Operational control for a multireservoir system - multiobjective approach

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Abstract

The real case of a complex multireservoir and multiobjective water reservoir system is presented. The basic elements of the used Two-Level Optimization Method is discussed in details. It is shown that the introduced optimization concept improves considerably the system performance in comparison with Standard Operation Rule for 90 year long historical data record.

1. Introduction

Technique for determining the yield of multireservoir water supply has been developed and applied to the system serving the industrial region of the Upper Vistula River. The major objectives of this particular system are to supply water for the industrial and municipal water-users, the steel works, the chemical plan and the fish farms. At the same time, concentration of pollutants in the river should be maintained at the levels compatible with water quality requirements.
The unified methodology that enables to comprise a large class of acceptable solutions and to cover the wide range of specific conceptual approaches is presented. It enables the inclusion of the operator's preferences, intuition and experience. The presented technique may be reduced to the following conjunct parts: the optimization of a simplified quantitative model of the actual system and the multiobjective verification/comparison through simulation. The first part consists in constructing a relatively wide class of control structures based on the two-layer optimization technique method (Terlikowski[1]). The second part is based on the simulation performed for historical data over a long time horizon (ninety

years). This simulation is an active research and consists in testing and adapting the control rules by computation of many objective values. Several control schemes have been proposed in the form of computer programmes for the Upper Vistula Reservoir System. They have been compared for a large number of simulated years and for many objectives. One can see the ambiguity of different unified, aggregated evaluation methods in such a problem. The proposed control schemes are compared with the so called Standard Decision Rule, to present their undoubted advantages. The theoretical case of perfectly known future inflows is also tested to show the quality of the proposed control structure.

2. Description of the case system model

The water resources system concerned consists of two aggregated reservoirs located on two rivers (Soła River and Vistula River) and of five water users. The scheme of the system is shown in Figure 1.

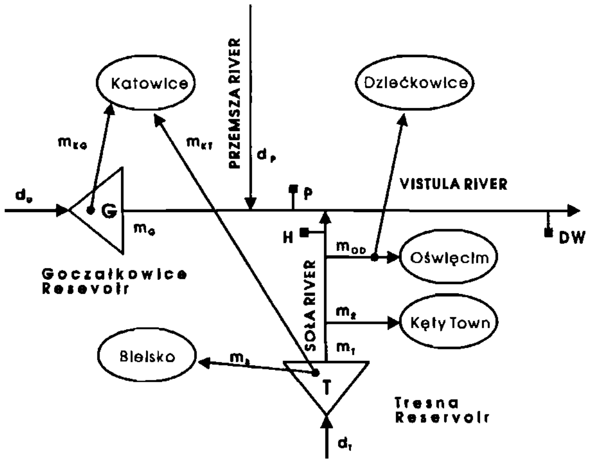

Figure 1: The Upper Vistula River System.

The major objectives of the system are to secure the water supply for the industrial and municipal water users, namely Katowice and Bielsko; to supply the steel works "Katowice" via the Dziećkowice Reservoir, and to supply water to the chemical plant Oświęcim and fish farms around the town of Kęty. At the same time, concentration of pollutants which are discharged mainly to the Vistula River downstream of the outlet of the Przemsza River should be maintained at the levels compatible with water quality requirements. Thc basic hydrological and reservoir characteristics are given in Table 1.

The purpose of the model is to describe relationships between flow rates in the rivers and in the conduits delivering water to users over a long time horizon (one year) with the discretization period of one decade (10 days). Therefore only the dynamics of the storage reservoir is considered, while effects of dynamics of flow in the river channels are neglected.

Reservoir	Tresna	Goczałkowice
historical inflows: the lowest the highest average	 1.18 m^3/s 1469 m^3/s 19.5 m^3/s	 0.36 m^3/s 581 m^3/s 7.75 m^3/s
catchment area	1095 km^2	522 km^2
total storage capacity V_{max}	139.7 mln m^3	202.8 mln m^3
dead storage V_{min}	13.6 mln m^3	20.0 mln m^3
flood control zone	27.0 mln m^3	30.4 mln m^3
max. outflow m_{max}	1730 m^3/s	935 m^3/s
min. outflow (biol. crit.) m_{min}	0.93 m^3/s	0.45 m^3/s

For brevity, the following notation is used:

