

Application of Deterministic Chaos and Neural Networks in Water Reservoir Management

Jaroslav J. Napiórkowski, and Tomasz Terlikowski

Institute of Geophysics, Ks. Janusza 64, 01-452 Warsaw, Poland, e-mail: jnn@igf.edu.pl

Abstract. The two-layer hierarchical technique with different prediction methods was applied to a part of the Wupper Reservoir System. The reservoir system consists of two reservoirs in series with additional inflow to the lower reservoir. The tasks of these reservoirs are flood control, recreation, hydropower and low flow augmentation with the aim of water quality improvement. Attention is focused on the implementation of anticipatory methods based on Deterministic Chaos and Artificial Neural Networks prediction of the inflows to the system that result in different operation rules. It is shown that the introduced optimisation concept improves considerably the system performance in comparison with the Standard Operation Rule.

Keywords: anticipation, reservoir management, deterministic chaos, neural networks, ARIMA model

INTRODUCTION

A method for determining the yield of a multireservoir water supply system has been applied to a part of the Wupper Reservoir System in Germany. The major objectives of this particular system are flood control, recreation, hydropower and low flow augmentation. The proposed technique may be reduced to the following associated parts: the optimisation of a simplified quantitative model of the actual system and the multiobjective verification and/or comparison through simulation. The first part consists in constructing a relatively wide class of control schemes based on the two-level optimisation technique method. We focus our attention on the implementation of a number of prediction techniques of the system inflow (ARIMA, Deterministic Chaos, Artificial Neural Networks) that result in different operation rules. The second part is based on the simulation performed for historical data over a long time horizon (39 years). This simulation consists in testing the control rules for chosen scalar objectives. The diagrams of frequency (reliability) criteria, calculated on the basis of simulation for a number of scalar criteria are analysed to obtain the final comparison results.

Several control schemes corresponding to the considered prediction models have been proposed in the form of computer programs. The simulations have been performed for a large number of years and for many objectives. To present advantages of the control schemes corresponding to the considered prediction systems, they are compared with so-called Standard Decision Rule (SDR).

DESCRIPTIONS OF THE CASE SYSTEM MODEL

The catchment of river Wupper is located in the southern part of North Rhine Westfalia. The hydrological features of this catchment are characterised by a massive rocky underground covered only by a small layer of soil and an average yearly precipitation of about 1300 mm per year. The absence of underground water storage leads to dangerous floods as well as to extreme droughts. To accommodate this problem several reservoirs were built. Here we are just interested in the management of the two reservoirs governing the discharges in the city of Wuppertal, which lies about 20 km downstream of reservoir No. 2. Figure 1 shows the simplified Wupper Reservoir System.

It contains two reservoirs located in series, the control centre at reservoir No. 2 and several runoff and rainfall gauges. The release of the reservoirs depends mainly on the runoff at the control gauge in Wuppertal. A runoff of 5 m³/s at this gauge is sufficient for the required water quality, runoff less than 3.75 m³/s should be avoided and the runoff less than 1 m³/s has to be regarded as ecologically disastrous. The basic hydrological and reservoir characteristics are given in Table 1.

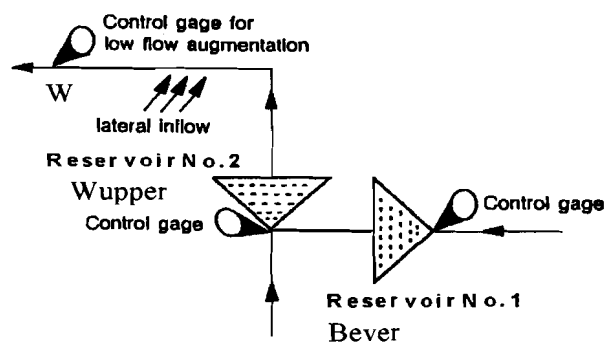


FIGURE 1. Basic structure of reservoir system.

