A C T A G E O P H Y S I C A P O L O N I C A

Vol. 53, no. 4, pp. 459-472 2005

ARTIFICIAL NEURAL NETWORKS AS AN ALTERNATIVE TO THE VOLTERRA SERIES IN RAINFALL-RUNOFF MODELLING

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Abstract

Methods of description of the non-linear effects in dynamic rainfall-runoff systems have been surveyed. Particular reference is given to such non-linear methods which do not require detailed topographical survey and determination of roughness parameters. To describe rainfall-runoff relation, alternative approaches to non-linear partial differential equations of mass and energy transfer have been discussed, namely conceptual and black-box models. In more details, application of Volterra net, Multi-Layer Perceptron Artificial Neural Network and Radial Basis Function Network is tackled. Illustrative numerical examples of rainfall-runoff simulation and river flow forecast are presented.

Key words: Volterra series, artificial neural network, rainfall-runoff model.

1. INTRODUCTION

Models describing the geophysical processes contributing to the hydrological cycle were developed in non-linear form in the nineteenth century. From their physical basis such models can simulate the complete runoff regime, providing such outputs as: river discharge, groundwater head and evaporation losses. Transfer of mass, momentum and energy are calculated directly from the governing partial differential equations which are solved using numerical methods, for example the St. Venant equations for surface flow and river flow, the Richards equation for unsaturated zone flow and the Boussinesq equation for ground water flow.

Hence, an accurate application of the hydraulic approach requires a detailed topographical survey and determination of roughness parameters. In order to avoid these difficulties, alternative non-linear approaches were developed, namely conceptual and black box models.

Conceptual rainfall-runoff models are designed to approximate within their structure the general internal sub-processes and physical mechanisms which govern the hydrological cycle. Such models usually incorporate simplified forms of physical laws. Until recently, for practical reasons most conceptual models are lumped.

In the second approach, known as black-box analysis, an attempt is made to extract, from the past records of input-output events on the system under examination, enough knowledge of the dynamics of the particular system, to provide a basis for predicting its output due to other specified outputs (Dooge and O'Kane, 2003). Black box analysis has been concerned with representing a system by a functional Volterra series in the form of a sum of convolution integrals (Volterra, 1930) or by some types of artificial neural networks.

In the paper, alternative approaches to describe rainfall-runoff relation have been discussed and compared, namely the application of Volterra net, Multi-Layer Perceptron Artificial Neural Network and Radial Basis Function Network.

2. CONCEPTUAL MODEL

Lumped conceptual models are formulated on the basis of a relatively small number of components, each of which is a simplified representation of one process element in the system being modelled. The conceptual model approach is to simulate the non-linear nature of the catchment response by a relatively simple model built up from simple elements, e.g. cascade of non-linear reservoirs. Each reservoir is responsible for part of the attenuation of the system response. This lumped dynamic model can be represented by a set of ordinary differential equations:

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t} S_1(t) = -f[S_1(t)] + x(t) \\
&\frac{\mathrm{d}}{\mathrm{d}t} S_2(t) = -f[S_2(t)] + f[S_1(t)] \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\frac{\mathrm{d}}{\mathrm{d}t} S_n(t) = -f[S_n(t)] + f[S_{n-1}(t)] \\
&y(t) = f[S_n(t)], \qquad \mathbf{S}(0) = \mathbf{S^0} \quad ,
\end{aligned} \tag{1}$$

where $x(t)$ is the input signal (rainfall), $S_i(t)$ is the storage in the i-th reservoir, $f(\,)$ represents the outflow-storage relation and $y(t)$ is the output signal (surface runoff).

The vector differential equation (1) can be considered as a definition of some non-linear operator P mapping a space of inflows into a space of corresponding storages. In order to determine how P operates for a given inflow hydrograph $x(t)$, it is necessary to solve the set of eq. (1) under the initial condition $\mathbf{S^0}$.

Conceptual model that is useful as representation of some particular system properties cannot be claimed as universal. It obviously exhibits some deficiencies; this is the price for its simplicity and low cost in terms of computing time and data requirements in comparison to rigorous hydrodynamic models.

