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Multiobjective approach to the operational control synthesis – The Wupper Reservoir System case study

J.J.Napiórkowski & T.S.Terlikowski
Institute of Geophysics, Polish Academy of Sciences, Warsaw, Poland

F.Wolbring
Institute of Hydrology, Water Resources Management and Environmental Techniques, Ruhr University Bochum, Germany

ABSTRACT: The two-layer hierarchical technique for real-time operation is presented along with the application to a part of the Wupper Reservoir System in the Federal Republic of Germany. The reservoir system consists of two reservoirs in series with additional inflow to the lower reservoir. The tasks of these reservoirs are flood control, recreation, hydropower and low flow augmentation with the aim of water quality improvement. It is shown that the introduced optimization concept improves considerably the system performance in comparison with the Standard Operation Rule for 38 year long historical data record.

1 INTRODUCTION

A technique for determining the yield of a multireservoir water supply has been applied to a part of the Wupper Reservoir System in Germany. The major objectives of this particular system are flood control, recreation, hydropower and low flow augmentation. The unified methodology comprises a large class of acceptable solutions and covers the wide range of specific conceptual approaches is presented. It enables the inclusion of the operator's preferences, intuition and experience. The presented technique may be reduced to the following conjunct parts: the optimization of a simplified quantitative model of the actual system and the multiobjective verification-comparison through simulation. The first part consists of constructing a relatively wide class of control structures based on the two-layer optimization technique method (Terlikowski 1993, Napiórkowski & Terlikowski 1996). The second part is based on the simulation performed for historical data over a long time horizon (39 years). This simulation is an active research and consists of testing and adapting the control rules by computation of many objective values. Several control schemes have been proposed in the form of computer programmes for the Wupper Reservoir System. They have been compared for a large number of simulated years and for many objectives. One can see the ambiguity of different unified, aggregated evaluation methods in such a problem. The proposed control schemes (TLM) are compared with the so called Standard Decision Rule (SDR), and Stochastic Dynamic Programming (SDP) to present their undoubted advantages. The theoretical case of perfectly known future inflows (OPT) is also tested to show the quality of the proposed control structure.

2 DESCRIPTIONS OF THE CASE SYSTEM MODEL

The catchment of river Wupper is located in the southern part of North Rhine Westfalia. The hydrological features of this catchment are characterized by a massive rocky underground covered only by a small layer and an average yearly precipitation of about 1300 mm per year. The missing ability of storing water underground leads to dangerous floods as well as to extreme droughts. To achieve the ability to handle this problem several reservoirs were built. Here we are just interested in the management of the two reservoirs governing the discharges in the city of Wuppertal which lies about 20 km downstream of reservoir No. 2. Figure 1 shows the simplified Wupper Reservoir System.

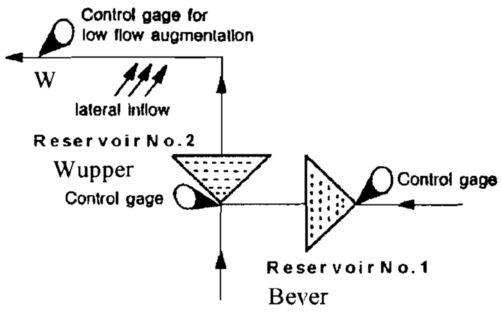

Figure 1. Basic structure of reservoir system

It contains two reservoirs located in series, the control center at reservoir No. 2, several runoff and rainfall gages. The release of the reservoirs depends mainly on the runoff at the control gage in Wuppertal. The runoff of 5 m^3/s at this gage is sufficient for the demanded water quality, runoff less than 3.75 m^3/s should be avoided and the runoff less than 1 m^3/s has to be regarded as ecologically disastrous. The basic hydrological and reservoir characteristics are given in Table 1.

