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Influence of Inflow Prediction on Performance of Water Reservoir System

J.J. Napiórkowski, A. Kozlowski *and* T. Terlikowski

ABSTRACT
The two-layer hierarchical technique with three different prediction methods was applied to a part of the Wupper Reservoir System. The reservoir system consists of two reservoirs in series with additional inflow to the lower reservoir. The tasks of these reservoirs are flood control, recreation, hydropower and low flow augmentation with the aim of water quality improvement. It is shown that the introduced optimisation concept improves considerably the system performance in comparison with the Standard Operation Rule.

1. INTRODUCTORY COMMENTS

A method for determining the yield of a multireservoir water supply system has been applied to a part of the Wupper Reservoir System in Germany. The major objectives of this particular system are flood control, recreation, hydropower and low flow augmentation. The proposed technique may be reduced to the following associated parts: the optimisation of a simplified quantitative model of the actual system and the multiobjective verification and/or comparison through simulation. The first part consists in constructing a relatively wide class of control schemes based on the two-level optimisation technique method. We focus our attention on the implementation of a number of prediction techniques of the system inflow (ARIMA, Deterministic Chaos, Artificial Neural Networks) that result in different operation rules. The second part is based on the simulation performed for historical data over a long time horizon (39 years). This simulation consists of testing the control rules for chosen scalar objectives. The diagrams of frequency (reliability) criteria, calculated on the basis of simulation for a number of scalar criteria are analysed to obtain the final comparison results.

Several control schemes corresponding to the prediction models considered have been proposed in the form of computer programs. The simulations have been performed for a large number of years and for many objectives. To present advantages of the control schemes corresponding to the prediction systems, they are compared with so-called Standard Decision Rule (SDR) and Stochastic Dynamic Programming (SDP).

2. DESCRIPTIONS OF THE CASE SYSTEM MODEL

The catchment of river Wupper is located in the southern part of North Rhine Westfalia. The hydrological features of this catchment are characterised by a massive rocky underground covered only by a small layer of soil and an average yearly precipitation of about 1300 mm per year. The absence of underground water storage leads to dangerous floods as well as to extreme droughts. To accommodate this problem several reservoirs were built. Here we are just interested in the management of the two reservoirs governing the discharges in the city of Wuppertal, which lies about 20 km downstream of reservoir No. 2. Figure 1 shows the simplified Wupper Reservoir System.

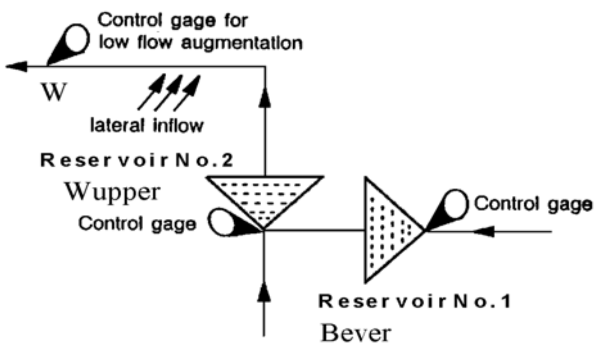

Fig. 1. Basic structure of reservoir system.

It contains two reservoirs located in series, the control centre at reservoir No. 2 and several runoff and rainfall gauges. The release of the reservoirs depends mainly on the runoff at the control gauge in Wuppertal. A runoff of 5 m^3/s at this gauge is sufficient for the required water quality, runoff less than 3.75 m^3/s should be avoided and the runoff less than 1 m^3/s has to be regarded as ecologically disastrous. The basic hydrological and reservoir characteristics are given in Table 1.

The purpose of the model is to describe relationships between flow rates in the rivers over a long time horizon (one year) with the discretization period of 10 days. Therefore, only the dynamics of the storage reservoir are considered, while effects of flow dynamics in the river channels are neglected.

For brevity, the following notation is used: j- number of 10-day intervals, V^j - state of the reservoir, d^j - natural inflow, u^j - flow in a given cross-section, z^j - water demand, m^j - outflow from the reservoir, *1,2* - denote the Bever and Wupper reservoirs, *3* - denotes the lateral inflow, *W*- cross-section at Wuppertal.

Table 1. The basic characteristics of the Wupper Reservoir System.