The purpose of the model is to describe relationships between flow rates in the rivers over a long time horizon (one year) with the discretization period of 10 days. Therefore, only the dynamics of the storage reservoir are considered, while effects of flow dynamics in the river channels are neglected.

For brevity, the following notation is used: j - number of 10-day intervals, v_j - state of the reservoir, x_j - natural inflow, u_j - flow in a given cross-section, z_j - water demand, o_j - outflow from the reservoir, 1, 2 - denote the Bever and Wupper reservoirs, 3 - denotes the lateral inflow, W - cross-section at Wuppertal.

According to the introduced notation, the state equations for the reservoir system and flow balance equation for the selected cross-section W are:

$$V_{j+1} = V_j - B * O_j + C * X_j \quad (1)$$

$$V = [v^1, v^2]; \quad O = [o^1, o^2]; \quad X = [x^1, x^2] \quad (2)$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \quad (3)$$

$$u_j^W = o_j^2 + x_j^3 \quad (4)$$

TABLE 1. The basic characteristics of the Wupper Reservoir System.

Reservoir	Bever	Wupper
Total storage V_{\max} (mln m ³)	23.70	25.90
Dead storage V_{\min} (mln m ³)	0.70	2.10
max. outflow (m ³ /s)	17.00	180.00
min. outflow (m ³ /s)	0.10	1.00
Annual average flow (m ³ /s)	0.94	3.51
catchment area km ²	25.7	212.00

THE OPTIMISATION PROBLEM

The objective function of the optimisation problem under consideration for any time instant k (for any 10-day period) and for annual time horizon TA can be written in the form of a penalty function [9]:

$$Q(O, V) = \sum_{j=k}^{k+TA} [a_j^+ (o_j^1 - z_j^1)^2 + a_j^+ (o_j^2 + x_j^3 - z_j^W)^2 + b_j^1 (v_j^1 - v_j^{*1})^2 + b_j^2 (v_j^2 - v_j^{*2})^2] \quad (5)$$

In Equation 5, symbols a and b with respective subscripts denote weighting coefficients. The performance index Q is expressed explicitly on controls O_j and the state trajectory V_j (reservoir contents) as follows:

$$Q(O, V) = \sum_{j=k}^{k+TA} Q(O^j, V^j) \quad (6)$$

The objective function during each 10-day period is subject to the constraints on the state of the system, controls and flows in given profiles:

$$\begin{aligned} V_{\min}^j &\leq V^j \leq V_{\max}^j \\ O_{\min}^j &\leq O^j \leq O_{\max}^j \end{aligned} \quad (7)$$

It is assumed that the operation of the reservoir system is carried out on annual basis in the following way:

- By late December, the reservoirs normally are returned to low level to prepare the system for the next flood season completing the annual cycle.
- The storage reservation for flood control on January 1 is determined for controlling the maximum probable flood. During the normal filling period, January-April, the reservoirs should be filled up completely.
- During the May-August period the first reservoir should be filled up to meet recreation requirements.
- During the May-November period the water stored in and released from the reservoirs is used for low flow augmentation and hydropower.

According to the general objective of the control problem, which is aimed at the rational protection against water deficits and at reaching the desired state at the end of April, the following values of weighting coefficients in the optimisation problem are used: $a_j^+ = 1$ if demands are greater than supply and $a_j^+ = 0.01$ otherwise, for $k=[1,36]$. As far as the second coefficient is concerned, in order to avoid a good performance in one year followed by a very poor performance in the next year $b_j = 0.01$ for $j=[1,12]$ (May-August), $b_j = 0.001$ for $j=[1,30]$ (September- February), $b_j = 0.004$ for $j=[31,33]$ (March) and $b_j = 0.01$ in April, for $j=[34,36]$.

