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Comment on "Identification of a Constrained Nonlinear Hydrological System Described by Volterra Functional Series" by J. Xia

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In his recent paper *Xia* [1991] presents an interesting penalty function technique for the identification of kernel functions for watershed systems which are represented by Volterra series. It is unfortunate that the published results of research that has been carried out in hydrology on rainfall-runoff modeling by means of Volterra series in the last 13 years were not taken into account. Hence, there are some errors and misconceptions in the paper.

Applicability of Volterra Series to Watershed Modeling

It is silently assumed in the paper by Xia that the watershed system can be represented by a lumped nonlinear operator P, which maps a space of lumped rainfalls into a space of corresponding flood waves at a measuring station on a river. If this unknown operator P is differentiable as many times as may be required in the Frechet sense, and the system is deterministic, time invariant, initially relaxed and nonanticipating, then its behavior can be approximated within the radius of convergence to any predetermined accuracy by a truncated Volterra series [*Volterra*, 1959]:

$$y(t) = \sum_{i=1}^{n} y_i(t) = \sum_{i=1}^{n} \left| \int_0^t \right|_i h_i(\tau_1, \tau_2, \cdots, \tau_i) \cdot \prod_{k=1}^{i} x(t - \tau_k)\, d\tau_k \qquad (1)$$

A watershed is a time-variable system and consequently the assumption of a time-invariant model may lead to unacceptable predictions of runoff. Nevertheless, in the identification process under review the author has not separated the effective rainfall and the base flow. Instead "the actual gain factor" for a particular catchment was introduced, which is storm dependent. As a result, the kernel functions are storm dependent and the mass conservation law is not fulfilled. The most distinct example is the runoff in Figure 2*a*, for May 17, 1975.

As has been stated by *Boneh and Diskin* [1973, p. 764],

> although it is true that base flow and rainfall excess are nonlinear effects, it does not necessarily follow that the Volterra series is the best tool for dealing with these nonlinearities.

The Physical Constraints

The Volterra series is suitable for representing lossless systems mainly due to necessary and sufficient conditions for losslessness which were introduced by *Boneh* [1972, 1982] and are given for symmetric kernels by

$$\int_0^\infty h_1(\tau)\, d\tau = 1 \qquad h_1(\tau) \geq 0 \qquad (2)$$

$$\int_0^\infty h_i(\tau,\ \tau + C_2, \cdots, t + C_i)\, d\tau = 0 \quad \text{for all } C_k \geq 0 \qquad (3)$$

The condition (3) means that the nonlinear components affect only the distribution of the predicted runoff ordinates and the total value of each of these components is zero.

Xia has changed condition (2) by introducing the actual gain factor G_a. One can accept some losses of water caused by infiltration ($G_a = 0.4$ for the Baihe watershed system), but the question arises if 50% amplification is a typical property of hydrologic systems ($G_a = 1.5$ for the Wulong watershed system). It seems the lumped rainfall was not properly determined.

Kernel Expansion in Orthogonal Functions

Napiórkowski [1978] and *Napiórkowski and Strupczewski* [1979, 1981] analytically obtained the first two kernels of the Volterra series for the cascade of identical nonlinear reservoirs. For that model the structure of the kernels was shown to be

$$h_1(\tau) = aH_n(\tau) \qquad (4)$$

$$h_2(\tau, \sigma) = b\{H_n(\tau) \sum_{i=1}^{n} H_i(\sigma) + H_n(\sigma) \cdot \sum_{i=1}^{n} H_i(\tau) - H_n[\max(\tau, \sigma)]\} \qquad (5)$$

in which n is the number of reservoirs, a and b are parameters resulting from the outflow-storage relation and

$$H_i(\tau) = \exp(-a\tau)(a\tau)^{i-1}/(i-1)! \qquad (6)$$

Note that the kernels (4)–(6) fulfill the conditions (2) and (3) and that the second-order kernel is not differentiable along the main diagonal. That property is characteristic of any system which combines the linear static and nonlinear dynamic characteristics.