Table 1. The basic characteristics of the Wupper Reservoir System

Reservoir	Bever	Wupper
total storage Vmax (mln m^3)	23.70	25.90
dead storage Vmin (mln m^3)	0.70	2.10
max. outflow (m^3/s)	17.00	180.00
min. outflow (m^3/s)	0.10	1.00
Annual average flow (m^3/s)	0.94	3.51
catchment area km^2	25.7	212.00

The purpose of the model is to describe relationships between flow rates in the rivers over a long time horizon (one year) with the discretization period of one decade (10 days). Therefore only the dynamics of the storage reservoir are considered, while effects of the dynamics of flow in the river channels are neglected.

For brevity, the following notation is used:

- j - number of decades
- V^j - state of the reservoir
- d^j - natural inflow
- u^j - flow in a given cross-section
- z^j - water demand
- m^j - outflow from the reservoir
- 1,2 - denote the Bever and Wupper reservoirs
- 3 - denotes the lateral inflow
- W - cross-section at Wuppertal

According to the introduced notation, we are able to write state equations for the system of reservoirs and flow balance equations formulated for the selected cross-section W. State equations are:

$$V^{j+1} = V^j - B*m^j + C*d^j \tag{1}$$

where

$$V=[V_1,\ V_2];\ m=[m_1,\ m_2];\ d=[d_1,\ d_2] \tag{2}$$

and

$$B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix};\quad C=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix} \tag{3}$$

The flow balance equation for the considered cross-section is as follows:

$$u_W^j = m_2^j + d_3^j \tag{4}$$

3 THE OPTIMIZATION PROBLEM

The objective function of the optimization problem under consideration for any time instant (for any decade) and for annual time horizon (T=36) can be written in the form of a penalty function:

$$Q(m,V)=\sum_{j=k}^{k+T}[a_1^{*j}(m_1^j-z_1^j)^2+a_2^{*j}(m_2^j+d_3^j-z_W^j)^2$$

$$+b_1^j(V_1^j-V_1^{*j})^2+b_2^j(V_2^j-V_2^{*j})^2] \tag{5}$$

In equation (5), symbols a and b with respective subscripts denote weighting coefficients. Other quantities which occur in its formulation are treated as parameters. The performance index Q is expressed explicitly on controls m^j and the state trajectory V^j (reservoir contents)

$$Q(m,V)=\sum_{j=k}^{k+T}Q(m^j,V^j) \tag{6}$$

The objective function during each decade is subject to the constraints on the state of the system, controls and flows in given profiles

$$V^j_{\min} \le V^j \le V^j_{\max}$$
$$m^j_{\min} \le m^j \le m^j_{\max} \qquad (7)$$

3.1 Required retention trajectory V^{*j}

It is assumed that the operation of the reservoir system is carried out on annual basis in the following way:

* by late December, the reservoirs normally are returned to low level to prepare the system for the next flood season completing the annual cycle.
* the storage reservation for flood control on January 1 was determined for controlling the maximum probable flood. During the normal filling period, January-April, the reservoirs should be filled up completely.
* during the May-August period the first reservoir should be filled up to meet recreation requirements.
* during the May-November period the water stored in and released from the reservoirs is used for low flow augmentation and hydropower.

3.2 Weighting coefficients a^{j+} and b^j

According to the general objective of the control problem, which is aimed at the rational protection against water deficits and at reaching the desired state at the end of April, the following values of weighting coefficients in the optimization problem are used: $a^{j+}=1$ if demands are greater than supply and $a^{j+}=0.01$ otherwise, for k=[1,36]. As far as the second coefficient is concerned, in order to avoid a good performance in one year followed by a very poor performance in the next year $b^j=0.01$ for j=[1,12] (May-August), $b^j=0.001$ for j=[1,30] (September-February), $b^j=0.004$ for j=[31,33] (March) and $b^j=0.01$ in April, for j=[34,36].

4 TWO-LEVEL OPTIMIZATION TECHNIQUE

To solve the aforementioned problem we adjoin the equality constraints (1) with the Lagrange multiplier sequence λ (prices). The Lagrangian function has the form:

$$L(m,V,\lambda)=\sum_{j=k}^{k+T}[Q(m^j,V^j)$$
$$+\lambda^j(V^{j+1}-V^j+B*m^j-C*d^j)] \qquad (8)$$

To include the state-variable and outflow constraints the above problem is solved by means of the two-level optimization method and in a decentralized (coordinated) fashion. At this stage we make use of the additivity of the Lagrangian function (9) and the possibility of separation of the decision variables.

