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The optimization of a third-order surface runoff model

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ABSTRACT The objective of the paper is to describe a method of representing the N-th order response of a surface runoff model by means of a conceptual model with (N + 1) parameters. The method is illustrated for the representation of the third order response by a four-parameter conceptual model. The conceptual model used is a cascade of n equal nonlinear reservoirs with arbitrary storage-outflow relationship whose first N derivatives at a reference outflow define the remaining model parameters. The outflow is shown to be linear in the parameters representing the second and higher order derivatives so that these parameters can be rapidly optimized for any choice of the first two parameters (number of reservoirs and first derivative). The third-order state model (TOSM), which is shown to be equivalent to a three term Volterra series, is fitted to the records of eight storms whose quadratic response was previously determined by Diskin & Boneh (1973) by direct determination of the 74 values in the linear and quadratic kernels.

L'optimisation d'un modèle d'écoulement de surface du troisième ordre
RESUME L'objectif de cette communication est la déscription d'une méthode pour représenter la réponse du $n^{ème}$ ordre d'un modèle d'écoulement de surface au moyen d'un modèle conceptuel avec N + 1 paramètres. On illustre cette méthode par la représentation de la réponse du troisième ordre au moyen d'un modèle conceptuel à quatre paramètres. Le modèle conceptuel utilisé est une cascade de N réservoirs égaux non linéaires avec une relation volume du stock/écoulement arbitraire dont les N premières dérivées pour un écoulement de référence définissent les paramètres restants. On montre que l'écoulement est linéaire en ce qui concerne les paramètres représentant les dérivées du second ordre et de l'ordre plus élevé de sorte que ces paramètres peuvent être rapidement optimisés pour un choix donné des deux premiers paramètres (nombre de réservoirs et premières dérivées). Le modèle d'état du troisième ordre (TOSM) que l'on montre équivalent à des séries de Volterra à trois termes est ajusté aux enregistrement des crues corréspondant à huit averses dont

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la réponse quadratique a été calculée auparavant par Diskin & Boneh (1973) au moyen de la détermination directe de 74 valeurs dans les kernels linéaires et quadratiques.

INTRODUCTION

In the case of runoff in a natural basin conditions are so complex that an accurate application of the hydraulic approach would require a topographical survey of great extent and complexity. Accordingly, two further types of approach to the rainfall-runoff problem have been developed in hydrology. These are the use of conceptual models and of black-box models. At first the representation of the catchment response by linear models was developed (Dooge, 1973) and more recently this was extended to representation by nonlinear conceptual models (Fleming, 1975) and nonlinear black-box analysis (Amorocho, 1973).

Nonlinear black-box analysis is concerned with representing the surface runoff system by a functional Volterra series in the form of a sum of convolution integrals. The conceptual model approach is to simulate a nature of the catchment response by a relative simple nonlinear model built up from simple nonlinear conceptual elements.

The method discussed in the paper combines the black-box model analysis with the conceptual model approach. To describe the nonlinear behaviour of the catchment a model in the form of third-order approximation of a cascade of nonlinear reservoirs is used. Such a model is called a Third-Order State Model (TOSM) and it can be shown to be equivalent to the three first terms of the Volterra series. The TOSM is derived from a somewhat more general state-space model, in order to show the general properties of the method proposed which can be applied to the higher order models.

THE THIRD-ORDER STATE MODEL

The surface runoff system is represented as a cascade of equal nonlinear reservoirs in which each nonlinear reservoir is responsible for part of the attenuation of the system response. This lumped dynamic model can be represented by a set of ordinary differential equations

$$\begin{aligned} \dot{S}_1(t) &= -f[S_1(t)] + I(t) \\ \dot{S}_2(t) &= -f[S_2(t)] + f[S_1(t)] \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{S}_n(t) &= -f[S_n(t)] + f[S_{n-1}(t)] \end{aligned} \tag{1a}$$

$$y(t) = f[S_n(t)] \tag{1b}$$

where n is the number of reservoirs, I(t) is the effective rainfall, $S_i(t)$ is the storage of the i-th reservoir, f[.] represents the outflow-storage relation of the individual reservoir, y(t) is the output from the model, the dot over the S indicates a first-order derivative with respect to time.

