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HYDROLOGIC INPUT SIGNALS: IDEALIZATION AND REALITY

Zbigniew W. KUNDZEWICZ, Roman J. BUDZIANOWSKI,
Henryk T. MITOSEK, Jarosław J. NAPIÓRKOWSKI

Institute of Geophysics, Polish Academy of Sciences
00-973 Warsaw, P. O. Box 155, Pasteura 3

Abstract

General analysis of hydrologic input signals in distributed and lumped forms is presented. The existence of considerable gap between idealization of input signals and reality is pointed out. The basic schematized standard input signals are reviewed and the relationships between system responses to particular standard inputs are given. Special cases of considerable differences between the dynamics of the input signal and of the system (the case of slow input and rapid system and the case of rapid input and slow system) are considered. The guidelines are given on the choice of input signals for the purpose of identification of parameters of certain hydrologic models. This problem emerges at least in two practical situations, that is when one can control the input signal or when the data on several events are available. The weakness of the existing procedures of determination of effective rainfall as the input signal to the rainfall-runoff model and of construction of design hyetograph are re-examined.

1. INTRODUCTION

Mathematical modelling of hydrologic systems has been progressing during the last decades. The art of modelling reached recently a very sophisticated stage of development and followed numerous methods and techniques developed in mathematics, system theory and in various applied fields remote from classical hydrology. The complexity of models being developed today seems to keep pace with the increasing power of modern computers.

It seems, however, that investigations of input signals for hydrologic models are severely underestimated. The main effort of modellers is laid on improving the techniques of mathematical description of complex natural phenomena occurring in three-dimensional nonhomogeneous, anisotropic and irregularly shaped environment, whereas the quality and quantity of the input data might not match sophisticated modelling techniques.

Let us consider, for example, the rainfall-runoff models, i.e. mathematical models for calculation of river flow caused by a certain storm rainfall event. The input signal used in modelling is effective rainfall, that is the part of rainfall that produces the surface runoff, directly and promptly contributing to the river flow. It is easy to see, however, that heuristic and empirical procedures of determination of the effective rainfall form a weak point in the modelling process. This is partly caused by the complex and unknown temporal-spatial structure of precipitation fields which is to be replaced by a representative lumped signal, and partly by the difficulties in estimating the amount of rainfall that will not cause the surface runoff (i.e. losses by infiltration, interception, evapotranspiration, surface detention storage etc.). This is in severe disharmony with the elegant technique of regular idealized standard input signals.

2. IDEALIZATIONS

2.1. Delta impulse input. The most important standard input signal used in modelling of linear systems is the Dirac delta-function (actually – generalised function, i.e. distribution) defined as

$$\delta(t)=0 \quad \text{for} \quad t\neq 0, \tag{1}$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt=1. \tag{2}$$

The concept of the delta-function has a clear physical sense. It represents an idealized impulse lasting for an infinitely short time (i.e. produced by a pulse generator of infinite power) and occurring in the time instant $t=0$. Since the effect of the action of the infinitesimal impulse (condition (2)) is different from zero, its amplitude must be infinite. The important property of the delta-function is its filtering ability, that is

$$\int_{T_1}^{T_2} f(\tau)\,\delta(t-\tau)\,d\tau=\begin{cases} f(t) & \text{for} \quad t\in[T_1, T_2], \\ 0 & \text{elsewhere}. \end{cases} \tag{3}$$

It is possible to approximate the Dirac delta-function by a sequence of realistic signals of finite amplitude and finite duration. From the point of view of application of the one-side Laplace transform, a nonsymmetrical, right side approximating sequence $\delta_n(t)$ is appropriate. Such a sequence meets the following conditions

$$\lim_{n\to\infty} \delta_n(t)=\delta(t), \tag{4}$$

$$\int_{0}^{+\infty} \delta_n(t)=1, \tag{5}$$

$$\delta_n(t)=0 \quad \text{for } t<0, \quad n=1,2,\ldots \tag{6}$$

An example of the sequence of approximating functions is given in below.