Reservoir	Bever	Wupper
total storage V_{max} (mln m^3)	23.70	25.90
dead storage V_{min} (mln m^3)	0.70	2.10
max. outflow (m^3/s)	17.00	180.00
min. outflow (m^3/s)	0.10	1.00
annual average flow (m^3/s)	0.94	3.51
catchment area km^2	25.7	212.00

According to the introduced notation, the state equations for the reservoir system and flow balance equation for the selected cross-section W are:

$$V^{j+1} = V^j - B * m^j + C * d^j \tag{1}$$

$$V = [V_1, V_2];\ m = [m_1, m_2];\ d = [d_1, d_2] \tag{2}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\quad C = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \tag{3}$$

$$u_W^j = m_2^j + d_3^j \tag{4}$$

3. THE OPTIMISATION PROBLEM

The objective function of the optimisation problem under consideration for any time instant k (for any 10-day period) and for annual time horizon T can be written in the form of a penalty function:

$$\mathbf{Q}(m,V) = \sum_{j=k}^{k+T} [\, a_1^{+j} (m_1^j - z_1^j)^2 + a_2^{+j} (m_2^j + d_3^j - z_W^j)^2 + b_1^j (V_1^j - V_1^{*j})^2 + b_2^j (V_2^j - V_2^{*j})^2 \,] \tag{5}$$

In equation (5), symbols *a* and *b* with respective subscripts denote weighting coefficients. The performance index **Q** is expressed explicitly on controls m^j and the state trajectory V^j (reservoir contents) as follows:

$$\mathbf{Q}(m,V) = \sum_{j=k}^{k+T} Q(m^j, V^j) \tag{6}$$

The objective function during each 10-day period is subject to the constraints on the state of the system, controls and flows in given profiles:

$$\begin{aligned} V^j_{\min} &\le V^j \le V^j_{\max} \\ m^j_{\min} &\le m^j \le m^j_{\max} \end{aligned} \tag{7}$$

3.1 Required retention trajectory V^{*j}

It is assumed that the operation of the reservoir system is carried out on an annual basis in the following way:

- by late December, the reservoirs are normally returned to low level to prepare the system for the next flood season, completing the annual cycle.
- the storage reservation for flood control on January 1 was determined for controlling the maximum probable flood. During the normal filling period, January-April, the reservoirs should be filled up completely.
- during the May-August period the first reservoir should be filled up to meet recreational requirements.
- during the May-November period the water stored in and released from the reservoirs is used for low-flow augmentation and hydropower.

3.2 Weighting coefficients a^{j+} and b^j

According to the general objective of the control problem, which is aimed at the rational protection against water deficits and at reaching the desired state at the end of April, the following values of weighting coefficients in the optimisation problem are used: $a^{j+}=1$ if demands are greater than supply and $a^{j+}=0.01$ otherwise,

for k=[1,36]. As far as the second coefficient is concerned, in order to avoid a good performance in one year followed by a very poor performance in the next year $b^j = 0.01$ for j=[1,12] (May-August), $b^j = 0.001$ for j=[1,30] (September-February), $b^j=0.004$ for j=[31,33] (March) and $b^j=0.01$ in April, for j=[34,36].

4. TWO-LEVEL OPTIMISATION TECHNIQUE

To solve the aforementioned problem we adjoin the equality constraints (1) with the Lagrange multiplier sequence λ (prices). The Lagrangian function has the form:

$$L(m,V,\lambda) = \sum_{j=k}^{k+T} [Q(m^j, V^j) + \lambda^j (V^{j+1} - V^j + B * m^j - C * d^j)] \qquad (8)$$

To include the state-variable and outflow constraints the above problem is solved by means of the two-level optimisation method in a decentralised (co-ordinated) fashion. At this stage we make use of the additivity of the Lagrangian function (9) and the possible separation of the decision variables.

The Lagrangian function has a saddle point which can be assigned by minimising L(**λ**,**V**,**m**) with respect to **V** and **m,** and then maximising with respect to **λ**. Finally, the optimisation problem can be expressed in the form:

$$\max_{\lambda} [\min_{V,m} L(\lambda, V, m)] \qquad (9)$$

with inequality constraints on state and control and no constraints on Lagrange multipliers. Figure 2 illustrates how the two-layer optimal control method works.