TWO-LEVEL OPTIMISATION TECHNIQUE

To solve the aforementioned problem we adjoin the equality constraints in Equation 1 with the Lagrange multiplier sequence λ (prices). The Lagrangian function has the form:

$$L(O, V, \lambda) = \sum_{j=k}^{k+TA} [Q(O^j, V^j) + \lambda^j (V^{j+1} - V^j + B * O^j - C * X^j)] \quad (8)$$

To include the state-variable and outflow constraints the above problem is solved by means of the two-level optimisation method and in a decentralised (co-ordinated) fashion. At this stage we make use of the additivity of the Lagrangian function (Equation 8) and the possibility of separation of the decision variables.

The Lagrangian function has a saddle point which can be assigned by minimising $L(\lambda, V, O)$ with respect to V and O , and then maximising with respect to λ . Finally, the optimisation problem can be expressed in the form:

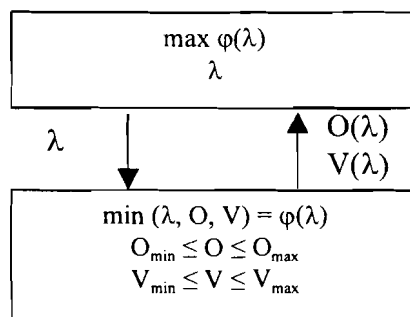


FIGURE 2. Two-level optimisation method.

$$\max_{\lambda} [\min_{V, O} L(\lambda, V, O)] \quad (9)$$

with inequality constraints on state and control and no constraints on Lagrange multipliers. Figure 2 illustrates how the two-layer optimal control method works.

At the *lower level* for given values of the Lagrange multipliers we look for the minimum of the Lagrange function. The necessary condition is the zero value of the gradient with respect to V and O . The task of the upper level is to adjust the prices λ in such a way that the direct control of the reservoir, affected by λ , results in the desired balance of the system (the mass balance Equation 1 is fulfilled satisfactorily). In the upper layer, in the maximisation of the Lagrange function with respect to λ , the standard conjugate gradient technique is used.

In the applied Two-Layer optimisation control method (TLM) illustrated in Figure 3 the solution of the two-level optimisation problem (Equation 9) is the essential "*upper layer part*".

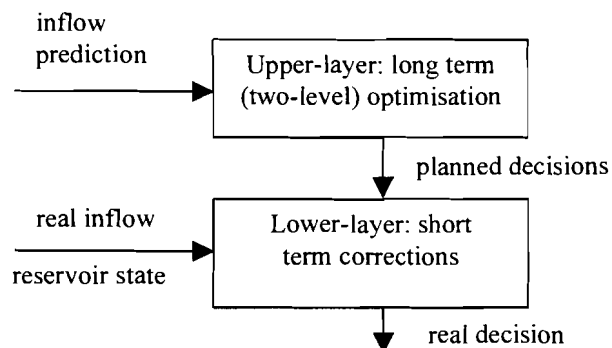


FIGURE 3. Two-layer control method.

INFLOW PREDICTION MODELS

Three inflow prediction techniques that were used for inflows predictions in the two-layer control method are briefly presented below.

Box-Jenkins ARMA Model

A classic multiplicative decomposition was applied to deseasonalise the observed data and then ARMA (Auto Regressive Moving Average) model was used in the prediction of inflows to the reservoir system. In the practical

calculations the set of appropriate procedures from Microsoft IMSL Library of Professional Edition of Microsoft Fortran Power Station v. 4.0. was adopted. These procedures enable to compute estimates of autoregressive and moving average parameters of ARMA(p,q) model and then calculate values of inflows estimates for specified number of points to be included in forecast of a fitted model. Calculations showed that the most effective model was ARMA(2,1).

$$x_t^T = p^1 x_{t-1} + p^2 x_{t-2} + a_t + q^1 a_{t-1} \quad (10)$$

where p, q are unknown parameters and a is the white noise process.

It enables better forecasts than the model in the form of average historical values for values for 20-days time horizon.