The Lagrangian function has a saddle point which can be assigned by minimizing **L(λ,V,m)** with respect to **V** and **m**, and then maximizing with respect to **λ**. Finally, the optimization problem can be expressed in the form:

$$\max_{\lambda}\ [\min_{V,\ m} L(\lambda,\ V,\ m)] \qquad (9)$$

with inequality constraints on state and control and no constraints on Lagrange multipliers. Figure 2 illustrates how the two-layer optimal control method works.

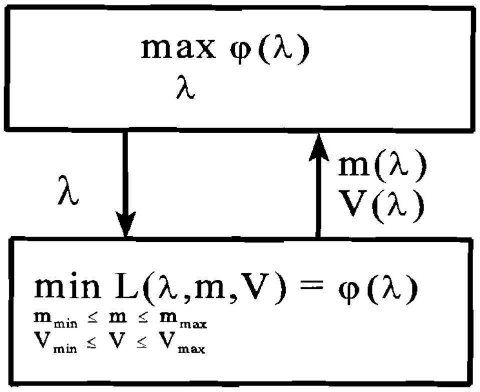

Figure 2: Two-level optimization method

At the *lower level* for given values of the Lagrange multipliers we look for the minimum of the Lagrange function. The necessary condition is the zero value of the gradient with respect to **m** and **V**. The task of the upper level is to adjust the prices **λ** in such a way that the direct control of the reservoir, affected by **λ**, results in the desired balance of the system (the mass balance equation (1) is fulfilled satisfactorily). On the upper layer, in the maximization of the Lagrange function with respect to **λ** the standard conjugate gradient technique is used.

In the applied Two-Layer optimization control method (TLM) the solution of the two-level optimization problem (9) is the essential "upper layer part". Note, that this planning layer "proposes" the sequence of T control variables $\{m^k,...m^{k+T}\}$ for a one year long time horizon. At the current decade they are taken into account by the lower layer that tries to apply them in the real conditions and are eventually subject to some additional operator's interventions.

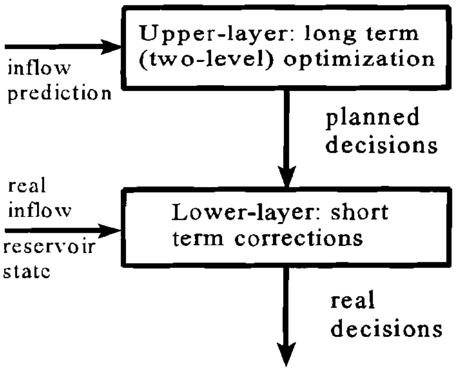

Figure 3. Two-layer control method

5 COMPARISON OF CONTROL METHODS THROUGH SIMULATION

The simulation of some chosen control methods has been carried out over the long time horizon of 39 years, with the real, historical data of natural inflows to the system. The methods under investigation have been partially discussed in the previous sections. Let us mention here once again those of them, which - after an initial stage of synthesis consisting in adjusting their parameter values - have been thoroughly compared by simulation.

1) TLM - Two-layer optimization method with:

a) the complex, long-term planning aiming at the optimization of all the particular goals in a compromising manner.

b) the realization of the planned decisions (water supplies and discharges) in the real, current conditions.

2) SDR -The standard decision rule was developed by simulation techniques on the basis of a historical record of 39 years and ten synthetical records of 50 years (Schultz & Harboe 1989). This operation rule takes account of the contents of the two reservoirs and the flow at the control gage in the city of Wuppertal. Therefore the contents of the two reservoirs were divided into five levels and four low flow augmentation targets between the desired runoff of 5 m^3/s and the disastrous runoff of 1 m^3/s at the control gage were defined.

3) SDP - Sequential Stochastic Dynamic Programming.