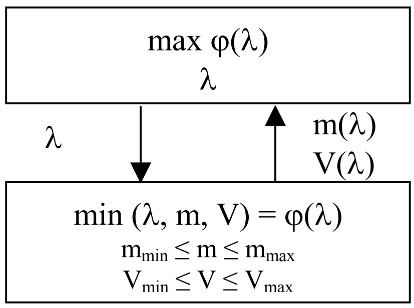

Fig. 2 Two-level optimisation method.

At the *lower level* for given values of the Lagrange multipliers we look for the minimum of the Lagrange function. The required condition is the zero value of the gradient with respect to **m** and **V**. The task of the upper level is to adjust the prices, **λ,** in such a way that the direct control of the reservoir, affected by **λ**, results in the desired balance of the system (the mass balance equation (1) is fulfilled satisfactorily). In the upper layer, in the maximisation of the Lagrange function with respect to **λ** the standard conjugate gradient technique is used.

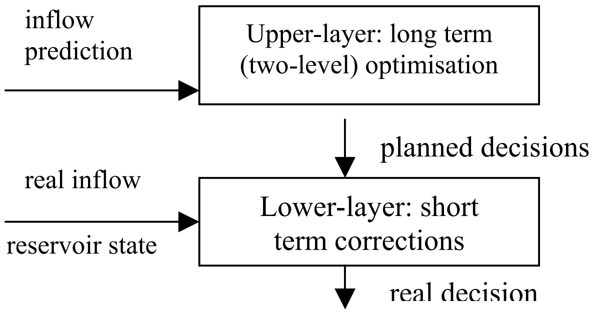

Fig.3 Two-layer control method.

In the applied Two-Layer optimisation control method (TLM) illustrated in Fig.3 the solution of the two-level optimisation problem (9) is the essential “upper layer part”.

5. INFLOW PREDICTION MODELS

Three inflow prediction techniques that were used for inflows predictions in the two-layer control method are briefly presented below.

5.1 Box-Jenkins ARMA model

A classic multiplicative decomposition was applied to deseasonalise the observed data and then ARMA (Auto Regressive Moving Average) model was used in the prediction of inflows to the reservoir system. In the practical calculations the set of relevant procedures from Microsoft IMSL Library of Professional Edition of Microsoft Fortran Power Station v. 4.0. was adopted.

These procedures enable the computation of estimates of auto-regressive and moving average parameters of the ARMA(p,q) model and then the calculation of values of inflow estimates for specified number of points to be included in the forecast of a fitted model.

Calculations showed that the most effective model was ARMA(2,1). It enables better forecasts than the model in the form of average historical values for 20-days time horizon.

5.2 Artificial Neural Network model

Inflow predictions based on the neural network simulation were generated with the help of the NeuroSolutions software package (NeuroDimension, Inc., 1997).

NeuroSolutions adheres to the so-called local additive model. A processing element (PE) simply multiplies an input by a set of weights and nonlinearly transforms the result into an output value. The principles of computation at the PE level are deceptively simple. The power of neural computation comes from the massive interconnection among the PEs which share the load of the overall processing task and from the adaptive nature of the parameters (weights) that interconnect the PEs. Under this model, each component can activate and learn using only its own weights and activations and those of its neighbours. The neural network architecture used is the multilayer perceptron (MLP) (Lippman, 1987).

The performance of an MLP is measured in terms of a desired signal and an error criterion. The output of the network is compared with a desired response to produce an error. NeuroSolutions uses an algorithm called back-propagation (Rumelhart et al., 1986). The network is trained by repeating this process many times. The goal of training is to reach an optimal solution based on the performance measurement.

The simulation results obtained justified 3 points as the maximum that can be included in the forecast with the following parameters of applied MLP:

- Hidden Layers = 1
- PEs = 8
- Transfer function = TanhAxon (hyperbolic tangent -1/+1)
- Learning Rule = Momentum (gradient and weight change, momentum = 0.7)
- Transfer function specified for output layer = LinearTanhAxon (piecewise linear -1/+1).

It should be noted that the ANN model gives the best predictions of inflows to the system.