- j - number of decade
- V^j - state of the reservoir at time j
- d^j - natural inflow to the reservoir or to the river at time j
- u^j - flow in a given cross-section
- z^j - water demands at time j
- m^j - outflow from the reservoir or water supply to a user, a control variable at time j
- S^j - pollutant load discharge at time j (kg/m^3)
- C^j - admissible pollutant concentration at time j
- T, G - denote the Tresna and Goczałkowice reservoirs
- H - control cross-section at Soła River
- P - control cross-section at Vistula River (down Przemsza River)
- DW - control cross-section at Vistula River

and the following subscripts are introduced:

- B refers to Bielsko
- T refers to Tresna
- O refers to Oświęcim
- R refers to fish farms;
- D refers to Dziećkowice

According to introduced notation, we are able to write state equations for the system of reservoirs and flow balance equations formulated for the selected cross-sections (H, P, DW). State equations are:

$$V^{j+1} = V^j - B*m^j + C*d^j \tag{1}$$

where

$$V=[V_T\ ,V_G];\quad m=[m_T\ ,m_G\ ,m_B\ ,m_{KT}\ ,m_{KG}];\quad d=[d_T\ ,d_G] \tag{2}$$

and

$$B=\begin{bmatrix}1 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 1\end{bmatrix};\quad C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{3}$$

The flow balance equations for the considered cross-sections are as follows:

$$u_P^j = m_G^j + d_P^j \tag{4}$$

$$u_H^j = m_T^j - m_R^j - m_{OD}^j \tag{5}$$

$$u_{DW}^j = u_P^j + u_H^j \tag{6}$$

3. Formulation of the optimization problem

The objective function of the optimization problem under consideration for any time instant (for any decade) and for annual time horizon (T=36) can be written in the form of a penalty function:

$$Q(m,V)=\sum_{j=k}^{k+T}[a_B^{+j}(m_B^j-z_B^j)^2+a_R^{+j}(m_R^j-z_R^j)^2+a_{OD}^{+j}(m_{OD}^j-z_{OD}^j)^2$$

$$+a_K^{+j}(m_{KT}^j+m_{KG}-z_K^j)^2+a_P^{+j}(u_P^j-S_P/C_P)^2+a_{DW}^{+j}(u_{DW}^j-S_{DW}/C_{DW})^2$$

$$+a_H^{+j}(u_H^j-z_H)^2+b_T^j(V_T^j-V_T^{*j})^2+b_G^j(V_G^j-V_G^{*j})^2] \tag{7a}$$

In equation (7), symbols a and b with respective subscripts denote weighting coefficients, while CP and C_{DW} denote values of pollutant concentration which should not be exceeded at the cross-sections P and DW. Inserting equations (4,5,6) into equation (7) the performance index Q can be expressed explicitly on controls m^j and the state trajectory V^j (reservoir contains). Other quantities which occur in its formulation are treated as parameters.

$$Q(m,V)=\sum_{j=k}^{k+T} Q(m^j,V^j) \tag{7b}$$

Required retention trajectory V^{*j}

It is assumed that the operation of the reservoir system is carried out on annual bases in the following way:

* by late December, the reservoirs normally are returned to low level to prepare the system for the next flood season completing the annual cycle.

* the storage reservation for flood control on January 1 was determined for controlling the maximum probable flood. During the normal filling period, January-April, the reservoirs should be filled up completely.

* during the May-November period the water stored in and released from the reservoirs is used for municipal, industrial and fish farms needs.

Weighting coefficients a^{j+} and b^j

According to the general objective of the control problem, which is aimed at the rational protection against water deficits and at reaching the desired state at the end of April, the following values of weighting coefficients in the optimization problem are used: $a^{j+}=1$ if demands are greater than supply and $a^{j+}=0.01$ otherwise, for k=[1,36]. As far as the second coefficient is concerned, in order to avoid a good performance in one year followed by a very poor performance in the next year b^j =.001 for j=[1,30] (May - February), b^j=.004 for j=[31,33] (March) and b^j=.01 in April, for j=[34,36].