3. VOLTERRA SERIES

The description of dynamic systems by the Volterra series is a generalization of the concept of the transfer function, which is of great importance in the analysis and design of linear systems. The Volterra series represents an explicit input-output relation for non-linear dynamic systems and consists of infinite series composed of the form of convolution integrals

$$y(t) = \int_0^\infty h_1(\tau_1)\, x(t-\tau_1)\mathrm{d}\tau_1 + \int_0^\infty\int_0^\infty h_2(\tau_1,\tau_2)\, x(t-\tau_1)\, x(t-\tau_2)\,\mathrm{d}\tau_1\mathrm{d}\tau_2 + \ldots$$
$$+\int_0^\infty\int_0^\infty \ldots \int_0^\infty h_n(\tau_1,\ldots,\tau_n)\, x(t-\tau_1)\ldots x(t-\tau_n)\,\mathrm{d}\tau_1\ldots\mathrm{d}\tau_n + \ldots \tag{2}$$

In the above equation $x(t)$ is the input to the system (rainfall), $y(t)$ is the output from the model (surface runoff), $h_1(\tau_1)$ is the first order kernel which reflects the linear properties of the system, $h_2(\tau_1, \tau_2)$ is the second order kernel which reflects the quadratic properties, $h_n(\tau_1,\ldots \tau_n)$ is the n-th order kernel and so on.

In the course of mathematical modelling of surface runoff systems, one deals with discrete signals at the stage of measurements. Then each function in eq. (2) is represented by a series of pulses at regular grid-points at the same interval Δt along the time axis. The second order kernel is represented by an array of pulses on a square grid at the same interval, Δt, as that used for the first-order functions, etc. With that discretization, the integrals are replaced by summation of products and the relationship between the pulses is given by the so-called Volterra net:

$$y(i) = \sum_{k=1}^{NX} H_1(k)\, x(i-k) + \sum_{k=1}^{NX}\sum_{l=1}^{NX} H_2(k,l)\, x(i-k)\, x(i-l) + \ldots\ , \tag{3}$$

where NX is the number representing the memory of the system, $i = 1,2,\ldots NT$, and NT is the number of observations. The values of weights H_i can be adaptively modified during the process of model calibration.

The identification of kernels of the Volterra series was discussed in detail by Napiórkowski and Strupczewski (1984). It was shown that:

(1) the identification of kernels of the Volterra series is a typical example of an ill-posed problem in the sense of Tikhonov (1963);

(2) in the case of slow varying input signals the solution is not unique;

(3) very good fitting of the output from the model to the observed data may be completely misleading, as far as identification of the system is concerned (see example in Napiórkowski and Strupczewski, 1984).

Note that the number of weights H_i increases geometrically with the number of terms in the Volterra net and the number NX representing the memory of the system. For example, number of weights H_1, symmetric H_2 for two-term series and $NX = 10$ is equal to $(NX^2 + 3NX)/2 = 65$.

The main reason why the problem of identification of the kernels of the Volterra series is ill-posed is that the class of functions within which the solution is sought is too wide. One has to reduce that class, on the basis of some mathematical and physical characteristics, to such a sub-class M for which the identification problem has a unique, stable solution in the case when the measurement values are contaminated with errors (Napiórkowski and Strupczewski, 1984). More precisely, M should be a subset for which the solution depends continuously on the measurements.

From the very beginning of its hydrological applications the Volterra series model was conceived to be of the black-box type, i.e. it could be regarded as a direct extension of the convolution integral technique accompanying the concept of instantaneous unit hydrograph. J. Amorocho, the pioneer in the field of Volterra series hydrologic modelling wrote in 1973 that "at this point no correspondence can be assumed … to exist between the components of the polynomial system (eq. 2) and any of the physical elements of the prototype" (Amorocho, 1973).

The first attempt to attribute some conceptual meaning to the black-box kernel of the Volterra series model used in hydrology was due to Diskin and Boneh (1972). The structure of their second-order kernel, however, was not directly related to any physical model. It was merely a hypothetical example of properties that theoretical second-order kernel of a conservative system should possess.