2.2. Idealized inputs of the polynomial or piece-wise type. An important class of idealized input signals are polynomial and piece-wise polynomial functions of time. The simplest of these is the unit step signal defined as

$$1(t)=\begin{cases} 0 & \text{for} \quad t<0 \\ 1 & \text{for} \quad t\geqslant 0. \end{cases} \tag{7}$$

Although this signal can be considered more realistic than the abstract Dirac delta-function, it also contains some amount of idealization. It is technically impossible (would require infinite power) to perform the switching operation (from 0 to 1) in an infinitesimally short time interval. It could be also impossible to maintain level 1 in the time interval from zero to infinity.

The former disadvantage disappears if a transient of finite steepness is assumed, that is

$$x(t)=\begin{cases}0 & \text{for} \quad t<0,\\ f(t) & \text{for} \quad 0\leqslant t<T,\\ 1 & \text{for} \quad t>T,\end{cases} \tag{8}$$

where $f(t)$ is an arbitrary practically realizable function attaining values 0 and 1 for the values of argument 0 and T, respectively.

The latter disadvantage disappears if the input signal is of finite volume, that is, for example, if the finite unit step is considered

$$\mathbf{1}(t,T)=1(t)-1(t-T). \tag{9}$$

This concept, introduced to hydrology some fifty years ago, is very useful. Each continuous signal can be conveniently approximated as a train of rectangular finite pulses (zero-order-splines). In the case of discrete mean value measurements (e.g. raingauges) the input signal is readily given in the form of rectangular pulses measuring the averaged behaviour of the signal.

A sequence of finite impulse functions (Fig. 1) can approximate the Dirac delta-function from the right hand side, according to the following scheme:

$$\delta_n(t)=n\cdot\mathbf{1}(t,1/n), \tag{10}$$

where $\mathbf{1}(t, 1/n)$ is defined by formula (9).

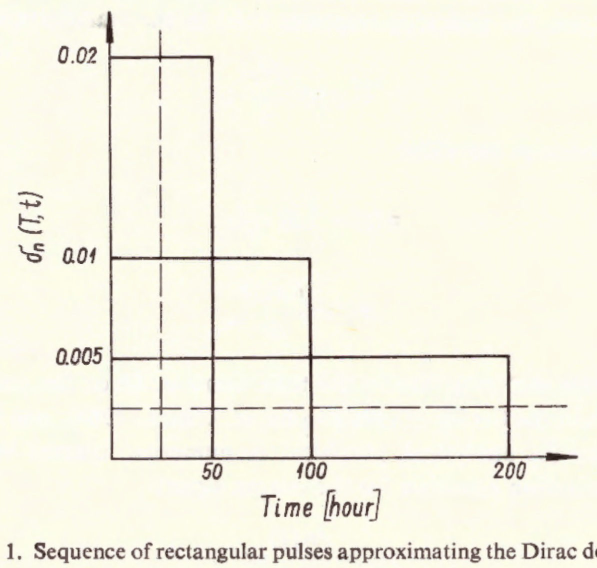

Fig. 1. Sequence of rectangular pulses approximating the Dirac delta-function

It could be also practical to consider polynomial or piece-wise polynomial inputs of higher order, that is – in the first order case – the unit ramp

$$x(t)=\begin{cases}0 & t<0,\\ t, & t\geqslant 0\end{cases} \tag{11}$$

or a hat function first-order-spline

$$x(t)=\begin{cases}0, & t<0,\\ t, & 0\leqslant t<T,\\ 2T-t, & T\leqslant t<2T,\\ 0, & t\geqslant 2T.\end{cases} \tag{12}$$

2.3. Relationships between system responses to different standard input signals. Once the response of a linear stationary system to a Dirac delta-function is known (in the time or operator domain), one can determine the system response to an arbitrary input signal. In the case of a non-anticipative (causal) initially relaxed system, the system response to an arbitrary input signal $x(t)$ is given by a convolution integral

$$y(t)=\int_0^t h(\tau)x(t-\tau)d\tau=\int_0^t h(t-\tau)x(\tau)d\tau=h(t)*x(t), \tag{13}$$

where $y(t)$ is the output signal, $h(t)$ is the system impulse function, and $*$ is the symbol of convolution operation.

In the complex domain this equation becomes

$$Y(s)=H(s)X(s), \tag{14}$$

where s is a complex variable, $Y(s)$, $H(s)$, $X(s)$ are the Laplace transforms of the output signal $y(t)$, the impulse response $h(t)$ and the input signal $x(t)$, respectively.