Artificial Neural Network Model

Inflow predictions based on the neural network simulation were estimated with the help of the NeuroSolutions software package[11].

NeuroSolutions adheres to the so-called local additive model. A processing element (PE) simply multiplies an input by a set of weights, and nonlinearly transforms the result into an output value. The principles of computation at the PE level are deceptively simple. The power of neural computation comes from the massive interconnection among the PEs which share the load of the overall processing task, and from the adaptive nature of the parameters (weights) that interconnect the PEs. Under this model, each component can activate and learn using only its own weights and activations, and the activations of its neighbours. The used neural network architecture is the multilayer perceptron (MLP) [7]. The performance of an MLP is measured in terms of a desired signal and an error criterion. The output of the network is compared with a desired response to produce an error. NeuroSolutions uses an algorithm called backpropagation [14]. The network is trained by repeating this process many times. The goal of the training is to reach an optimal solution based on the performance measurement.

The obtained simulation results justified 3 points as the maximum that can be included in forecast, with the following parameters of applied MLP: Hidden Layers =1, PEs =8, transfer function = TanhAxon (hyperbolic tangent -1/+1), Learning Rule = Momentum (Gradient and Weight Change, Momentum = 0.7), transfer function specified for output layer = LinearTanhAxon (picewise linear -1/+1). It should be noted that ANN model gives the best predictions of inflows to the system.

Model Based On Deterministic Chaos Concept

The concept of deterministic chaos has been recently developed to analyse many processes observed in the natural environment. In „deterministic chaos” one tries to unify two contradictory concepts: that of *chaos* and of *determinism*. In fact, a deterministic chaos phenomenon concerns actually deterministic dynamical processes. However, the evolution process in such a case has some special properties, combined one with another, which yield that the resulting observed outputs do not have an appearance of a „normal” deterministic phenomenon. Let us precise the mentioned characteristic features.

The fundamental notion and an object occurring in any deterministic chaos process, is *attractor*. Let us consider a dynamical process defined by a function $\varphi : T \times X \times T \rightarrow X$ and such that:

$$\forall \underline{X} \in X \quad \forall t_0, t \in T \quad : \underline{X}(t) = \varphi(t_0, \underline{X}(t_0), t) \quad (11)$$

where X is a (metric) space of state values, $T = Z$ in discrete case or R^n in continuous case (Z means the set of positive integer numbers). \underline{X} is a trajectory (evolution curve) i.e. a function $\underline{X} : T \rightarrow X$, X is a set of trajectories. If X satisfies (11), then X is called deterministic set.

If the state space X includes a minimal subset \mathcal{M} , $\mathcal{M} \subseteq X$, being a compact smooth differential manifold, such that:

\mathcal{M} is invariant, i.e. if $\underline{X}(t_0) \in \mathcal{M}$ for some $\underline{X} \in \mathbf{X}$ and some t_0 , then $\underline{X}(t) \in \mathcal{M}$ for $t \geq t_0$;

\mathcal{M} is attractive for \mathbf{X} , i.e. $\forall \underline{X} \in \mathbf{X}$, $\underline{X}(t)$ converges to \mathcal{M} (in the metric ρ of space X) as $t \rightarrow \infty$, then \mathcal{M} is called attractor of dynamic process φ or of the set \mathbf{X} .

The phenomenon of attractor occurs in many areas and in many known real cases [2, 3, 4, 10, 13, 16, 17, 19]. An attractor has usually lower dimension (e.g. Hausdorff dimension D) than the topological dimension of the state space X . In many cases we deal with *strange* attractors of the *fractal* structure [5], with a non-integer dimension.

The fact that \mathcal{M} is attractive and invariant has many important practical implications. Such a process can be considered as limited to \mathcal{M} only, thus, to a lower dimension space. All other features characteristic to a deterministic chaos are strongly related to the existence of attractor. These are: unstability and non-linearity, as well as an apparent „chaotic” behaviour of the considered dynamic system. Moreover, every trajectory of the dynamic process attains, after a respective time, any point of \mathcal{M} . The mentioned features occur always together in every known realisation/example of deterministic chaos.