Napiórkowski and co-authors (Napiórkowski, 1978; 1983; Napiórkowski and Strupczewski, 1979; 1981; Napiórkowski and O'Kane, 1984) aimed to establish a relationship between the non-linear conceptual model described in Section 2 (in the state space framework) and the Volterra series. It was shown that if the function $f(\,)$ in eq. (1) is differentiable as many times as required, the state-space equation (1) can be approximated by the Volterra series (eq. 2) by means of Taylor series expansion for operators. The structure of the first two kernels was shown to be

$$h_1(\tau_1) = a\,\psi_n(\tau_1)\ , \tag{4}$$

$$h_2(\tau_1,\tau_2) = b\left\{\psi_n(\tau_1)\sum_{k=1}^{n}\psi_k(\tau_2) + \psi_n(\tau_2)\sum_{k=1}^{n}\psi_k(\tau_1) - \psi_n[\max(\tau_1,\tau_2)]\right\}\ , \tag{5}$$

where

$$a = \left.\frac{\mathrm{d}f}{\mathrm{d}S}\right|_{S=0} , \quad b = \frac{1}{2}\left.\frac{\mathrm{d}^2 f}{\mathrm{d}S^2}\right|_{S=0} , \quad \text{and} \quad \psi_k(\tau) = \frac{(a\tau)^{k-1}}{(k-1)!}\exp(-a\tau) \ . \tag{6}$$

The main feature of the two-term Volterra series model based on the cascade of non-linear reservoirs (4)–(6) is the small number of parameters (*n, a, b*) to be determined in comparison with the method based on direct optimisation of the ordinates.

Note that analytical derivation of the kernels of the Volterra series is not another methodological approach, but it helps in the correct formulation of the identification problem and enables the verification of the optimisation procedures.

4. MULTI-LAYER PERCEPTRON ARTIFICIAL NEURAL NETWORK

Artificial Neural Networks (ANN) application for rainfall-runoff modelling has undergone much investigation during last decade (ASCE, 2000). Although there are several types of neural networks, without doubts the most popular in hydrological sciences is Multi-Layer Perceptron (MLP) network. This is due to its simple form, flexible structure and easy calibration of parameters by means of gradient-based algorithms.

In most applications networks composed of three layers are sufficient to approximate relations between input and output variables (Brath *et al.*, 2002). Each layer comprises the proper number of nodes. The number of input and output nodes is equal to the number of input and output variables. There is no effective rule for estimation of quantity of hidden nodes, which is to be evaluated empirically.

The MLP nodes in neighbouring layers are linked via weighted connections (see Fig. 1). Shortly, Multi-Layer Perceptron networks applied to rainfall-runoff modelling operate in the following way: signals of delayed in time measured rainfall pulses $x(i\text{-}p)$ and delayed in time runoff pulses $z(i\text{-}q)$ ($p = 1,\ldots NS$, $q = 1,\ldots NY$, $i = 1,\ldots NT$) from the input nodes (e.g. values normalized to 0-1 interval) are multiplied by proper weights $w(l, k)$, connecting the neuron from which signal has been dispatched and a suitable neuron in the hidden layer.

In the second layer, in each of K nodes the weighted sum of all the inputs and weights $w(0, k)$ representing threshold values is computed and then transformed by proper function (e.g. logistic function), giving the value dispatched by k-th neuron:

$$\phi(k) = \left[1 + \exp\left(-w(0,k) \ - \sum_{l=1}^{NX+NY} w(l,k)\, I(l)\right)\right]^{-1} \ . \tag{7}$$

Afterwards the signals $\phi(k)$ multiplied by proper weights $v(k)$ are transferred to the neuron of the third layer. In this final stage the new weighted sum is computed