Since, by definition, the unit step response $U(s)$ in the transform domain is

$$U(s)=\frac{H(s)}{s}, \tag{15}$$

the following relationships are valid:

$$u(t)=\int_0^t h(\tau)d\tau, \tag{16}$$

$$h(t)=\frac{du(t)}{dt}, \tag{17}$$

where $u(t)$ is the unit step response in the time domain. Once the unit step response is given (S-hydrograph in the theory of unit graphs, cf. Chow, 1964), one has to differentiate it with respect to time (by analytical, graphical or numerical means) in order for to use the convenient convolution equation for the output signal

$$y(t)=\frac{du(t)}{dt}*x(t). \tag{18}$$

The system response to a finite rectangular pulse can be readily evaluated in terms of the unit step response as

$$w(t, T)=u(t)-u(t-T), \tag{19}$$

where $w(t, T)$ is the system response to $\mathbf{1}(t, T)$, and is also called the T-unit hydrograph (TUH).

Equation for the TUH can be also expressed in terms of the impulse responses of the system. By inserting the respective unit step responses to equation (19) one obtains

$$w(t, T)=\frac{1}{T}\left[\int_{0}^{t} h(\tau)d\tau-\int_{\max(0,T)}^{t} h(\tau-T)\,d\tau\right]=\frac{1}{T}\int_{\max(0,t-T)}^{t} h(\tau)d\tau. \tag{20}$$

As an example, let us consider the Kalinin-Milyukov flood routing model. Its TUH can be established directly from equation (20) for integer n by direct integration of the impulse response

$$h(t)=\frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1}\exp\left(-\frac{t}{K}\right), \tag{21}$$

according to the following formula that should be used n times (integration by parts)

$$\int t^{n}\exp\left(-\frac{t}{K}\right)dt=-Kt^{n}\exp\left(-\frac{t}{K}\right)+nK\int t^{n-1}\exp\left(-\frac{t}{K}\right)dt. \tag{22}$$

A more general handy notation of the TUH for the Kalinin-Milyukov model makes use of the incomplete gamma function (Reed et al., 1975)

$$w(t, T)=\frac{1}{T}\left\{I\left(n, \frac{t}{K}\right)-I\left(n, \frac{t-T}{K}\right)\right\}, \tag{23}$$

where n and K are the model parameters (n is a real number of characteristic reaches, K is the time constant of a characteristic reach) and I is the incomplete gamma function given as

$$I\left(n, \frac{t}{K}\right)=\frac{1}{\Gamma(n)}\int_{0}^{t/K}\left(\frac{t}{K}\right)^{n-1}\exp\left(-\frac{t}{K}\right)d\left(\frac{t}{K}\right). \tag{24}$$

2.4. Harmonic input. The Dirac delta-function, polynomial functions and splines are not the only schematized standard input signals of importance in analyses of systems of dynamic hydrology. In several cases (cf. Dooge and Kundzewicz, 1984) it is more appropriate to consider harmonic input signals (i.e. pure sine waves). This is true, for instance, if the flow in a river reach is influenced by tides.

Another reason for considering harmonic signals stems from the Fourier series resolution. Every smooth continuous function, either periodic (with period T) or defined in a finite interval T (i.e. amenable to periodic extension) can be developed into a Fourier series

$$f(t)=\sum_{k=-\infty}^{+\infty} g_k\exp\left(i2\pi k\frac{t}{T}\right), \tag{25}$$

where

$$g_k = \frac{1}{T}\int_0^T f(t)\exp\left(-i2\pi k\frac{t}{T}\right)dt \tag{26}$$

and i is the imaginary unit.

It should be mentioned, however, that the relative importance of this approach in hydrology is considerably smaller than in other subjects.

Examples of periodic hydrologic input functions are any phenomena with annual (e.g. rainfall, flow), weakly (water consumption and wastewater disposal) or daily periodicity (discharge from a peak-hour power station, glacier originated flow). On the other hand, several hydrologic input signals can, in fact, be considered as events that attain their non-zero values in finite intervals only.