The function $f[S_i(t)]$ is not prescribed. We assume only that it is differentiable for $S_i(t) \geq 0$ as many times as there are terms in the Volterra series. In this study we are concerned with initially relaxed systems, so the initial condition is $\underline{S}(0) = 0$.

We may divide $S_i(t)$ and $y(t)$ into linear, quadratic, cubic parts and a residual error, i.e.

$$S_i(t) = \delta S_i(t) + \delta^2 S_i(t) + \delta^3 S_i(t) + e(S_i) \quad (2)$$

$$y(t) = \delta y(t) + \delta^2 y(t) + \delta^3 y(t) + e(y) \quad (3)$$

In order to compute the linear (δ), quadratic (δ^2) and cubic (δ^3) components of $y(t)$ and $S_i(t)$ we make use of the Taylor expansion of the outflow-storage relation about steady state $S_i(t) = S_o$ and $y(t) = y_o = f(S_o)$. In our case $S_o = 0$ and $y_o = 0$. Thus the expansion is:

$$f[S_i(t)] = aS_i(t) + b[S_i(t)]^2 + c[S_i(t)]^3 + e(f) \quad (4)$$

where

$$a = \frac{\partial f}{\partial S_i}, \qquad b = \frac{1}{2}\frac{\partial^2 f}{\partial S_i^2}, \qquad c = \frac{1}{6}\frac{\partial^3 f}{\partial S_i^3} \quad (5)$$

Substituting equations (2), (3), (4) and (5) into equation (1) and neglecting second and higher order terms gives the set of equations for the linear approximation as

$$\delta\dot{\underline{S}}(t) = a\phi\delta\underline{S}(t) + [1,0,\ldots,0]^T\, I(t) \quad (6a)$$

$$\delta y(t) = a\delta S_n(t) \quad (6b)$$

where

$$\phi = \begin{bmatrix} -1, & 0, & \ldots, & 0 \\ 1, & -1, & \ldots, & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0, & 0, & \ldots, & -1 \end{bmatrix} \quad (7)$$

When second-order increments are taken into account we get the additional equations

$$\delta^2\dot{\underline{S}}(t) = a\phi\delta^2\underline{S}(t) + b\phi[\delta\underline{S}(t)]^2 \quad (8a)$$

$$\delta^2 y(t) = a\delta^2 S_n(t) + b[\delta S_n(t)]^2 \quad (8b)$$

which are linear in $\delta^2\underline{S}(t)$ and $\delta^2 y(t)$. When third-order increments are also taken into account we have the further equations

$$\delta^3\underline{S}(t) = a\phi\delta^3\underline{S}(t) + 2b\phi\delta\underline{S}(t)\delta^2\underline{S}(t) + c\phi[\delta\underline{S}(t)]^3 \quad (9a)$$

$$\delta^3 y(t) = a\delta^3 S_n(t) + 2b\delta S_n(t)\delta^2 S_n(t) + c[\delta S_n(t)]^3 \quad (9b)$$

which are linear in $\delta^3\underline{S}(t)$ and $\delta^3 y(t)$.

It should be noted that the argument of the forcing function for equation (8) is the solution of equation (6) and that the argument of the forcing functions for equation (9) are solutions of equations (6) and (8). It should also be noted that the input of effective precipitation I(t) occurs only in equation (6). Consequently the addition of the components $\delta^2 y(t)$ and $\delta^3 y(t)$ effects only the distribution of the predicted runoff ordinates and the total value of each of these components is zero. A more detailed description of this approach can be found in Napiórkowski (1978), Napiórkowski & Stupczewski (1979, 1981) and in Napiórkowski & O'Kane (1984).

The first, second and third-order components described by equations (6), (8) and (9) form the Third-Order State Model (TOSM) which is assumed to represent the catchment response.