As far as practical, quantitative recognition of the process is concerned, many difficult problems/tasks still arise. The first one is the determination of dimension of \mathcal{M} ; the final one is the exact form of function φ . In fact, we are rarely keen of obtaining a whole, precise description of the process, it is usually enough to determine good approximation of φ . Moreover, we are usually not interested in the whole vector $\underline{X}(t)$, but in its few components.

The classical situation is the following. A sequence $\{x_i\}$, $x_i \in \mathbf{R}^1$, $i = 1, \dots, N$, of observed/measured values is given, where x_i is an element of the state vector $\underline{X}(t)$, i.e.: $\underline{X}(i) = \underline{X}_i = (\dots, x_i, \dots)$. In order to find a (partial) approximation of the deterministic relationship φ of eq.1 (for an autonomous/stationary process), one considers the function F defined by the following relationship:

$$y_m^{i+\Delta} = F(y_m^i) = F(x_i, x_{i-1}, \dots, x_{i-(m-1)}) \quad (12)$$

with a properly adjusted number m , called *embedding dimension* and a given time delay Δ . Hence, the so called pseudo-phase space \mathcal{P} composed of m -element sub-sequences y_m^i of $\{x_i\}$ is considered:

$$y_m^i = (x_i, x_{i-1}, \dots, x_{i-(m-1)}) \in \mathcal{P} \quad (13)$$

The function F in Equation 12 is a dynamical process in the space \mathbf{R}^m , which – according to the embedding approach [19] forms an attractor \mathcal{M}' in \mathbf{R}^m , if the original process is a deterministic chaos. It results from the Takens theorem [19] that \mathcal{M}' has the same topological properties as the original attractor \mathcal{M} of the dynamic process φ in the space X . Takens theorem states also that m can be put as: $m = 2n + 1$, where n is the topological dimension of X , thus the number of elements of vector $\underline{X}(t)$.

However, the second practical problem arises now: how to determine, disposing only of a sequence $\{x_i\}$, the embedding dimension m , if we do not know the topological dimension n of X . This problem has been considered and solved by several authors [1, 3, 4].

The proposed approach is as follows. We measure the spatial correlation of the points that evolve on the attractor in \mathcal{P} with the *correlation integral*

$$C_N(l, m) = 1/N^2 \sum_{i, j=1, \dots, N} \theta(l - r_{ij}(m)) \quad (14)$$

defined for sequences y_m^i in $\mathcal{P} = \mathbf{R}^m$, determined by the given sequence $\{x_i\}$, where $r_{ij} = \|y_m^i - y_m^j\|$, l is the radius of the sphere centred in y_m^i , θ is Heaviside's function, $\|\bullet\|$ is the Euclidean norm, and N is the number of elements of $\{x_i\}$. Hence, $C_N(l, m)$ determines the averaged relative number of points y_m^j whose distance from y_m^i is less than l .

Then correlation dimension ν is determined as the slope of function $\ln(C_N(l, m))$ with respect to $\ln(l)$, in a respective range of sufficiently small l , such that the function behaves as a linear one. The value m is taken as the smallest one for which the above occurs. The value ν determined in this way is a good approximation and a lower bound of Hausdorff dimension D of the considered attractor \mathcal{M}' , [4] and hence of \mathcal{M} [19]. Finally, taking into account that $n = \min\{k \in \mathbb{Z} : k \geq \nu\}$ determines the topological dimension n of the space including attractor \mathcal{M} , and using Takens theorem, one put: $m = 2n + 1$ as the searched embedding dimension.