In the first method, requiring solution of the optimization problem (9), the average values of real historical data for the period 1946-1983 have been taken as the long-term prediction of inflows. Furthermore, to compare and investigate the „power" of optimizing methods, the variant denoted OPT has been considered, which differs from the optimizing methods only in the fact that real current values of inflows are put in place of predicted values. It is worth noting that it is possible to make use of a more precise knowledge of future inflows only in the methods including long-term planning.

Each method is evaluated through many different performance indices - equivalent, in a way, to degree of realization of conflicting goals. Hence, the indices reflect only the partial, not global, effects of system performance. In our model, each performance index is represented as a function of time:

- for the cross-sections W this is the deficit function expressed with a respective time unit (decades).
- for the reservoirs this is the function of storage level and control (we are interested mainly in the average value of water area of Bever reservoirs in the summer period and hydropower).

Each index is evaluated through many scalar criteria. In order to define them precisely, let us consider the deficit in meeting the needs at the cross-section W in a period of 1 year. The function u_W^j, where j corresponds to decade - together with z_W^j (representing the target) - characterize this one particular index in the most complete manner. However, in order to compare in a clear, well ordered manner the results of different controls, and the results of the others control techniques, we introduce some scalar criteria depending on these functions.

The following criteria have been proposed for the functions which represent the system performance (j is included in a given period of 1 year, i.e. of 36 decades):

- global deficit time TD:

$$TD = Card(\{ j:\ u_W^j < z_W^j \}) \qquad (10)$$

- average relative deficit AvD:

$$AvD = \sum_{j=1}^{36} \frac{(z_W^j - u_W^j)_+}{z_W^j} \frac{1}{36} \qquad (11)$$

- maximum relative deficit MxD;

$$MxD = \max(\{\frac{(z_W^j - m_W^j)_+}{z_W^j} : j=1,...,36\}) \qquad (12)$$

At the same time the trajectories V_1^j, V_2^j are described by 2 criteria: average losses in recreation area (a nonlinear function f_V of storage volume) in the summer period for Bever Reservoir:

$$RE_B Av = \sum_{j=1}^{12} \frac{REmax - f_1(V_B^j)}{REmax} \frac{1}{12} \qquad (13)$$

where REmax corresponds to maximum possible water area; and the losses in hydropower production (a nonlinear function $f_{V,m}$ of storage and outflow from Wupper reservoir)

$$EN_W Av = \sum_{j=1}^{36} \frac{ENmax - f_{V,m}(V_W^j)}{ENmax} \frac{1}{36} \qquad (14)$$

where Enmax corresponds to maximum possible performance of turbines at Wupper dam.

As a result, we obtain a sequence of 5 numbers, characterizing system performance in a synthetic way. This could be sufficient to evaluate and compare the different functions for one year, e.g. with the aid of any multiobjective optimization method. However, it is more complicated, because we have to compare the control effects not for a particular year, but a for 39 years long historical record.

To solve such a problem it is necessary to use a specific approach, which is arbitrary to some extent and makes use of intuition. To obtain the final comparison results we analyze the diagrams of s.c. frequency (reliability) criteria, calculated on the basis of simulation for 5 scalar criteria (10-14).

Those frequency criteria are also functions, but defined over the set of values of respective scalar criteria. Their values represent the number of years, for which the respective scalar criterium has its values in a given range. Formally, e.g. for MxD we have:

$$f_{MxD}(x) = Card(\{I: x-\Delta \le MxD^I \le x\}) \qquad (15)$$

$$F_{MxD}(x) = Card(\{I: MxD^I \le x\}) \qquad (16)$$

where MxD^I denotes the value of criterium MxD for the year I, and Δ is the step of discretization of values of MxD. As it is seen, f corresponds to the notion of density function and F - of comulative distribution function of the „random variable" MxD^I, when I is treated as representing the elementary events.