THE RELATION BETWEEN THE TOSM AND THE VOLTERRA SERIES

The description of any dynamic system by a Volterra series is a generalization of the concept of the transfer function which is of great importance in the analysis, design and control of linear systems. The Volterra series represent an explicit input-output relation for nonlinear systems (Wiener, 1942) and consist of an infinite series composed of terms of the form of convolution integrals.

$$y(t) = \int_0^t h_1(r)\, I(t - r)dr + \int_0^t\int_0^t h_2(r_1,r_2)\, I(t - r_1)I(t - r_2)dr_1dr_2$$
$$+ \int_0^t\int_0^t\int_0^t h_3(r_1,r_2,r_3)I(t - r_1)I(t - r_2)I(t - r_3)dr_1dr_2dr_3 + \dots \qquad (10)$$

This type of series was applied for the first time by Volterra in 1887 on functional equations (Volterra, 1930).

The solution of the linear set of equations (6a) describing the linear part of the storage trajectory is

$$\delta\underline{S}(t) = \int_0^t \exp(a\phi r)\ [1,0,\dots,0]^T I(t - r)dr \qquad (11)$$

where $\exp(a\phi r)$ is the transition matrix for equation (6). One can see that the *linear* component of the state of the cascade of nonlinear reservoirs can be described as the first term of the Volterra series

$$\delta\underline{S}(t) = \int_0^t \underline{K}_1(r)\ I(t - r)dr \qquad (12a)$$

where the vector of linear response kernels is given by

$$\underline{K}_1(r) = \exp(a\phi r)[1,0,\dots 0]^T \qquad (13a)$$

From equation (6b) one can see that the linear part of the outflow trajectory is

$$\delta y(t) = \int_0^t h_1(r)\ I(t - r)dr \qquad (12b)$$

with

$$h_1(r) = a\ K_{1,n}(r) \tag{13b}$$

where $K_{1,n}$ is the linear state kernel for the n-th reservoir in the cascade.

The solution of the linear set of equations (8a) describing the quadratic part of the storage trajectory is

$$\delta^2\underline{S}(t) = \int_0^t \exp(a\phi\tau)\ b\phi[\delta\underline{S}(t - \tau)]^2 d\tau \tag{14}$$

and the transition matrix for equation 8(a) is the same as for equation (6a). Having the solution for $\delta\underline{S}(t)$ from equation (12a) we can insert $[\delta\underline{S}(t - \tau)]^2$ in equation (14). The double change of the order of integration between τ and r_1, r_2 results in the second term of the Volterra series

$$\delta^2\underline{S}(t) = \int_0^t\int_0^t \underline{K}_2(r_1,r_2)\ I(t - r_1)I(t - r_2)dr_1dr_2 \tag{15a}$$

where

$$\underline{K}_2(r_1,r_2) = \int_0^{\max(r_1,r_2)} b\exp(a\phi\tau)\ \phi\ \underline{K}_1(r_1 - \tau)\underline{K}_1(r_2 - \tau)d\tau \tag{16a}$$

is the vector of the second order state kernels.

From equation (8b) one can see that the *quadratic* part of outflow trajectory is

$$y(t) = \int_0^t\int_0^t h_2(r_1,r_2)\ I(t - r_1)I(t - r_2)dr_1dr_2 \tag{15b}$$

with

$$h_2(r_1,r_2) = aK_{2,n}(r_1,r_2) + bK_{1,n}(r_1)K_{1,n}(r_2) \tag{16b}$$

where $K_{2,n}(r_1,r_2)$ is the second-order state kernel for the n-th reservoir and $K_{1,n}(r)$ is the first-order kernel already found in the linear approximation.