One can proceed then to the stage of determining the prediction model for the relationship F in Equation 12. It occurs that this is possible for deterministic chaos case: as the process is really deterministic and due to existence of an attractor (thus of a bounded set). On the other hand however, the unstability contradicts the possibility of making good (precise) prediction on a long time horizon, but for a short horizon it is quite possible.

The considered prediction model has the form of a function, belonging to a given class F , such that it approximates the function F , or even less – a „component” of F , being prediction of a future value of state $\{x_{i+T}\}$:

$$x_{i+T} = f_T(x_i, x_{i-1}, \dots, x_{i-(m-1)}) \quad (15)$$

where T is a prediction horizon. Such a function is denoted here by f_T^i , since it depends on the time instant i of making prediction, and on the horizon T of this prediction. Thus, we search for a function $f_T^i : \mathcal{P} \rightarrow \mathbb{R}^1$:

$$f_T^i(y_m^i) = f_T^i(x_i, x_{i-1}, \dots, x_{i-(m-1)}) \quad (16)$$

that would determine a good approximation of the value x_{i+T} of the given sequence $\{x_i\}$.

Applying the *local model* concept, [1, 13], we proceed as follows: for a given point $y_m^i \in \mathcal{P}$ and a number K one determines the K -element set of nearest neighbours of y_m^i (in the \mathbb{R}^m norm), denoted by $K(i)$. Then, the function $f_T^i \in F$ is adjusted as being solution of the *approximation problem*:

$$\min_{f_T^i \in F} \sum_{y_m^i \in K(i)} |x_{i+T} - f_T^i(y_m^i)|^2 \quad (17)$$

The class F may be taken as the set of quadratic functions, thus:

$$f_T^i(x_i, x_{i-1}, \dots, x_{i-(m-1)}) = (y_m^i)^T \cdot C \cdot (y_m^i) + \langle \underline{a}, y_m^i \rangle + b \quad (18)$$

or as a set of linear functions ($C = \underline{0}$ in Equation 18).

In the above formulae, \underline{a} is m -element vector, $b \in \mathbb{R}^1$ and C is $m \times m$ matrix; $\langle \cdot, \cdot \rangle$ denotes scalar product in \mathbb{R}^m . The number K , adjusted experimentally, should satisfy some conditions to assure the uniqueness of the approximation problem (17) solution.

Numerous computations for inflows in the Wupper Reservoir System were performed, in order to verify the hypothesis of deterministic chaos and to find the embedding dimension m . It has been shown, that the data represent the chaotic dynamics of dimension $m = 7$. Then, two prediction models (18) have been built. The best quality of forecast (the minimum error between forecasts and the original data) was obtained with $m = 4$ and $m = 5$; thus, less than $m=7$ [18][. The linear approximation model (with prediction horizon $T=1, 2, 3$) showed better results than the quadratic model and the model in the form of average historical values.

COMPARISON OF CONTROL METHODS BY SIMULATION

The simulation of some of the chosen control methods were carried out over the long time horizon of 39 years, with the real, historical data of natural inflows to the system. The methods under investigation have been partially discussed in the previous sections. Let us mention here once again those, which - after an initial stage of synthesis consisting of adjusting their parameter values - have been thoroughly compared by simulation.

1) TLM - Two-layer optimisation method with:

a) the complex, long-term planning aiming at the optimisation of all the particular goals in a compromising manner.

b) the realisation of the planned decisions (water supplies and discharges) in the real, current conditions.

2) SDR - The standard decision rule was developed by simulation techniques on the basis of a historical record of 39 years and ten synthetical records of 50 years [15].

In the first method, requiring solution of the optimisation problem (9), the long-term prediction of inflows (for 36 10-day periods) consists of two parts. For 10-day periods $j = [1,3]$ the results of one of the discussed inflow prediction models were used and for $j = [4,36]$ the average values of historical data were applied. Furthermore, to compare and investigate the 'power' of optimising methods, the variants denoted OPT and AVR have been considered, which differs from the optimising methods only in the fact that real/average values of inflows are put in place of predicted values.