2.5. Idealized standard stochastic signals. In mathematical modelling of hydrologic and water resources systems the input signals are often considered as stochastic processes. In such a case, the systems can be analysed by tracking the propagation of some conventional signals of stochastic structure applied to the model input. This concept has been introduced by Yule (1927) who postulated that signals, in the form of stochastic processes with strongly dependent successive elements, can be considered as the processes which are generated by another process of the mutually independent impulses. These impulses are usually treated as a realization of the same random variable which is normally distributed with the zero mean, $\mu_x=0$, and the variance σ_x^2. The important property of this stochastic process — usually called the white noise — is the following equation for autovariance:

$$R_{xx}(\tau)=\sigma_x^2\,\delta(\tau), \tag{27}$$

where τ is the lag time.

Another standard stochastic signal is the simple Markovian process. The simple Markovian process (i.e. Markovian noise) is obtained on the output of the single linear reservoir model with the white noise on its input. In stochastic hydrology, however, it is often convenient to assume that inflows form the simple Markovian process, i.e. the stationary and normal Markovian noise with the autovariance given by

$$R_{xx}(\tau)=\sigma_x^2\exp(-c\tau), \tag{28}$$

where c is a constant.

Analyses of the transformation of stochastic processes in several mathematical models in hydrology and water resources systems are described by several authors. Moran (1959) presented the theory of storage with the white noise input signals, and Kaczmarek (1963) and Lloyd (1963) developed, independently of one another, the theory of storage with the Markovian input signals. The transformation of standard stochastic signals in linear hydrologic models (of rainfall-runoff and flood routing type) was analysed by Strupczewski et al. (1975). The analysis of stochastic structure of the river flow process was given by Mitosek (1984b).

3. LINEAR SYSTEM RESPONSE TO RAPID AND SLOW INPUT SIGNALS

It may happen in modelling hydrologic phenomena that the temporal scale of dynamics of the input signal is very much different from the system dynamics. The dynamics of the input signal is significantly more rapid than the one of the system in the following examples:

a) input signal — storm of short duration, system — catchment (rainfall-runoff model);

b) input signal — lumped source of effluents to a river, system — river reach (propagation of the concentration of pollutants);

c) input signal — discharge from a peak-hour power station, system — river reach of sufficient length (open channel flow).

The dynamics of the input signal is significantly slower than the one of the system for the case of a long unimodal slowly-varying wave propagating along a short river reach.

Let us consider first the system whose dynamics is considerably slower than the variability of the input signal. The system impulse response in equation (13) can be conveniently expanded into the Taylor series

$$h(t-\tau)=h(t)-\tau h'(t)+\tfrac{1}{2}\tau^2 h''(t)-\dots \tag{29}$$

We can assume that the rapidly varying input signal can be represented as an isolated short pulse of the duration T. The model response then is

$$y(t)=h(t)\int_0^{\min(t,T)} x(\tau)d\tau-h'(t)\int_0^{\min(t,T)} \tau x(\tau)d\tau+\tfrac{1}{2}h''(t)\int_0^{\min(t,T)} \tau^2 x(\tau)d\tau-\dots \tag{30}$$

For $t>T$ the above equation becomes

$$y(t)=\langle x(t)\rangle_0^{0,T}h(t)-\langle x(t)\rangle_1^{0,T}h'(t)+\tfrac{1}{2}\langle x(t)\rangle_2^{0,T}h''(t)-\dots \tag{31}$$

where

$$\langle x(t)\rangle_i^{0,T} \overset{\mathrm{df}}{=} \int_0^T t^i x(t)dt, \quad i=1,2,\dots \tag{32}$$

If, because of slow dynamics of the system and rapid dynamics of the input, the following inequalities are true

$$\langle x(t)\rangle_0^{0,T}h(t)\gg\langle x(t)\rangle_i^{0,T}h^{(i)}(t), \quad i=1,2,\dots \tag{33}$$

where $h^{(i)}$ denotes i-th temporal derivative of h, then

$$y(t)\approx\langle x(t)\rangle_0^{0,T}h(t), \tag{34}$$

that is the response of the system is similar to the Dirac delta impulse response multiplied by the volume of the input pulse. This means that in such a case one can measure the impulse response in the field as the system reaction to a short lasting pulse of any shape, subject to the scaling condition

$$\int_0^\infty h(t)dt=1. \tag{35}$$

The responses of the Kalinin-Milyukov flood routing model to sequences of rectangular pulses with unit volume are presented in Fig. 2. The details of the analysis of pulse responses in the Kalinin-Milyukov model are given in chapter 2.3.