5.3 Model Based On Deterministic Chaos Concept

The real process in its whole complexity is defined by a given generalized-state evolution mapping: $\mathbf{t} \mapsto \boldsymbol{X}_\mathbf{t} = (\underline{\mathbf{X}}_\mathbf{t}, \ldots)$, where $\boldsymbol{X}_\mathbf{t}$ is a generalised state value, often of unknown character and dimension. If the dimension of $\boldsymbol{X}_\mathbf{t}$ is infinite, we deal with a case of *chaos*. However, according to the concept of *deterministic* chaos, a distinguished "sub-vector" $\underline{\mathbf{X}}_\mathbf{t}$ of $\boldsymbol{X}_\mathbf{t}$ may satisfy a functional relationship between precedent and the next state value. $\underline{\mathbf{X}}_\mathbf{t}$ is assumed to evolve on the so-called *attractor*. The phenomenon is of deterministic nature: there exists a functional relationship: $\underline{\mathbf{X}}_{\mathbf{t+T}} = \mathbf{F}(\underline{\mathbf{X}}_\mathbf{t})$ strongly non-linear and unstable. $\underline{\mathbf{X}}_\mathbf{t}$ evolves on an *attractor* $\mathbf{M} \subseteq X$, in *the* space $X = \mathbf{R}^n$ (Takens, 1981), where $\mathbf{M}$ is a smooth, compact manifold of specific properties, see also (Soukhodolow et al., 1996).

The dimension of attractor $\mathbf{M}$, e.g. the so-called Hausdorff dimension, occurs to be relatively small in practical applications. It is usually not topological dimension (is less than n) and may be not an integer (*strange attractor*).

The fundamental task is to determine the proper prediction model, that is, to construct a good approximation of function $\mathbf{F}$. Practically, we are faced with measurement and approximation errors that grow exponentially in time, due to process unstability. Hence, it is possible to have a prediction of a given accuracy only for a short time horizon. The second difficulty results from the limited observability of the process: one disposes of a sequence $\{x_i\}$ of measured scalar values only (time series) and not of a whole vector $\underline{\mathbf{X}}_\mathbf{i} \in \mathbf{R}^\mathbf{n}$. Therefore, while constructing a prediction model from a given inflow time series, we apply the so-called *embedding approach*. The idea of this concept (Takens, 1981) consists in determining the relationship between the value of state $\boldsymbol{X}_{j+T}$ at time instant j+T and a finite sequence y^j_m of its $\mathbf{m}$ past values.

$$x_{i+T} = f_T(y^i_m) = f_T(x_i, x_{i-1}, \ldots, x_{i-(m-1)}) \qquad (10)$$

According to the embedding theorem of Takens, it is possible to adjust the value **m**, called *embedding dimension*, such that that the resulting evolution (10) of $\boldsymbol{x}_i$ reconstructs the topological properties of the original attractor **M**. The value **m** is closely related to the attractor Hausdorff dimension. To find it, we use the so-called correlation integral concept, introduced and developed in (Grassberger and Procaccia, 1983; Packard et al., 1980). In Takens, the embedding dimension **m** can be defined as: $\mathbf{m=2m^*(\upsilon)} + 1$, where $\mathbf{m}^*(\upsilon)$ is the minimum integer number greater than attractor dimension υ.

When constructing an approximation of function $\boldsymbol{f}_T$ in relation (10), almost all authors (e.g Porporato and Ridolfi, 1997; Casdagli, 1989), propose an approximation of $\boldsymbol{f}_T$ by polynomials of a given order (*local model* concept):

for a given "embedding point" $\boldsymbol{y}^i_{\mathbf{m}}$ the set $\mathbf{Y}^i = \{\boldsymbol{y}^j_{\mathbf{m}}\mathbf{:\ j} \in \mathbf{K}_i\}$ is determined, as the set of K nearest (in the norm $\|\bullet\|$) *neighbours* to y^i_m. Then, function $\boldsymbol{f}_T$ is adjusted, such as to obtain: min $\sum \mid \mathbf{x}_{j+T} - \boldsymbol{f}_T(\boldsymbol{y}^j_{\mathbf{m}}) \mid^2$, over $j \in K_i$, where $\mathbf{x}_{j+T}$ is the first component of vector $\boldsymbol{y}^{j+T}_{\mathbf{m}}$.