Finally, the solution of the linear set of equations (9a) describing the cubic part of the storage trajectory is

$$\delta^3\underline{S}(t) = \int_0^t \exp(a\phi\tau)\phi\{2b\delta\underline{S}(t - \tau)\delta^2\underline{S}(t - \tau) + c[\delta\underline{S}(t - \tau)]^2\}d\tau \tag{17}$$

where the transition matrix for equation (9a) is the same as for equations (6a,8a). Having the solutions for $\delta\underline{S}(t)$ from equation (12a) and for $\delta^2\underline{S}(t)$ from equation (15a) we can insert $\delta\underline{S}(t - \tau)\delta^2\underline{S}(t - \tau)$ and $[\delta\underline{S}(t - \tau)]^3$ in equation (17). The triple change of the order of integration between τ and r_1, r_2, r_3 results in the third term of the Volterra series

$$\delta^3\underline{S}(t) = \int_0^t\int_0^t\int_0^t \underline{K}_3(r_1,r_2,r_3)I(t - r_1)I(t - r_2)I(t - r_3)dr_1dr_2dr_3 \tag{18a}$$

where

$$\underline{K}_3(r_1,r_2,r_3) = \int_0^{\max(r_1,r_2,r_3)} \exp(a\phi\tau)\phi[2b\underline{K}_1(r_1-\tau)K_2(r_2-\tau,r_3-\tau)$$
$$+ c\underline{K}_1(r_1-\tau)\ \underline{K}_1(r_2-\tau)\ \underline{K}_1(r_3-\tau)]d\tau \qquad (19a)$$

From equation (9b) one can see that the *cubic* part of outflow trajectory is

$$\delta^3 y(t) = \int_0^t\int_0^t\int_0^t h_3(r_1,r_2,r_3)\ I(t-r_1)I(t-r_2)I(t-r_3)dr_1dr_2dr_3 \qquad (18b)$$

with

$$h_3(r_1,r_2,r_3) = a\underline{K}_{3,n}(r_1,r_2,r_3) + 2b\underline{K}_{1,n}(r_1)\underline{K}_{2,n}(r_2,r_3)$$
$$+ c\underline{K}_{1,n}(r_1)\ \underline{K}_{1,n}(r_2)\ \underline{K}_{1,n}(r_3) \qquad (19b)$$

where $K_{3,n}$ is the third-order kernel for the n-th reservoir.

This proves that the TOSM as represented by equations (6,8,9) is equivalent to the sum of three first terms of the Volterra series given by equations (12b,15b,18b).

THE IDENTIFICATION PROBLEM FOR THE TOSM

The problem to be solved is to fit the TOSM to a given surface runoff system for which data are available. This fitting may be carried out for several records of storms by comparing the corresponding direct surface runoff with the output from the model and manipulating parameters of the TOSM until a best-match is found in the sense of least-squares. So, we are looking for the parameters minimizing the objective function

$$J(n,a,b,c) = \Sigma_{i=1}^{M} \int_0^{T_i} [O_i(t) - \delta y_i(t) - \delta^2 y_i(t) - \delta^3 y_i(t)]^2 dt \qquad (20)$$

where M is the number of independent records, i is the number of storms, T_i is the length of the i-th outflow record, $O_i(t)$ is the i-th direct surface runoff.

From the computational point of view it is convenient to divide the cubic components $\delta^3 y(t)$ and $\delta^3\underline{S}(t)$ in equation (9) into two sub-components

$$\delta^3\underline{S}(t) = \delta^3\underline{S}'(t) + \delta^3\underline{S}''(t) \qquad (21a)$$

$$\delta^3 y(t) = \delta^3 y'(t) + \delta^3 y''(t) \qquad (21b)$$

based on the two parts of the forcing function. Since equation (9) is linear in $\delta^3\underline{S}(t)$, this decomposition and subsequent superposition involves no assumption or approximation.

The term $\delta^3\underline{S}'(t)$ is that part of the cubic component which results from the forcing by the cube of the linear storage component $\delta\underline{S}(t)$ and may be referred to as the cubic-linear sub-component. It is governed by the vector state transition equation:

$$\delta^3\dot{\underline{S}}'(t) = a\phi\delta^3\underline{S}'(t) + c\phi[\delta\underline{S}(t)]^3 \tag{22a}$$