It is easy to see that, since for the sequence of functions approximating the Dirac delta the following condition is valid

$$\lim_{n\to\infty} \delta_n(t)=\delta(t), \tag{36}$$

the sequence of responses has the property

$$\lim_{n\to\infty} [y_n(t)=\int_0^t h(\tau)\delta_n(t-\tau)d\tau]=\int_0^t h(\tau)\delta(t-\tau)d\tau=h(t). \tag{37}$$

The above condition, illustrated also in Fig. 2, can be interpreted that for the input signal short and rapid (delta-like) the system response does not considerably differ from the impulse response. The system response depends entirely on the properties of the system and not on the details of the input signal.

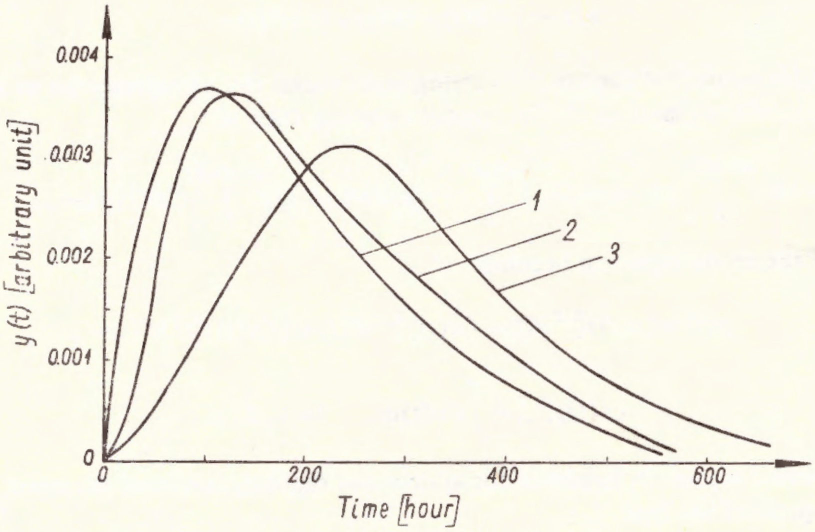

Fig. 2. Responses of flood routing model to sequences of rectangular pulse inputs

1 – impulse response, *2* – response to rectangular pulse of duration 50 hours, *3* – response to rectangular pulse of duration 200 hours

Another application of the presented findings is the problem of pulse discrimination.

The physical system of propagation of a constant volume flood wave has diffusive properties (attenuation of the amplitude and increase of the time base with the wave propagation). Therefore a practical question of discrimination of the consecutive input signals can occur. This is particularly important for water systems containing tributaries which can significantly contribute to the flood wave. The safety condition for the reach below the confluence is the lack of synchronism between the bulks of flows in the main channel and in tributaries. The idea of discrimination is also of interest in large reaches, where a