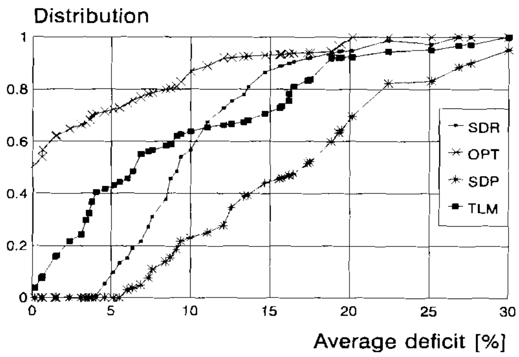

Figure 4. Average relative deficit at W cross-section.

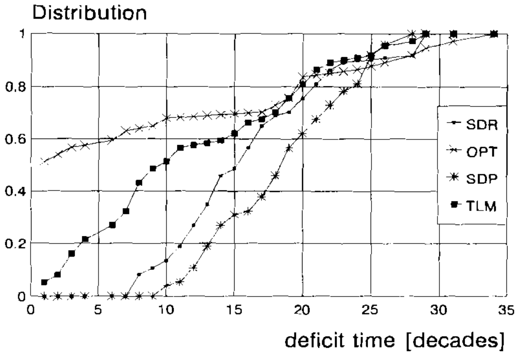

Figure 5. Global deficit time at the W cross-section.

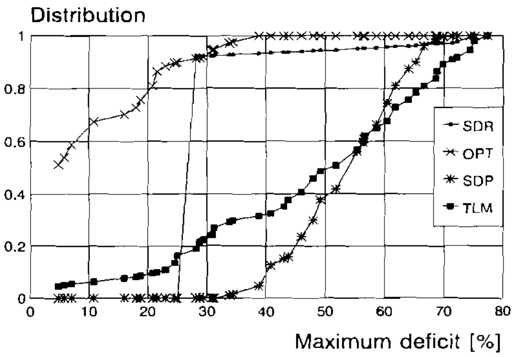

Figure 6. Maximum relative deficit at W cross-section.

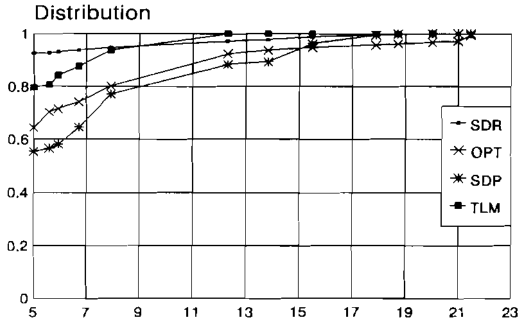

Figure 7. Losses in recreation area for Bever Reservoir.

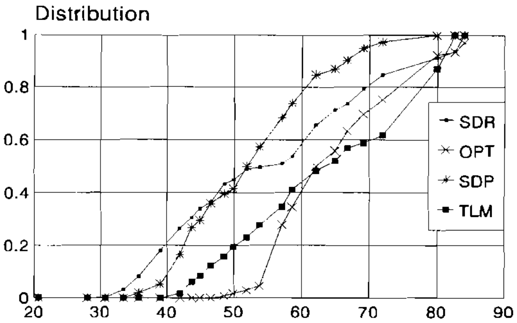

Figure 8. Energy losses in hydropower.

RESULTS AND CONCLUSIONS

Some of the simulation results for the considered control methods, namely SDR, TLM, SDP and OPT are presented below by means of the reliability criterium F. Figures (4-8) show the diagrams of distribution F corresponding to the criteria (10-14). The advantage of TLM (and of course OPT) is evident in the sense of the first three considered scalar criteria, namely (10, 11, 12). TLM gives the worst result in the case of the energy losses criterion. It results from the fact that this particular criterium is not directly included in the objective function (5).

SDP gives the worst results for all but energy losses criteria. This results from the character of this technique. SDP requires the discretization of both inflows and storages and due to "curse of dimentionality" that discretization can not be to dense.

It appears that SDR is rather to conservative and gives results between TLM and SDP.

The TLM takes into account the cooperation of the whole system and better coordinates the partial decisions when compared with the other methods discussed.

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