The output due to the cubic-linear sub-component is given by

$$\delta^3 y'(t) = a\delta^3 S_n'(t) + c[\delta S_n(t)]^3 \tag{22b}$$

The second sub-component of the cubic response results from the forcing by the product of the linear storage component and the quadratic storage component and may be referred to as the cubic-quadratic sub-component. It is governed by the state transition equation

$$\delta^3\dot{\underline{S}}''(t) = a\phi\delta^3\underline{S}''(t) + 2b\phi\delta\underline{S}(t)\delta^2\underline{S}(t) \tag{23a}$$

and the output for the cubic-quadratic sub-component is given by

$$\delta^3 y''(t) = a\delta^3 S_n''(t) + 2b\delta S_n(t)\ \delta^2 S_n(t) \tag{23b}$$

Let us denote by $y^2(t)$ and $y^3(t)$ the solution of equation (8) and of equation (23) respectively for $b = 1$ and by $y^4(t)$ the solution of equation (22) for $c = 1$. Then due to linearity of equation (8,22, 23) the following relations are fulfilled

$$\delta y(t) = y^1(t) \tag{24a}$$

$$\delta^2(t) = by^2(t) \tag{24b}$$

$$\delta^3 y(t) = b^2 y^3(t) \tag{24c}$$

$$\delta^4 y(t) = cy^4(t) \tag{24d}$$

and the objective function takes the form

$$J(n,a,b,c) = \Sigma_{i=1}^{M}\int_0^{T_i}[O_i(t) - y_i^1(t) - by_i^2(t) - b^2y_i^3(t) - cy_i^4(t)]^2 dt \tag{25}$$

Note that on the right-hand side of equation (25) the functions $y^1(t)$, $y^2(t)$, $y^3(t)$ and $y^4(t)$ depend on the parameters a and n but do not depend on the parameters b and c. So, the problem of identification can be reduced to optimization with respect to two variables only, n and a. For given n and a two other parameters b and c result from the necessary condition for optimum $\partial J/\partial b = 0$ and $\partial J/\partial c = 0$.

The following steps are therefore required in the overall optimization of the model:

(a) assuming $b = 0$ and $c = 0$ compute initial values of parameters n and a as in linear analysis of catchment response, e.g. by moment matching (Nash, 1959; Dooge, 1973);

(b) assuming an integral value of the parameter n close to n and a suitable value of the scale parameter (a) compute the functions $y^1(t)$, $y^2(t)$, $y^3(t)$ and $y^4(t)$ solving equations (6,8,22,23) for $b = 1$ and $c = 1$;

(c) compute directly the optimal values of the parameters b and c from the necessary condition for optimum $\partial J/\partial b = 0$, $\partial J/\partial c = 0$;

(d) maintaining the same value of the parameter n and varying the parameter a repeat the procedure of steps (b) and (c) to determine the optimal set of values of (a,b,c) for the assumed integral value of n;

(e) assuming a range of values of n, repeat the procedure of steps (b) and (c) for each of n to determine the optimal set of the four parameters (n,a,b,c,).

THE RESULTS OF A NUMERICAL EXAMPLE

An example which illustrates the applicability of the TOSM is presented below. The objective is to solve the problem of identifying the four parameters n,a,b and c of the model for a watershed previously described and used by Diskin & Boneh (1973) in identification of the first two kernels of the Volterra series by means of direct optimization of the ordinates of the kernels. The catchment is that of the Cache River at Forman in southern Illinois which is 630 km^2 in extent with mild slopes and a well developed drainage network. The data of effective rainfall represented as rectangular pulses with time interval of one day and surface runoff for eight storms observed between 1935 and 1951 are given in Table 1 and 2 respectively. The optimal values of the parameters (n,a,b,c) of the TOSM were found to be

$n = 3$

$a = 0.677$ (1/day)

$b = 5.58 \times 10^{-3} (day^{-1} mm^{-1})$

$c = 83.6 \times 10^{-6} (day^{-1} mm^{-2})$

The outputs simulated by the TOSM for the optimal values of the parameters are given in Table 3. An example of the degree of fit to the real runoff by the TOSM is shown in Fig.1 for one of eight storms (storm no. 1). The separate linear, quadratic and cubic components for this particular storm are plotted in Fig.2.

In Table 4 a comparison is made between the optimal linear (b = c = 0), optimal quadratic (c = 0) and the cubic model based on cascade of equal nonlinear reservoirs.