Function $\boldsymbol{f}_T$ is searched as a linear function (2-nd order approximation), or as a polynomial of 2-nd degree (3-rd order approximation). We put then respectively:

$$f_T(x_i, x_{i-1}, \ldots, x_{i-(m-1)}) = < a, y^i_m > + b \tag{11}$$

$$f_T(x_i, x_{i-1}, \ldots, x_{i-(m-1)}) = (y^i_m)^T C(y^i_m) + < a, y^i_m > + b \tag{12}$$

where 'a' is a vector, 'b' is a constant; C is **m×m** matrix; < > is inner product.

Numerous computations for inflows in the Wupper Reservoir System were performed, in order to verify the hypothesis of deterministic chaos and to find the embedding dimension **m**. It has been shown, that the data represent the chaotic dynamics of dimension **m** = 7. Then, two prediction models (11) and (12) have been built. The best quality of forecast (the minimum error between forecasts and the original data) was obtained with **m** = 4 and **m** = 5; thus, less than **m**=7. The quadratic approximation model (with prediction horizon **T**=3) showed better results than the linear model and the model in the form of average historical values.

6. COMPARISON OF CONTROL METHODS BY SIMULATION

The simulation of some of the chosen control methods were carried out over the long time horizon of 39 years, with the real, historical data of natural inflows to the system. The methods under investigation have been partially discussed in the previous sections. Let us mention here once again those, which - after an initial stage of synthesis consisting of adjusting their parameter values - have been thoroughly compared by simulation.

1) TLM - Two-layer optimisation method with:

a) the complex, long-term planning aiming at the optimisation of all the particular goals in a compromising manner.

b) the realisation of the planned decisions (water supplies and discharges) in the real, current conditions.

2) SDR -The standard decision rule was developed by simulation techniques on the basis of a historical record of 39 years and ten synthetic records of 50 years (Schultz and Harboe, 1989).

3) SDP - Sequential Stochastic Dynamic Programming (Napiórkowski et al., 1997).

In the first method, requiring solution of the optimisation problem (9), the long-term prediction of inflows (for 36 10-day periods) consists of two parts. For 10-day periods j = [1,3] the results of one of the discussed inflow prediction models were used and for j = [4,36] the average values of historical data were applied. Furthermore, to compare and investigate the 'power' of optimising methods, the variants denoted OPT and AVR have been considered, which differs from the optimising methods only in the fact that real/average values of inflows are put in place of predicted values.

In order to compare in a clear, well-ordered manner the results of different controls and the results of the other control techniques, we introduce the following scalar criteria goals (Napiórkowski and Terlikowski, 1996):

-global deficit time TD:

$$TD = Card(\{\, j : u_W^j < z_W^j \,\}) \tag{13}$$

- average relative deficit AvD:

$$AvD = \sum_{j=1}^{36} \frac{(z_W^j - u_W^j)_+}{z_W^j} \frac{1}{36} \tag{14}$$

- maximum relative deficit MxD;

$$MxD = \max(\{\frac{(z_W^j - m_W^j)_+}{z_W^j} : j = 1,...,36\}) \tag{15}$$

- average losses in recreation area in the summer period for Bever Reservoir:

$$RE_B\,Av = \sum_{j=1}^{12} \frac{REmax - f_V(V_B^j)}{REmax} \frac{1}{12} \tag{16}$$

where REmax corresponds to maximum possible water area.

As a result, we obtain a sequence of 4 numbers, characterising system performance in a synthetic way. This could be sufficient to evaluate and compare the different functions for one year, e.g. with the aid of any multiobjective optimisation method. However, it is more complicated, because we have to compare the control effects not for a particular year, but for 39 years long historical record.

To solve such a problem it is necessary to use a specific approach, which is arbitrary to some extent and makes use of intuition. To obtain the final comparison results we analyse the diagrams of s.c. frequency (reliability) criteria calculated on the basis of simulation for 4 scalar criteria (13-16).

Those frequency criteria are also functions, but defined over the set of values of respective scalar criteria. Their values represent the number of years, for which the respective scalar criterion has its values in a given range. Formally, e.g. for MxD we have:

$$F_{MxD}(x) = Card(\{ I : MxD^I \leq x \}) \tag{17}$$

where MxD^I denotes the value of criterion MxD for the year I. As it is seen, F corresponds to the notion of cumulative distribution function of the "random variable" MxD^I, when I is treated as representing the elementary events.