In order to compare in a clear, well-ordered manner the results of different controls and the results of the other control techniques, we introduce the following scalar criteria goals [8]:

- global deficit time TD:

$$TD = Card(\{j : u_w^j < z_w^j\}) \quad (19)$$

- average relative deficit AvD:

$$AvD = \sum_{j=1}^{36} \frac{(z_w^j - u_w^j)_+}{z_w^j} \frac{1}{36} \quad (20)$$

- maximum relative deficit MxD;

$$MxD = \max\left\{\frac{(z_w^j - u_w^j)_+}{z_w^j} : j = 1, \dots, 36\right\} \quad (21)$$

- average losses in recreation area in the summer period for Bever Reservoir:

$$RE_B Av = \sum_{j=1}^{12} \frac{REmax - f_v(V_B^j)}{REmax} \frac{1}{12} \quad (22)$$

where REmax corresponds to maximum possible water area.

As a result, we obtain a sequence of 4 numbers, characterising system performance in a synthetic way. This could be sufficient to evaluate and compare the different functions for one year, e.g. with the aid of any multiobjective optimisation method. However, it is more complicated, because we have to compare the control effects not for a particular year, but for a long historical record.

To solve such a problem it is necessary to use a specific approach, which is arbitrary to some extent and makes use of intuition. To obtain the final comparison results we analyse the diagrams of so-called frequency (reliability) criteria calculated on the basis of simulation for 4 scalar criteria (19-22).

Those frequency criteria are also functions, but defined over the set of values of respective scalar criteria. Their values represent the number of years, for which the respective scalar criterion has its values in a given range. Formally, e.g. for MxD we have:

$$F_{MxD}(x) = Card(\{I : MxD^I \leq x\}) \quad (23)$$

where MxD^I denotes the value of criterion MxD (Equation 21) for the year I . As it is seen, F_{MxD} corresponds to the notion of cumulative distribution function of the "random variable" MxD^I , when I is treated as representing the elementary events.

RESULTS AND CONCLUSIONS

Some of the simulation results for the considered control methods, namely SDR, TLM, and OPT are presented below by means of the reliability criteria F , similar to Equation 23. Fig.(4-7) show the diagrams of distributions F corresponding to the criteria (19-22).

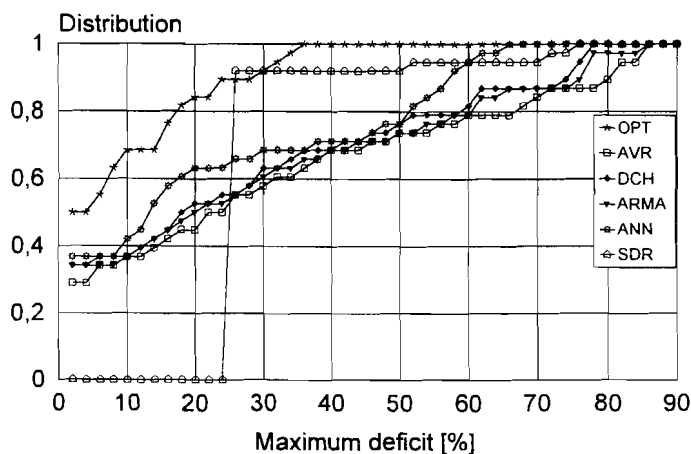


FIGURE 4. Maximum relative deficit at W cross-section.

The advantage of TLM, for all considered inflow prediction models (AVR, DCH, ARMA, ANN), but especially for ANN (the best forecast) and DCH, is evident in the sense of MxD criterion (Fig. 4). It results from the fact, that TLM takes into account the co-operation of the whole system and better co-ordinates the partial decisions.