Having determined the kernels of the Volterra series in an

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analytical manner, different methods of identification based on input-output data were examined [*Napiórkowski*, 1978, 1986, 1988; *Napiórkowski and Strupczewski*, 1984]. The conclusion was that the standard expansion of the second-order kernel in orthogonal functions may give good results in the case of systems with nonlinear (quadratic) characteristics, but in the case of closed systems, it cannot lead to satisfactory results. There is a "contradiction" between the approximation theory and practice. In theory, any two-dimensional function can be approximated to any predetermined accuracy by orthogonal functions, provided the number of functions used is large enough. In hydrological practice this number is limited to 5 or 6 (15 or 21 unknown parameters). The larger number of parameters leads to ill-conditioned optimization problems.

On the basis of (6) the subset

$$h_2(\tau, \sigma) = \sum_{i=0}^{M} \sum_{j=0}^{M} a_{ij}\phi_i(\tau)\phi_j(\sigma) + \sum_{i=0}^{M} b_i\phi_i[\max(\tau, \sigma)] \quad (7)$$

in which $\phi_i(\tau)$ are orthonormal polynomials, was recommended for identification of closed or open systems with nonlinear dynamic and quasi-linear static behavior.

The author fully agrees with Xia as far as the use of Laguerre orthornormal polynomials (or their discrete analog) is concerned. However, it is suggested to use them in time-scaled form:

$$\phi_i(t) = \sqrt{\alpha} \exp(-\alpha t/2) \sum_{k=0}^{i} (-1)^k \binom{i}{k} (\alpha t)^i/i! \quad (8)$$

The presence of the exponential damping term in (8) means that for practical purposes the memory of the model is time T for which

$$\int_0^T [\phi_i(t)]^2 \, dt \geq 0.99 \quad (9)$$

One can see that the memory of the model has nothing to do with the memory of the system unless a time-scaled parameter is introduced, which is calibrated during identification.

Xia used unscaled Meixner functions for both daily and hourly data with the same number of approximating functions. Moreover, the assumed memory length of the model is certainly too small ($m = 10$ in Table 1 or $m = 15$ in Figure 1*b* (which one is correct ?)), because the first-order kernel function does not converge to 0 for $t = m$.

Identification of Kernels of Volterra Series as an Ill-Posed Problem

The determination of the kernels of the Volterra series using a finite length of input and output data is a typical example of an ill-posed problem in the sense of Tikhonov [*Napiórkowski*, 1978, 1986, 1988; *Napiórkowski and Strupczewski*, 1984]. It follows from the above that one may find large errors of kernel estimates even if measurement errors are very small. This is caused by a too wide class of functions within which the solution is sought. That class has to be reduced on the basis of some mathematical and physical characteristics, to such a subclass for which the identification problem has a solution that depends continuously on the measurement data.

By using the physical constraints, Xia reduced the class of functions within which the solution is sought, but it seems that this was not enough to get a correct solution. The oscillation in the first-order kernels in Figure 1 is characteristic of ill-posed problems, rather than of Baihe and Wulong catchments (see analytical example of *Napiórkowski* [1978]) and should be removed by means of the regularization method [*Tikhonov and Arsenin*, 1974].

The author would like to know more about the shape of the second-order kernel.

Model Copositivity

Any Volterra series can be applied only within a range of convergence. However, when only the estimates of the first two or three kernel functions are known after identification it is impossible to determine the radius of convergence for practical applications. As a practical means of overcoming this problem *Boneh and Golan* [1978] and *Boneh* [1982] derived a general criterion for maximum amplitude of input and/or maximum volume of input which when applied at least ensures a positive output response to a positive input signal.

This aspect of modeling of hydrologic systems has not been referred to by Xia. The author's practical experience is that one gets negative outflow when the amplitude of rainfall for the calibration period is less than the amplitude of rainfall for the verification period. I would like to ask Xia to verify this conclusion by using the data from Figure 2*b* for calibration and data from Figure 2*a* for verification.

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