7. RESULTS AND CONCLUSIONS

Some of the simulation results for the control methods considered, namely SDR, TLM, SDP and OPT are presented below by means of the reliability criterion F. Fig.(4-7) show the diagrams of distribution F corresponding to the criteria (13-16).

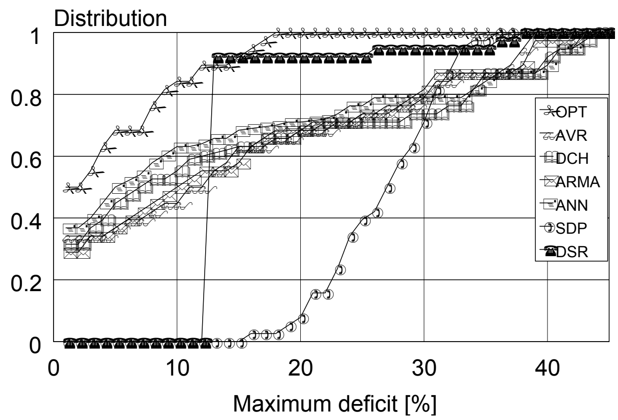

Fig.4 Maximum relative deficit at W cross-section.

The advantage of TLM, for all considered inflow prediction models (AVR, DCH, ARMA, ANN), but especially for ANN (the best forecast) and DCH, is evident in the sense of MxD criterion (Fig.4). It stems from the fact that TLM takes into account the co-operation of the whole system and better co-ordinates the partial decisions when compared with the other methods discussed.

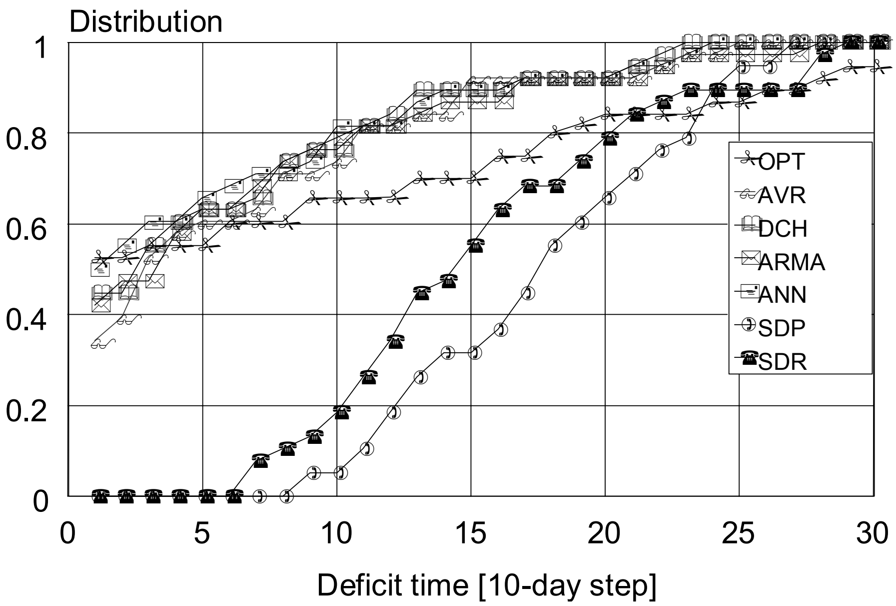

Fig.5 Global deficit time at the W cross-section.

For TD criterion (Fig.5) the plot of OPT is below the plots of ANN, DCH, ARMA and AVR models. It reflects the fact that the "system" prefers longer and small deficits rather than short and deep ones and of course the knowledge of future inflows guarantees the lowest maximum deficit.

For the criterion AvD (Fig.6) the differences between diagrams corresponding to 4 prediction models are smaller, but the method TLM is still shown to be better than SDP. Moreover, these diagrams are then closer to the "optimal" ones (those for OPT).

