is realized as follows: after assuming certain values of parameters, simulation of the reservoirs operation and flow transformation in the whole river basin until the predicted end of the flood ($t_{cf}^{'}$) is performed. Then, the value of the overall performance index $\Psi_n(a)$ related to the given vector is computed.

3.3 Numerical experiments

A case study of the Upper Vistula river basin system was considered. This system consists of three reservoirs, located on tributaries to the main river, and of three uncontrolled side inflows (see Fig.1). Calculations were performed for a set of data containing eleven hydrographs of historical floods, which occurred between the years 1960-1974 and several hypothetical, so-called scenarios. The central operator performed optimization of 6 parameters forming vector a, using several optimization methods.
Many calculations were performed for every method. The question was how the global algorithms influence the optimization results and thus influence the issues of the operation of a multireservoir system during flood. The most interesting numerical results obtained for three floods: two historical, which occurred in 1970 and 1972 and one hypothetical scenario, are collected in Tables 2 and 3. Table 2 presents the best (i.e., the lowest) and the worst (i.e., the highest) optimal values of the performance index (10) obtained during 10 runs of each optimization algorithm and the reduction of the performance index with respect to the Nelder-Mead's algorithm. The results obtained by the Nelder-Mead, ES, CRS2 and CRS3 methods are compared. The available numerical results indicate that the global optimization algorithms give better results than the standard Nelder-Mead simplex algorithm. In most cases the best results were obtained by ES but the time required to compute a solution was longer than in CRS methods. The CRS3 method provided better results with respect to CRS2. However, the reduction of cost with respect to CRS2 method was not very big.

optim. method	*hypothetical flood*		*his. flood 1970*		*his. flood 1972*	
	*the best /**	*the worst*	*the best /*	*the worst*	*the best /**	*the worst*
Nelder Mead	1587	1587	1989	1989	773	773
CRS2	1574 *0.81%*	1587	1892 *4.87%*	1907	765 *1.03%*	768
CRS3	1578 *0.56%*	1583	1877 *5.61%*	1892	764 *1.17%*	770
ES	1515 *4.51%*	1572	1773 *10.86%*	1792	722 *10.81%*	770

/ -reduction of criterion w.r.t. Nelder-Mead method*

Table 2: Optimization results of different methods for historical data

The results presented in Table 2 and discussed above are results of the optimization process, when the objective was to calculate the optimal coordination parameters for current inflow forecasts. The CRS2 and CRS3 methods were used in the simulation of an on-line reservoir management during flood. The whole control system is to be capable of working in real time so it is necessary to take into account the uncertainty of the inflow forecasts. Because of that all tasks have to be solved repetitively using actual measurements. We assume that the central operator of the whole system solves his problem at time $t_c^{'}$. Each local operator of the reservoir changes his decisions more frequently, at times $t_l^{''}, t_{l+1}^{''}, ...$ until the new coordination decision at time $t_{c+1}^{'}$. It is obvious, that $\Delta t = t_{c+1}^{'} - t_c^{'}$ must contain several shorter time intervals $\Delta t = t_{l+1}^{''} - t_l^{''}$ related to the use of the local mechanism. Because of that the central operator repeats the optimization process several times, up to the end of the flood. The simulations were performed for several historical floods. Table 3 presents the minimal values of the loss function obtained for two historical floods from 1970 and 1972. Percent reduction of the criterion with respect to unregulated flood is given in parentheses.

3.4 Conclusions

In general, the available results of numerical experiments indicate that global optimization algorithms can bring benefits in the case of complex control problems. The considered

optimization method	*historical flood 1970*	*historical flood 1972*
Nelder Mead	1904 (*32.71%*)	792 (*44.69%*)
CRS2	1872 (*33.83%*)	763 (*46.72%*)
unregulated flood	2829	1432

/ -reduction of criterion w.r.t. unregulated flood*

Table 3: Numerical results of flood control simulation

global optimization procedures have given better results than the standard Nelder-Mead's simplex method. The best results were obtained by ES method but it was connected with longer computation time. It can involve decision delays, which can not be neglected in operational control (see [6]). This was the reason why the CRS methods were chosen for simulation of on-line flood control.
It should be pointed out, that the optimization problem solved by us was rather difficult. It is so, because the multireservoir system in the Upper Vistula river basin (Fig. 1) is located in the mountain region, where the flood situation changes very rapidly. Due to the specific topography, it is impossible to expand the retention capacity. The efficient use of reservoir storage volume during conservation periods between floods (water distribution and power generation) can be achieved only through decreasing the mandatory storage capacity reserved for usage in flood emergency periods. Because of that, for big floods in the Upper Vistula river system, the coordination of the local operators activities itself may bring an additional improvement not greater than 10%, even when very accurate information about the future inflows is available. Hence, the influence of the optimization method used for determination of the coordination parameters is limited.

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