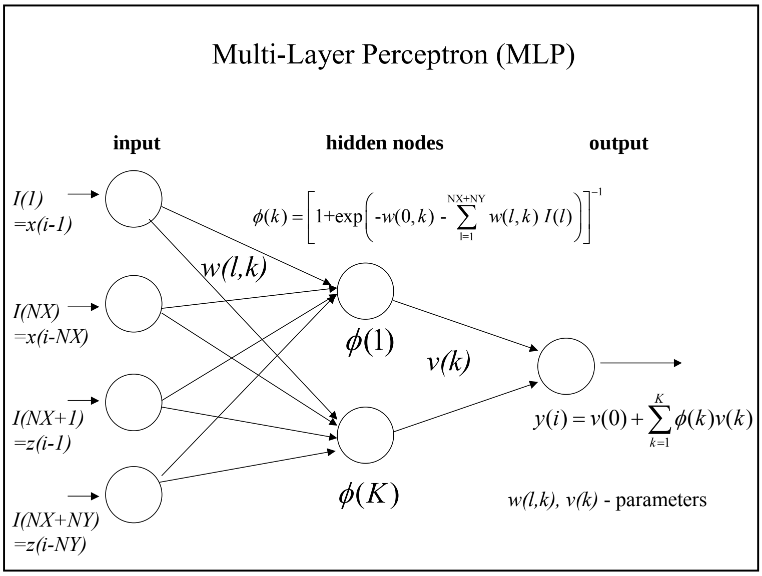

Fig. 1. A two-layer feedforward neural network.

$$y(i) = v(0) + \sum_{k=1}^{K} v(k)\,\phi(k) \tag{8}$$

and after de-normalization of output, the sought (forecasted) value *y(i)* is determined. MLP are feed-forward networks, which means that there is only one direction of the propagation of information, from the input to the output layer.

The values of weights $w(l, k)$ and $v(k)$ can be adaptively modified during the process of training the network. In this study, due to the relatively simple architecture of all the networks, Levenberg–Marquardt non-linear optimisation algorithm was adopted (Press *et al*., 1989).

It should be stressed that in the distinction from the Volterra net, MLP network can have both rainfall and runoff observations in the input layer.

5. RADIAL BASIS FUNCTION NETWORK

Radial Basis Function (RBF) Neural Networks gain more popularity in hydrological sciences in recent years (Jayawardena and Fernando, 1998; Dawson *et al*., 2002; Moradkhani *et al*., 2004). RBF network architecture depicted in Fig. 2 differs from MLP ones and includes one hidden layer of special units that pre-process the input and feed a single-layer perceptron (e.g., Haykin, 1994; Silipo, 2003).

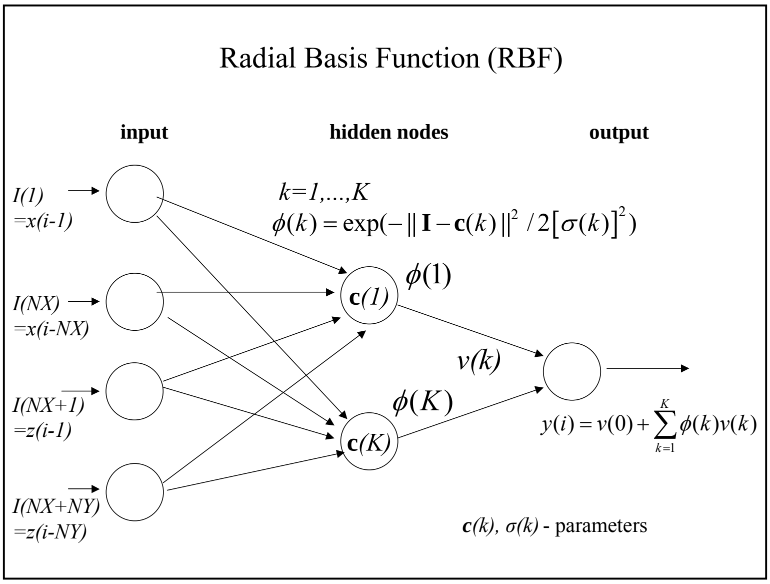

Fig. 2. The architecture of a Radial Basis Function network.