The objective function during each decade is subject to the constraints on the state of the system, controls and flows in given profiles

$$V_{\min}^j \le V^j \le V_{\max}^j\ ;\quad m_{\min}^j \le m^j \le m_{\max}^j\ ;\quad u_{\min}^j \le u^j \le u_{\max}^j \tag{8}$$

2. TWO-LEVEL OPTIMIZATION TECHNIQUE

To solve the aforementioned problem we adjoin the equality constraints (1) with Lagrange multiplier sequence λ (prices). The Lagrangian function has the form:

$$L(m,V,\lambda)=\sum_{j=k}^{k+T}[Q(m^j,V^j)+\lambda^j(V^{j+1}-V^j+B*m^j-C*d^j)] \tag{9}$$

To include the state-variable and outflow constraints the above problem is solved by means of the two-level optimization method and solved in a decentralized (coordinated) fashion. At this stage we make use of the additivity of the Lagrangian function (9) and the possibility of separation of the decision variables.

The Lagrangian function has a saddle point which can be assigned by minimizing **L(λ,V,m)** with respect to **V** and **m,** and then maximizing with respect to **λ**. Finally, the optimization problem can be expressed in the form:

$$\max_{\lambda} \left[\min_{S\ u} L(\lambda, V, m)\right] \tag{10}$$

with inequality constrains on state and control and no constrains on Lagrange multipliers. Figure 2 illustrates how the two-layer optimal control method works.

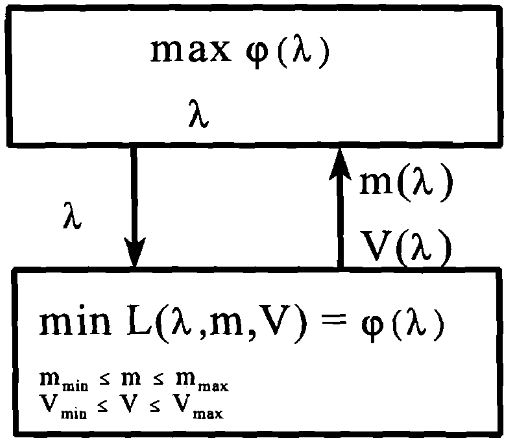

Figure 2: Two-level optimization method

At ***lower level*** for given values of the Lagrange multipliers we look for the minimum of the Lagrange function. The necessary condition is the zero value of the gradient with respect to **m** and **V**. The task of **the upper level** is to adjust the prices λ in such a way, that the direct control of the reservoir, affected by λ, results in the desired balance of the system (the mass balance equation (1) is fulfilled satisfactorily). On the upper layer, in the maximization of the Lagrange function with respect to λ the standard conjugate gradient technique is used.

In the applied Two-Layer optimization control method (TLM) the solution of the two-level optimization problem (10) is the essential "upper layer part". Note, that this planning layer "proposes" the sequence of T control variables $\{m^k,...m^{k+T}\}$ for one year long time horizon. At the current decade they are taking into account by the lower layer that tries to apply them in the real conditions and eventually subject to some additional operator's interventions.

4. Comparison of control methods through simulation

The simulation of some chosen control methods has been carried out over the long time horizon of 90 years, with the real, historical data of natural inflows to the system. The methods under investigation have been partially discussed in the previous sections. Let us mention here once again those of them, which - after an initial stage of synthesis consisting in adjusting their parameter values - have been thoroughly compared by simulation.

1. SDR -The natural, standard decision rule realizing mainly the following principle: „take as much as you need if you can" for any particular water user.
2. TLM - Two-layer optimization method with:
   1) the complex, long-term planning aiming at the optimization of all the particular goals in a compromise manner.
   2) the realization of the planned decisions (water supplies and discharges) in the real, current conditions.
3. SS - The method TLM supplemented by some elements of standard, direct decision rules.

In the last two methods, requiring solution of the optimization problem (10), the average values of real historical data for the period 1901-1990 have been taken as the long-term prediction of inflows. Furthermore, to compare and investigate the „power" of optimizing methods, the variant denoted REAL has been considered, which differs from the optimizing methods only in the fact that real current values of inflows are put in place of predicted values. It is worth to note that it is possible to make use of a more precise knowledge of future inflows only in the methods including a long-term planning.

Each method is evaluated through many different **performance indices** - equivalent, in a way, to degree of realization of conflicting goals. Hence, the indices reflect only the partial, not global, effects of system performance. In our model, each performance index is represented as a function of time:

- For water users (B, R, OD, K and the cross-sections H, P, DW) this is the **deficit** function expressed with respective time unit (decades).
- For the reservoirs this is the function of **storage level** (we are interested mainly in its average value in the summer period).