In Table 5 a comparison is made between the models based on a

TABLE 1 Values of effective rainfall (mm day^{-1})

Time (days)	*Number of storm:* 1	2	3	4	5	6	7	8
1	18.8	2.5	0.3	0.3	33.0	1.0	69.9	2.5
2	95.3	3.8	0.3	38.1	23.1	5.1	8.6	0.0
3	19.1	17.5	17.0	19.1	0.0	49.5	0.0	48.0
4	0.0	78.7	56.1	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	13.7	0.0	0.0	0.0	0.0	0.0

TABLE 2 Values of direct runoff (mm day^{-1})

Time (days)	*Number of storm:* 1	2	3	4	5	6	7	8
1	0.8	0.0	0.0	0.0	2.3	0.0	3.6	0.0
2	4.8	0.0	0.0	2.5	7.6	1.8	9.1	0.5
3	35.8	0.0	0.8	9.1	10.4	6.4	18.5	2.3
4	40.6	7.6	5.1	14.0	12.7	9.9	16.8	6.4
5	22.1	30.0	16.8	11.2	8.9	11.7	11.7	12.7
6	12.2	25.4	21.1	7.6	5.6	8.4	7.4	10.9
7	7.6	16.3	16.0	4.8	3.8	6.1	4.6	6.4
8	3.6	9.4	10.7	3.3	2.0	4.1	2.5	4.3
9	2.5	5.3	6.1	2.0	1.3	3.0	1.8	2.5
10	1.5	3.3	3.8	1.3	0.8	2.0	1.0	1.5
11	0.8	1.8	2.3	0.8	0.5	1.3	0.8	1.0
12	0.5	1.3	1.8	0.5	0.3	0.8	0.5	0.8
13	0.3	1.0	1.3	0.3	0.0	0.3	0.3	0.5
14	0.0	0.5	0.8	0.0	0.0	0.0	0.0	0.5
15	0.0	0.5	0.5	0.0	0.0	0.0	0.0	0.3
16	0.0	0.3	0.3	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0

nonlinear cascade and linear and quadratic Volterra models based on direct optimization of the ordinates as presented by Diskin & Boneh (1973).

Although the agreement between the observed and computed output

TABLE 3 Outputs predicted by TOSM (mm day^{-1})

Time (days)	*Number of storm:* 1	2	3	4	5	6	7	8
1	0.6	0.1	0.0	0.0	1.2	0.0	3.2	0.1
2	9.3	0.4	0.0	1.5	6.5	0.3	14.9	0.3
3	34.6	1.6	0.7	7.3	11.5	3.0	19.0	2.4
4	39.0	9.0	5.6	12.0	11.3	10.3	14.9	8.9
5	24.2	25.7	17.0	11.4	8.6	12.2	9.7	10.8
6	11.2	25.5	20.6	8.5	6.0	9.9	6.1	9.1
7	5.0	16.5	15.9	5.9	4.0	7.0	3.9	6.5
8	2.8	9.3	10.2	3.9	2.6	4.7	2.5	4.4
9	1.9	5.3	6.3	2.6	1.7	3.1	1.6	2.9
10	1.4	3.3	4.0	1.6	1.1	2.0	1.0	1.9
11	1.0	2.1	2.6	1.0	0.7	1.3	0.6	1.2
12	0.7	1.4	1.7	0.7	0.4	0.8	0.4	0.8
13	0.4	0.9	1.1	0.4	0.2	0.5	0.2	0.5
14	0.3	0.6	0.7	0.2	0.1	0.3	0.1	0.3
15	0.2	0.3	0.4	0.1	0.1	0.2	0.1	0.2
16	0.1	0.2	0.2	0.1	0.1	0.1	0.0	0.1
17	0.1	0.1	0.1	0.0	0.0	0.1	0.0	0.1