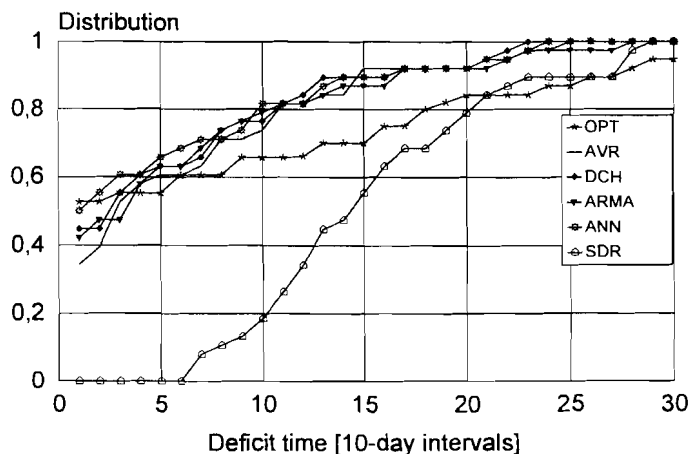


FIGURE 5. Global deficit time at the W cross-section.

For TD criterion (Fig. 5) the plot of OPT is below the plots of ANN, DCH, ARMA and AVR models. It reflects the fact that "system" prefers longer and small deficits rather than short and deep ones and, of course, the knowledge of future inflows guarantees the lowest maximum deficit.

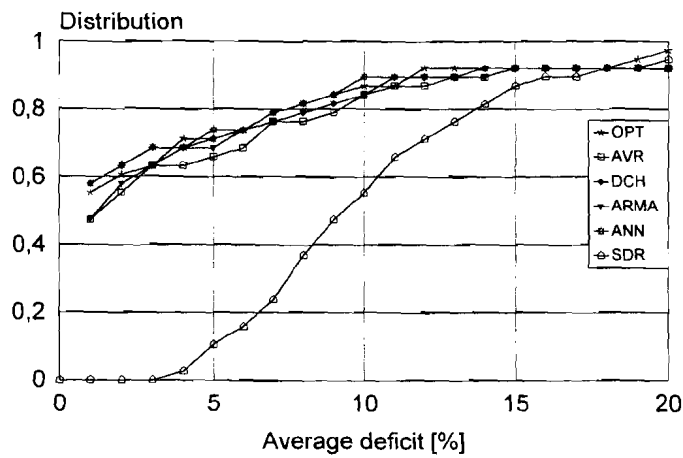


FIGURE 6. Average relative deficit at W cross-section.

For the criterion AvD (Fig. 6) the differences between diagrams corresponding to 4 prediction models are smaller, but the method TLM shows still to be better than SDR. Moreover, these diagrams are then closer to the "optimal" ones (those for OPT).

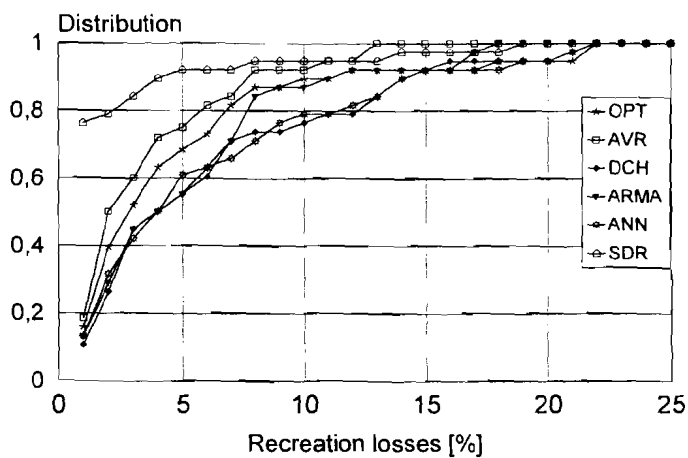


FIGURE 7. Losses in recreation area for Bever reservoir.

SDR gives the worst results for all but recreation losses criterion (Fig. 7).

Recapitulating, the method called TLM proved to be the best for reservoir system simulation with short time prediction obtained by means of ANN.

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