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THE VOLTERRA SERIES AS SPECIAL CASE OF ARTIFICIAL NEURAL NETWORK MODEL

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The geophysical processes contributing to the hydrological cycle are described by theoretically sound non-linear partial differential equations of mass and energy transfer. The hydrodynamic equations describing hydrological processes were developed in non-linear form in the nineteenth century. In the case of surface runoff from a natural catchment or flow in an open channel, an accurate application of the hydraulic approach requires a detailed topographical survey and determination of roughness parameters. In order to avoid these difficulties, alternative approaches e.g. via conceptual models and black box models were developed in the second half of the last century. The conceptual model approach is to simulate the nature of the catchment response or the channel response by relatively simple non-linear model built up from simple non-linear elements, e.g. cascade of non-linear reservoirs. Each non-linear reservoir is responsible for part of the attenuation of the system response. This lumped dynamic model can be represented by a set of ordinary differential equations:

$$\dot{S}_1(t) = -f[S_1(t)] + x(t)$$
$$\dot{S}_2(t) = -f[S_2(t)] + f[S_1(t)]$$
$$\ldots$$
$$\dot{S}_n(t) = -f[S_n(t)] + f[S_{n-1}(t)]$$
$$y(t) = f[S_n(t)]$$

where x is the input signal(rainfall or flow at the upstream end of the channel), Si is the storage in the i-th reservoir, f(.) represents the outflow-storage relation and y is the output signal (surface runoff or flow at the downstream end of the channel). Non-linear black box analysis is concerned with representing a system by a functional Volterra series in the form of a sum of convolution integrals:

y(t) = $\int_0^t h_1(\tau)x(t-\tau)d\tau + \int_0^t\int_0^t h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2$
$+$
$\int_0^t\int_0^t\int_0^t h_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + ...$

In the above equation x is the input to the system, y is the output from the model, is the first order kernel which reflects the linear properties of the system, is the second order kernel which reflects the quadratic properties , and so on. Modelling of hydrological processes by means of a Volterra series has been developed independently of other methods of describing dynamic systems, in particular by state equation formulation. The problem of series identification has been solved by numerical methods applied to input record and its corresponding output by means of kernel expansion in orthonormal polynomials. It can be shown (Napiórkowski and Strupczewski, 1979; Napiórkowski and O'Kane, 1984) that if the function f in eq.(1) is differentiable as many times as required the state-space equation (1) can be approximated by the Volterra series (2) by means of Taylor series expansion for operators. The structure of the first two kernels was shown to be

h$_1(\tau) = aH_n(\tau)$

h$_2(\tau_1,\tau_2) = b\left\{H_n(\tau_1)\sum_{k=1}^{n} H_k(\tau_2) + H_n(\tau_2)\sum_{k=1}^{n} H_k(\tau_1) - H_n[\max(\tau_1,\tau_1)]\right\}$

where

Y(i) = $\sum_{k=1}^{N_s} H_1(k)X(i-k) + \sum_{k=1}^{N_s}\sum_{l=1}^{N_s} H_2(k,l)X(i-k)X(i-l) + ...$

where Ns is the number representing the memory of t he system i.e. k, l =1,...Ns, NT is the number of observations, i.e. i=1,2,...NT. It is easy to see that discrete Volterra series described by eq.(6) is a special case of artificial network model that is called in resent publications as Volterra net. In the paper we focus our attention on of the existence and uniqueness of the solution of the described above identification problem.

Literature Napiórkowski J.J., O'Kane P., 1984. A new nonlinear conceptual model of flood waves. Journal of Hydrology, 69, 43 58. Napiórkowski J.J., Strupczewski W.G., 1979. The analytical determination of the kernels of the Volterra series describing the cascade of nonlinear reservoirs. Journal of Hydrological Sciences, vol.6, no.3 4, 121 142.