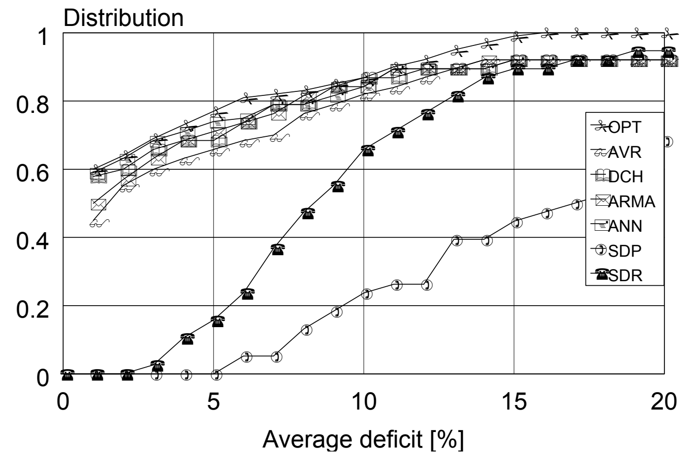

Fig.6 Average relative deficit at W cross-section.

SDR gives results between TLM and SDP; the latter giving the worst results for all but the recreational loss criterion (fig.7).

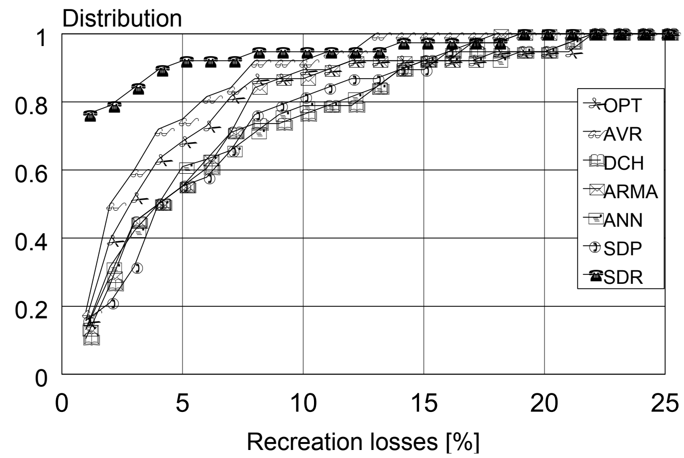

Fig.7 Losses in recreation area for Bever reservoir.

SDP gives the worst results for all criteria. This results from the character of this technique. SDP requires the discretization of both inflows and storages and due to "curse of dimensionality" that discretization cannot be too dense.

To recap, the method called TLM proved to be the best for reservoir system simulation with short time prediction obtained by means of ANN.

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REFERENCES

Casdagli M., 1989. Nonlinear prediction of chaotic time series, Physica D 35, 335-356.

Grassberger P. and Procaccia I., 1983. Characterization of strange attractors, Physical Review Letters, Vol. 50, No 5, 346-349.

Lippman R. 1987. An introduction to computing with neural nets. IEEE Trans., ASSP Magazine 4, 4-22,.

Napiórkowski, J.J. & T.S. Terlikowski 1996. Operational control for a multireservoir system - multiobjective approach. In P. Zannetti & C.A. Brebbia (eds.) *Development and Application of Computer Techniques to Environmental Studies VI*, Computational Mechanics Publications, Southhampton, Boston.

Napiórkowski J.J., Terlikowski T.S., Wolbring F., 1997. Multiobjective Approach to the operational control synthesis - The Wupper Reservoir System Case Study. In Operational Water Management, Eds. Refsgaard and Karalis, 165-170, A.A.Balkema/Rotterdam/Brookfield.

NeuroDimension, Inc., 1997. The Neural Network Simulation Environment, Manual Version 3.

Packard N.H., Crutchfield J.P., Farmer J.D. and R.S. Shaw., 1980. Geometry from a time series, Phys. Rev. Lett., 45, 712-716.

Porporato A. and Ridolfi L., 1997. Nonlinear analysis of river flow time sequences, Water Resources Research, Vol. 33, No 6, 1353-1367.

Rumelhart D., Hinton G. and Williams R. "Learning internal representations by error propagation." In Parallel Distributed Processing, (eds. Rumelhart and McClelland), MIT Press, 1986.

Schultz, G.A. and R. Harboe 1989. Development of reservoir operation rules for Wupper Dam during floods and low flow periods. Wasserwirtschaft 79, 1-4.

Sukhodolov A., Napiórkowski J.J., Rowinski P., 1996. Applicability of Flow Prediction Based on Attractors. Acta Geophysica Polonica, XLIV, No.3, 277-286.

Takens F., 1981, Detecting strange attractors in turbulence. In: D.A. Rang and L.S. Young (eds.), “Topics in Dynamical Systems and Turbulence”, 366-381, Springer-Verlag, Warwick.