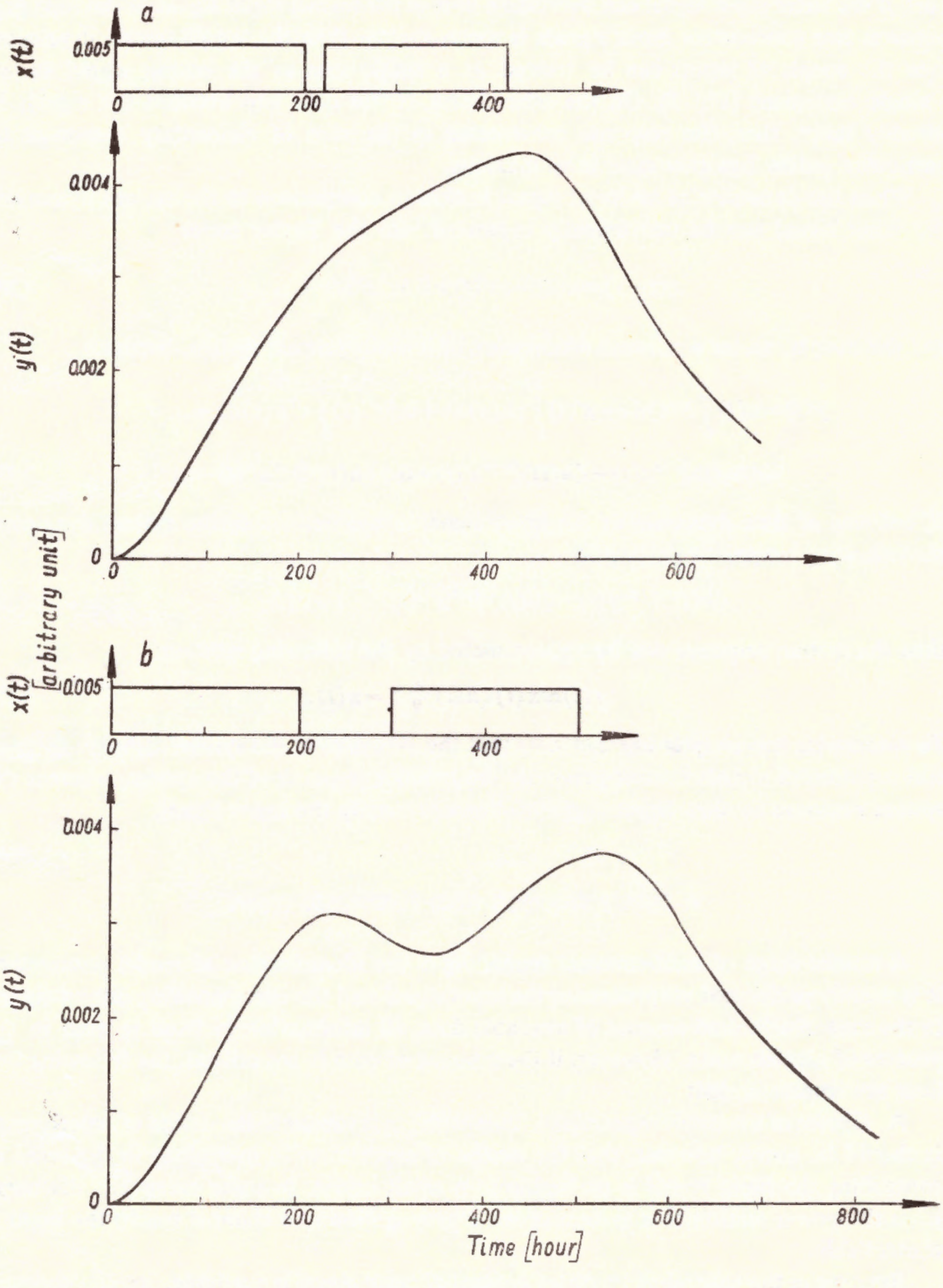

Fig. 3. Discrimination of rectangular pulse inputs with the time gap between two consecutive signals. (*a*) much shorter than the lag of system dynamics, (*b*) suitably long

part of the latter input signal can overtake a part of the former input signal. If the time gap between two consecutive input signals is much shorter than the lag of system dynamics, then the couple of inputs can be treated as a single one as the output signal shows a single maximum (Fig. 3). On the other hand, if the time gap between the signals is suitably large, then the response constructed as the sum of elementary responses to particular pulses has two clear local maxima.

When the dynamics of an input signal is slow in comparison to the system dynamics, the input signal can be conveniently expanded into the Taylor series

$$x(t-\tau)=x(t)-\tau x'(t)+0.5\tau^2 x''(t)-\dots \tag{38}$$

and the output, under a relieved assumption of initially relaxed system, becomes

$$y(t)=x(t)\int_0^\infty h(\tau)d\tau-x'(t)\int_0^\infty \tau h(\tau)d\tau+0.5x''(t)\int_0^\infty \tau^2 h(\tau)d\tau-\dots=$$
$$=\langle h(t)\rangle_0^{0,\infty}x(t)-\langle h(t)\rangle_1^{0,\infty}x'(t)+0.5\langle h(t)\rangle_2^{0,\infty}x''(t)-\dots \tag{39}$$

If, because of slow dynamics of the input signal in comparison to the system dynamics, the following inequalities are valid

$$\langle h(t)\rangle_0^{0,\infty}x(t)\gg\langle h(t)\rangle_i^{0,\infty}x^{(i)}(t), \quad i=1,2,\dots \tag{40}$$

then the system response can be approximated by the following relation:

$$y(t)\simeq x(t)\langle h(t)\rangle_0^{0,\infty}=x(t). \tag{41}$$

To summarize this chapter one can state that the dynamics of the system response to short and rapid input signals follows primarily the impulse response of the system, whereas the system response to slow input signals follows primarily the input signal.