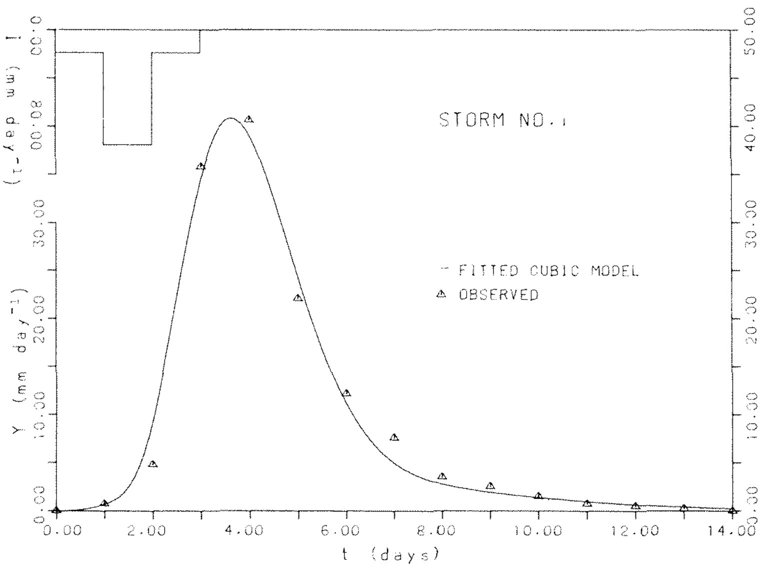

FIG.1 Comparison of observed runoff and that predicted by the TOSM for storm no. 1.

for the case of quadratic Volterra model based on direct optimization of the ordinates of the kernels is better than for the case of cubic TOSM, the latter has one big advantage. It has only four parameters, which ensures that the identification problem is well-conditioned and ensures the robustness of the solution in the presence of error in the input-output measurements. The small number of parameters makes the model suitable as the basis of a

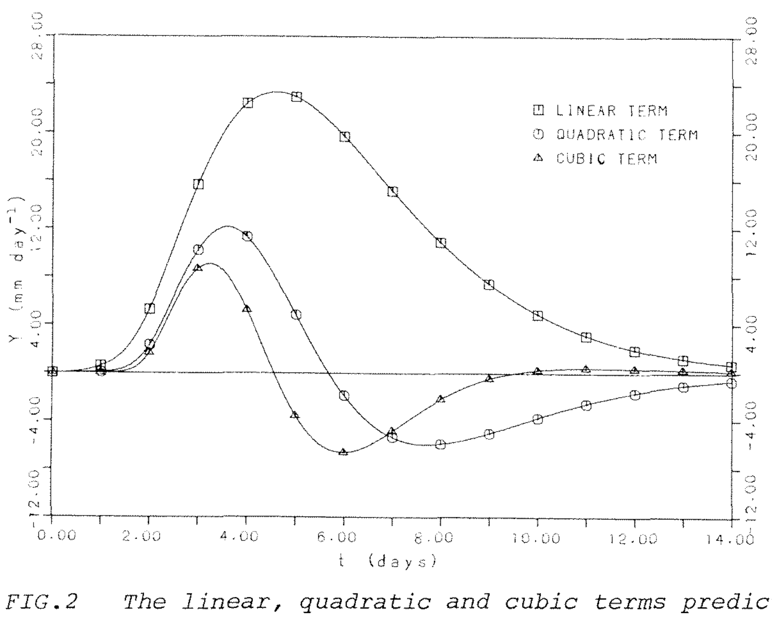

FIG.2 The linear, quadratic and cubic terms predicted by the TOSM for storm no. 1.

TABLE 4 Optimal parameters and values of objective function for models based on nonlinear cascade

	Linear	Quadratic	Cubic
n	4	3	3
a	1.32	0.75	0.677
$b \times 10^{-3}$	0	6.84	5.58
$c \times 10^{-6}$	0	0	83.6
J	445	233	154

control system in the operation of a water resource system.

TABLE 5 Comparison between models based on nonlinear cascade and ordinates

	Based on nonlinear cascade:			Based on ordinates:	
	Linear	Quadratic	Cubic	Linear	Quadratic
Number of parameters	2	3	4	14	74
Objective function	445	233	154	354	53

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