At the same time, each index is evaluated through many **scalar criteria.** In order to define them precisely, let us consider the deficit in meeting the needs of a given water user, e.g. R (fish farms) in a period of 1 year. The function m_R^j, where j corresponds to decade - together with z_R^j (representing the needs of fish farms) - characterize this one particular index in the most complete manner. However, in order to compare in a clear, well ordered manner the results of different controls m_R^j, and furthermore with the results of the others controls, we introduce some scalar criteria depending on these functions.

The following criteria have been proposed for the functions which represent the **water users** performance (i, j, k are included in a given period of 1 year, i.e. of 36 decades):

- global deficit time TD:

$$TD = Card(\{ i:\ m_R^j < z_R^j\}) \tag{11}$$

- maximal continuous deficit time TDc:

$$TDc = \max(\{|k \le l|:\ k \le l \wedge [\forall\ k \le j \le l;\ m_R^j < z_R^j]\}) \tag{12}$$

- average relative deficit AvD:

$$AvD = \sum_{j=k}^{k+T} \frac{(z_R^j - m_R^j)_+}{z_R^j} \, \frac{100\%}{T} \tag{13}$$

- maximal average relative continuous deficit AvDc;

$$AvDc = \max(\{ \sum_{j=k}^{l} \frac{z_R^j - m_R^j}{z_R^j} \, \frac{100\%}{T} : \forall \; k \le j \le l \;\; m_R^j < z_R^j \}) \tag{14}$$

- maximum relative deficit MxD;

$$MxD = \max(\{ \frac{(z_R^j - m_R^j)_+}{z_R^j} \, \frac{100\%}{T} : \; k \le j \le k+T \}) \tag{15}$$

Any function defining particular water user supply (m_B, m_R, m_{OD}, m_K), as well as the flow in the cross section (H, P, DW) is characterized for a given one year period by 5 numbers as defined above.

At the same time the trajectories V_T^j, V_G^j are described by 2 criteria. For example average water content in the Summer period for Goczałkowice Reservoir is:

$$V_G Av = \sum_{j=1}^{12} \frac{V_G^j}{12} \tag{16}$$

As a result, we obtain for each user/goal the sequence of 5 numbers, characterizing a given performance index function in a synthetic way. This could be sufficient to evaluate and compare the different functions for one, fixed index, e.g. with the aid of any multiobjective optimization method. However, it is more complicated, because we have to compare the control effects for 9 "users" and not for particular year, but for 90 years long historical record.

To solve such a problem it is necessary to use a specific approach, which is arbitral to some extent and makes use of intuition. To obtain the final comparison results we analyze the diagrams of s.c. **frequency (reliability) criteria**, calculated on the basis of simulation for each of nine „users" and for each of 5 or 2 scalar criteria (11-16).

Those frequency criteria are also functions, but defined over the set of values of respective scalar criteria TD, ..MxD, ..etc. Their values represent the number of years, for which the respective scalar criterium has its values in a given range. Formally, e.g. for MxD we have:

$$f_{MxD}(x) = Card(\{ I: \; x - \Delta \le MxD^I \le x \}) \tag{17}$$

$$F_{MxD}(x) = Card(\{ I: \; MxD^I \le x \}) \tag{18}$$

where MxD^I denotes the value of criterium MxD (15) for the year I, and Δ is the

step of discretization of values of MxD (e.g. 2 per cent). As it is seen, f corresponds to the notion of density function and F - of commutative distribution function of the „random variable" MxD^I, when I is treated as representing the elementary events.

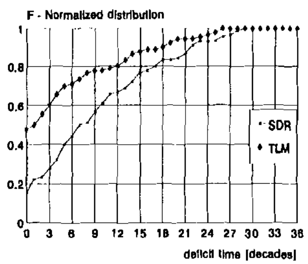

Figure 3: TD reliability criterion for Katowice

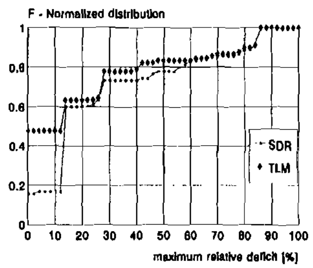

Figure 4: MxD reliability criterion for Katowice.