Similarly to MLP network, input layer of RBF may contain rainfall pulses $x(i\text{-}p)$ and runoff pulses $z(i\text{-}q)$ ($p = 1,\ldots NS$, $q = 1,\ldots NY$, $i = 1,\ldots NT$). Each of K units in the hidden layer contains the centre $\mathbf{c}(k)$ of the given region of the input space. The corresponding non-linear activation function $\phi\,(k)$ expresses, by means of distance measure, the similarity between any vector $\mathbf{I}$ of input variables and the prototype $\mathbf{c}(k)$. The most commonly adopted Basis Function is Gaussian:

$$\phi(k) = \exp(-\| \mathbf{I} - \mathbf{c}(k) \|^2 / 2[\sigma(k)]^2) \ . \tag{9}$$

Parameter values $\mathbf{c}(k)$, $\sigma\,(k)$ and $v(k)$ are to be optimised. Popular self-organized selection of $\mathbf{c}(k)$ by k-nearest neighbour approach is applied in this paper, rather than the well-known gradient-based algorithms or global optimisation techniques for supervised selection of centres, or simple fixed centre selection at random.

After localizing centres, $\sigma\,(k)$ values have to be evaluated, these may differ for each $\mathbf{c}(k)$ or possess the same value for all centres. The second option, when each $\sigma\,(k)$ is the same for each k, is much simpler and reduces number of parameters, unfortunately flexibility of the model also diminishes. Nonetheless, this option was chosen in present paper because of small number of data and high uncertainty of measurements. In the paper heuristic rule (Haykin, 1994) was applied

$$\sigma = b \frac{p}{\sqrt{2m}} \ . \tag{10}$$

In this rule p is the distance between the most distant centres, m is a number of centres and b is unknown value deciding how peak or flat σ should be, in present work it was empirically verified that b should be set to 5.

The set of linear weights $v(k)$ was optimised by pseudoinverse method (Haykin, 1994).

6. COMPARISON OF THE VOLTERRA SERIES AND MLP NETWORK

Two-term Volterra series and Multi-Layer Perceptron Artificial Neural Network were fitted to the first six (out of eight) "classical" storms used many times for comparison of different models by Diskin and Boneh (1973). The watershed from which the data was obtained is that of Cache River at Forman in southern Illinois. The area of the watershed is 630 km^2, the topography being fairly flat with gentle slopes and well-developed drainage network. Daily rainfall data comprised ordinates of effective rainfall and direct surface runoff for eight storms observed between 1935 and 1951.

The optimal values of the Volterra net parameters were found to be $n = 3$, $a = 0.75$, $b = 0.01$, and the optimal structure of the MLP network with 49 parameters can be described as 10–4–1 (10 inputs, 4 hidden nodes and 1 output). It should be underlined that only rainfall data were used as input both for Volterra net and MLP network.

The results of comparison of the Volterra net with the MLP network for Diskin and Boneh (1973) data are presented in Table 1.

The MLP network performs better for the calibration data – the root mean squared error (RMSE) is lower, but the 3-parameter Volterra net gives slightly better results for verification records. Note that the kernels described by eqs. (5-7) automatically guarantee mass conservation by the Volterra net and for the case of MLP network the maximum error in mass conservation is about 4%.

Table 1

Comparison of the Volterra and MLP networks
for Diskin and Boneh (1973) data

Storm number	1	2	3	4	5	6	7	8
Storm status	Calibration						Verification	
RMSE for Volterra net	2.27	1.48	0.57	0.47	0.60	1.04	1.83	1.02
RMES for MLP	0.62	0.71	0.43	0.60	0.67	1.03	1.85	1.05
Total runoff [mm]	133	103	87.4	57.4	56.1	55.6	78.5	50.7
Total MLP runoff [mm]	139	103	83.8	55.7	54.5	55.1	78.1	50.3
Maximum runoff [mm/day]	40.6	30.0	21.1	14.0	12.7	11.7	18.5	12.7

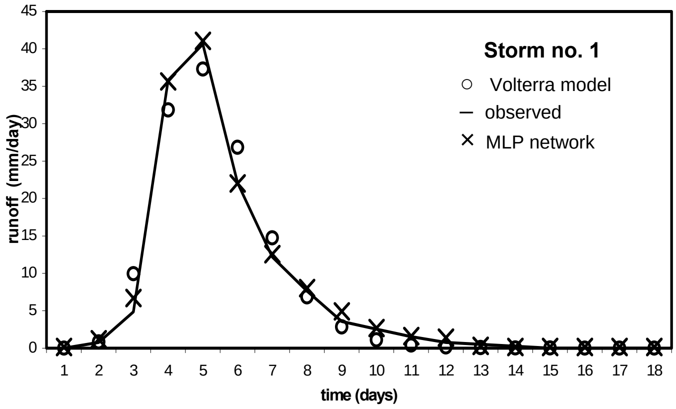

Fig. 3. Comparison of observed runoff and that predicted by the Volterra model and MLP network for calibration data.

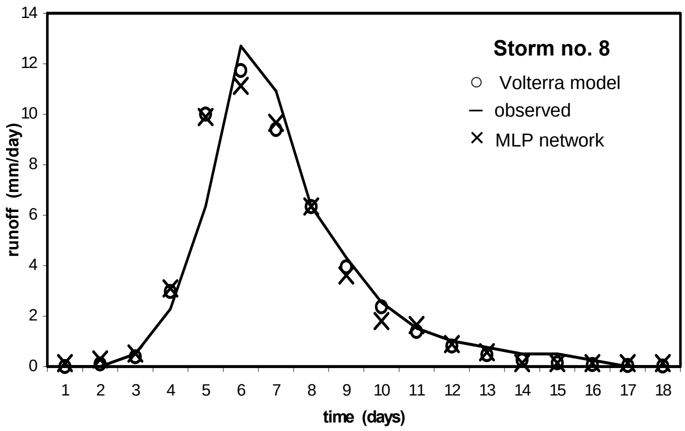

Fig. 4. Comparison of observed runoff and that predicted by the Volterra model and MLP network for independent data.

The degree of fit to observed runoff by the Volterra series and MLP network for the first storm used for calibration is shown in Fig. 3 and for one of the storms used for verification is shown in Fig. 4.

7. APPLICATION OF THE RBF AND MLP NETWORKS TO RUNOFF FORECASTING

The main disadvantage of the Volterra net is that observations of previous runoff cannot be used in calculations of future runoff. So in this section devoted to runoff forecasting in Nysa Kłodzka catchment only results of application of the RBF and MLP networks are presented.

The case study area, namely the Nysa Kłodzka catchment in the southern part of Poland, is considered to be of high importance. This river is a tributary of the Odra, the second biggest river in the country. Here we are concerned with the forecasting of the flow at the Bardo cross-section, which closes the catchment of an area of 1744 km^2 (see Fig. 5).

Calibration of both networks was based upon hydro-meteorological measurements recorded with time interval $\Delta t = 3$ hours. The hydrographs of the total flow discharge recorded during the highest water levels in the selected cross-section were delivered by the Institute of Meteorology and Water Management. Five 100-element series of 3-hour flows were selected: one for the year 1965, and two series for the years 1977 and 1997 (see Table 2).

Three-hour precipitation depths were recorded in five pluviograph stations situated in: Kłodzko, Lądek Zdrój, Międzylesie, Słoszów and Mieroszów (see Fig. 5). The