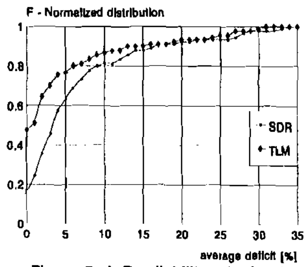

Figure 5: AvD reliability criterion for Katowice

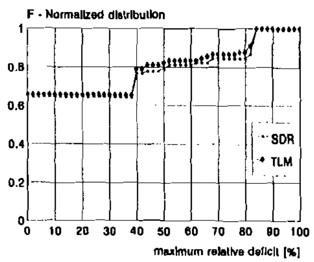

Figure 6: MxD reliability criterion for Bielsko

5. Results and conclusions

Some of the simulation results for the considered control methods, namely SDR, TLM, SS and REAL are presented below by means of the reliability criterium F.

Figures (3-5) show the diagrams of normalized distribution F corresponding to the criteria TD, MxD and AvD for the deficit in Katowice. Two methods of control are compared: Standard Decision Rule (SDR) and Two-Layer optimization Method (TLM). For the water user Katowice, the advantage of TLM is evident in the sense of all the considered scalar criteria. This results from the character of this particular user, taking water from the both parts of the system: Tresna Reservoir and Goczałkowice Reservoir. The TLM takes into account the cooperation of the whole system and coordinates the partial decisions. It gives distinctively better results.

It appears that for the users taking the water directly from Soła River (R, OD and B) the optimizing method also gives better results than SDR. The difference is, however, less evident (see Figure 6). Generally, the obtained results show that the TLM improves the control quality in comparison to SDR for any "real" water user (K, R, OD, B), especially for the MxD criterium of maximum deficit value.

In the case of flows required in the chosen control cross-sections the comparison of SDR with TLM and SS methods does not give so univocal conclusion. For the H cross-section at Soła River the SDR method gives better results than TLM or SS in most cases. This may be explained by the local character of this "user". Nevertheless, if we apply the TLM technique with the perfect knowledge of future inflows - as in REAL method - we get considerable improvements in all indices. This is shown

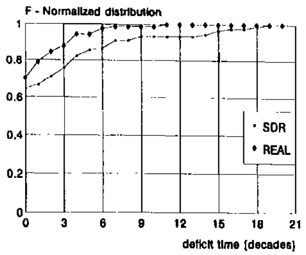

Figure 7: TD reliability criterion for the H cross-section (Sola River).

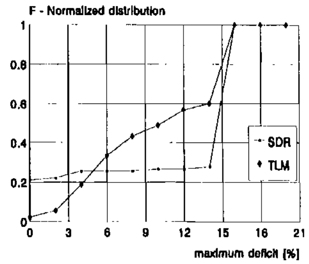

Figure 8: MxD criterion for the P cross-section (Vistula River).

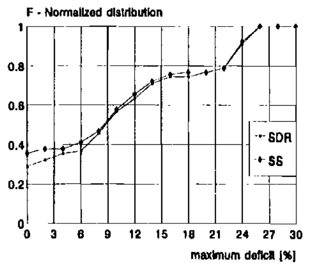

Figure 9: MxD criterion for the WP cross-section (Vistula River).

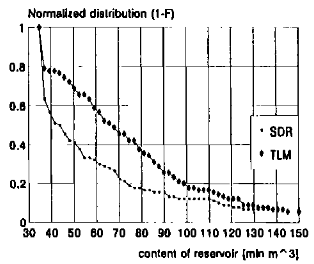

Figure 10: Average water content of Goczalkowice Reservoir in Summer.

in Figure (7), which presents the diagrams of F_{TD} criterion.

For the P cross-section at Visula River the TLM appears to be better than SDR even in real situation (uncertainty in future inflows) but not for any scalar criterion. For the F_{TD} criterium SDR produces generally worse results while for F_{mxD} - maximum deficit criterium SDR is often more advantageous as is shown in Figure (8).

For the DW cross-section (see Figure (9)) the TLM technique occurs to be better than SDR with respect to any scalar criterium. The above conclusion reflects one more fact that the optimizing method improves particularly those partial effects of control which is more structurally connected to the system as a whole and in consequence depend more on a proper coordination of the system.

Finally, Figure (10) shows the exemplary diagrams of reliability criteria for reservoirs. The idea of these criteria is, in a way, inverse to (18), because we prefer possibly large values of reservoir contents. Namely we have, e.g. for $A_G Av$ (see (16)):

$$F_{V_G Av}(x)=Card(\{ I:\ \ V_G Av^I \geq x \})$$

One can see that the TLM allows to keep the given water contents more frequently, even for relatively high values (90-120 mln m^3).

Acknowledgments

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