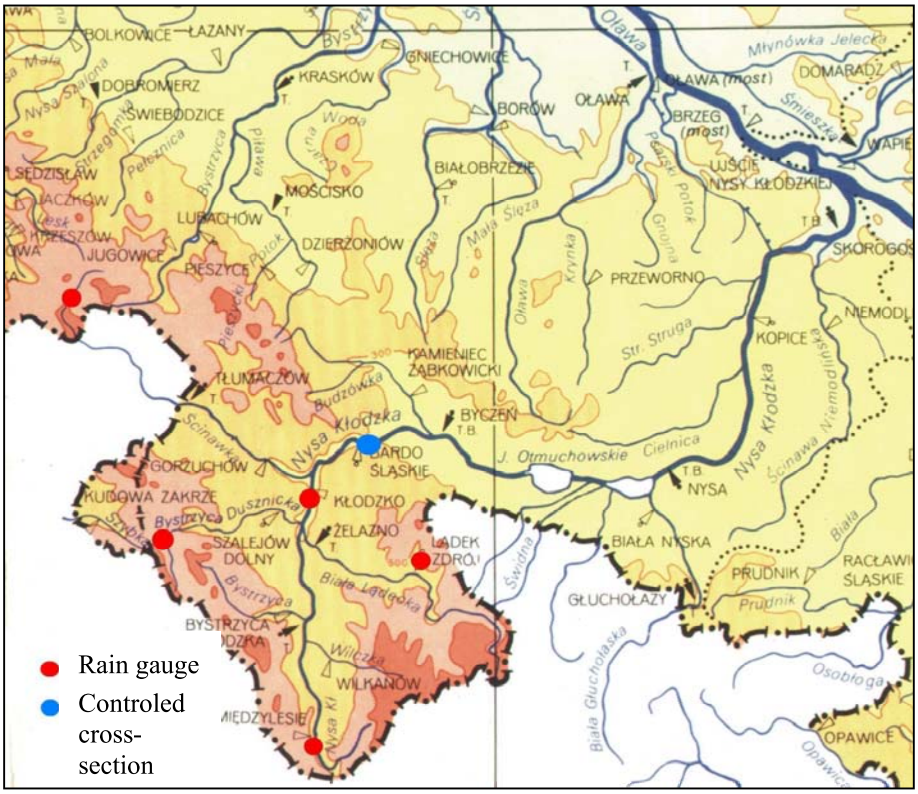

Fig. 5. Nysa Kłodzka catchment.

Table 2

Comparison of the MLP and RBF networks – Nysa Kłodzka case study

Flood wave	Wave status	MLP RMSE	RBF RMSE	Peak flow
97a	training	43.71	60.77	1718
97b	training	38.87	43.79	451
77a	training	24.62	27.65	488
77b	*verification*	*22.43*	*27.85*	*423*
65	training	34.07	40.94	823

data were corrected so as to make particular daily depths calculated from pluviograph records concordant with daily depths calculated similarly from records of all available rainfall stations.

Optimal network structure of MLP-ANN designed for runoff forecast with 12-hour leading time may be expressed as 24-14-1, i.e. 365 parameters. 24 inputs are represented by 4 delayed consecutive rainfall records at 5 pluviograph stations and 4 delayed flow records at Bardo cross-section. Due to large number of parameters, 4 flood records were chosen for model calibration and only one remaining record as the validation set.

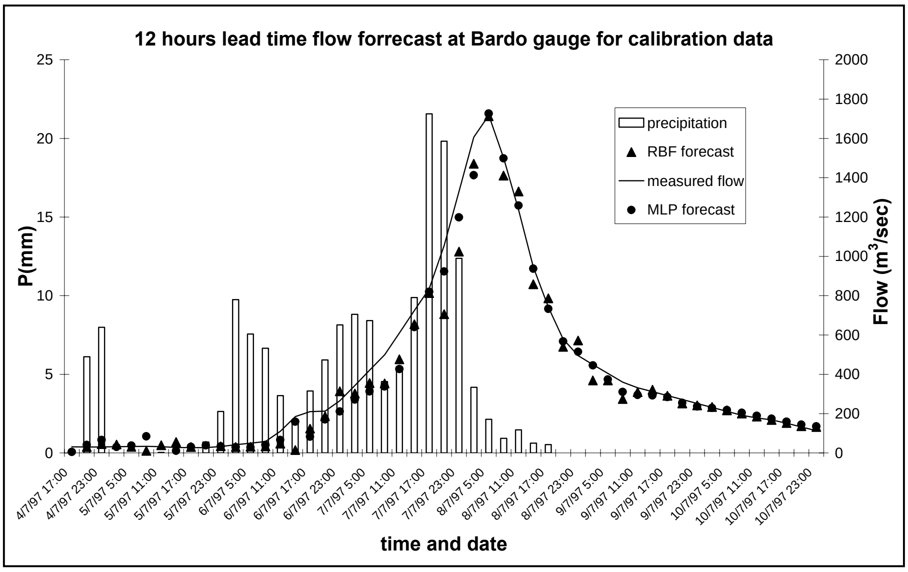

Fig. 6. Comparison of observed runoff and that predicted by MLP and RBF networks for calibration data.

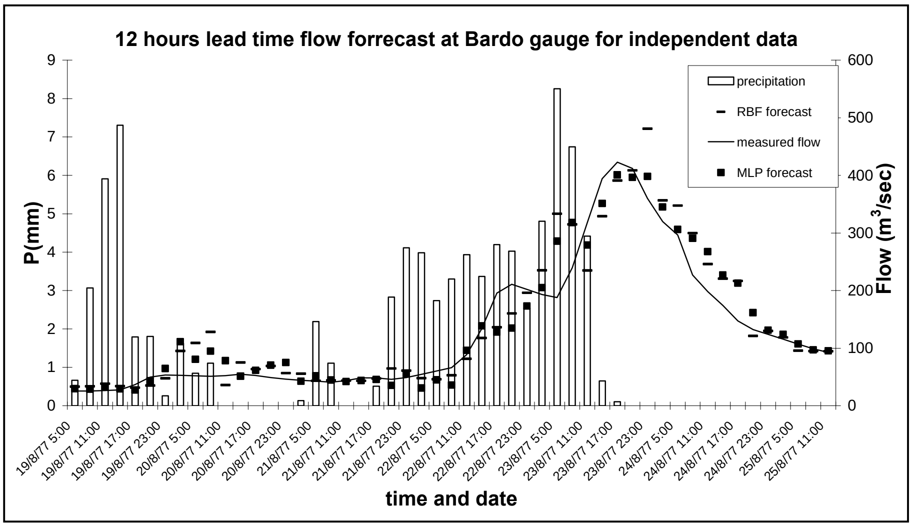

Fig. 7. Comparison of observed runoff and that predicted by MLP and RBF networks for verification data.

Optimal network structure of RBF is formed by 7 inputs (4 delayed lumped rainfall records and 3 delayed runoff records) and 22 centers in the hidden layer. Due to the course of dimensionality, mean areal 3-hour precipitation depths were used. They were calculated with the method of equal rainfall polygons (de Thiessen) upon measurements carried out in five pluviograph stations. So with one parameter σ and 22 linear weights, the total number of parameters is equal to 46.

The degree of fit to the observed runoff by MLP and RBF networks for one of the storms used for calibration is shown in Fig. 6 and for the verification storm in Fig. 7.

8. CONCLUSIONS

Non-linear lumped black-box models can be used to describe runoff from a catchment, when application of theoretically sound non-linear partial differential equations of mass and energy transfer is difficult or impossible due to the lack of required data.

Volterra series and Multi-Layer Perceptron Neural Networks are able to simulate rainfall-runoff relations with similar performance for the data set described in section 6. For events treated as training set, which includes two storms twice bigger than validation ones, MLP highly outperforms Volterra series. Note that there is a wide discrepancy in number of parameters to be optimized for both models (3 for Volterra series, 49 in the case of MLP).

Comparing MLP with Radial Basis Function Networks for real-world flood forecasting situation for Nysa Kłodzka catchment, MLP networks showed a bit better performance. But one should bear in mind that precipitations from 5 stations were taken into account separately for MLP, giving 24 inputs, whereas in the case of RBF networks due to course of dimensionality a lower number of input values is allowed. Lumped precipitations were treated as input variables, giving the total number of them as low as 7. This resulted in large differences in the number of parameters in both networks – 365 in MLP and 46 in RBF case.

It should be noted that in this particular catchment of Nysa Kłodzka the forecast is reasonably good up to 12-hour lead-time. When trying to use the ANN models to forecast the flow at the controlled cross-section with longer time horizon, the performance decreased rapidly.

Acknowledgements. This work was supported in part by grants 2 P04D 008 29 and 2 P04D 009 29 from Ministry of Science and Information Society Technologies.

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Received 15 July 2005
Accepted 30 August 2005