4. INPUT SIGNALS AND IDENTIFICATION

A crucial stage of mathematical modelling is the identification of model parameters. It is usually performed via processing the input signal and the corresponding output signal. In the case of a limited amount of available data, all information in hand is used in the process of identification. However, if the amount of data is not critical, the following two lines are recommended:

A. Choice of a suitable input signal. This problem is similar to the design of experiments. It should be noted, however, that the design of experiment in hydrology in the sense of controlling the input signal is realizable to a small extent. In some cases (e.g. discharge from the power station to a system of wave propagation in a channel reach) the input is controllable.

But even in the case when no direct control of input signal is available, one can choose a pair of corresponding input and output events from the available data. The choice of the suitable set should follow the requirements easing the identification. The remaining data can be used, for instance, for testing the model with identified values of parameters.

B. Determination of the optimum impulse response (or optimum set of parameters of the impulse response) for all data can be achieved using the least squares criterion (Diskin and Boneh, 1975).

4.1. Deterministic identification of linear systems. In this chapter the impact of the type of input signal on the stability of solution of the deterministic identification problem is considered.

Let us assume that a linear, non-anticipative, initially relaxed system, as given by equation (13), is to be identified from the series of concurrent values of input and output variables. This sort of identification enables to establish the system operator (kernel function $h(t)$) without the knowledge of the system inner mechanism. The identified kernel can be subsequently used, owing to the attribute of linearity, for prediction of outputs caused by arbitrary input signals.

However, the requirement of initially relaxed situation must be relieved in several cases. In many practical situations the system is not completely at rest at the time $t=0$ and to overcome the difficulty with non-zero initial conditions an assumption is made that the system considered has a finite settling time T_s. The settling time of the system ("memory") is the time interval over which the past input history has the effect on the present output.

Thus, the problem to be solved is to find the best estimate of the kernel function $h(\tau)$, $\tau \in [0, T_s]$ of the system with the settling time T_s

$$y(t)=\int_0^{T_s} h(\tau)x(t-\tau)d\tau, \tag{42}$$

which minimizes the mean square residual error, using the output record $y(t)$ observed in a finite interval $[0, T]$ larger than the system memory and the input record $x(t)$ observed in an interval $[-T_s, T]$

$$J(h)=\int_0^T [z(t)-y(t)]^2 dt. \tag{43}$$

4.1.1. *Orthonormal expansion.* The problem of determining the kernel function is mathematically equivalent to the inversion problem, which can be solved by several methods, among which kernel expansions in orthonormal functions are widely used (e.g. Dooge, 1965; Papazafiriou, 1976). In that method the approximate solution takes the form

$$h(t)=\sum_{i=1}^{N} a_i \varphi_i(t), \tag{44}$$

where $\varphi_i(t)$ is the sequence of orthonormal function in $[0, T_s]$. Any set of functions orthonormal over a finite interval may be used for the expansion. However, as advocated by Dooge (1965), orthonormal polynomials with exponentially decreasing weighting function are particularly suitable in hydrological applications. The coefficients of the polynomials are generated by the three-term recurrence relation (Davis and Rabinowitz, 1975)

$$p_{n+1}(t)=(t-a_n)\,p_n(t)-b_n\,p_{n-1}(t), \tag{45}$$

where

$$p_{-1}(t)=0, \quad p_0(t)=1,$$

$$a_n=\frac{\langle tp_n, p_n\rangle_\beta}{\langle p_n, p_n\rangle_\beta}, \quad n=0,1,\ldots,$$

$$b_n=\frac{\langle tp_n, p_{n-1}\rangle_\beta}{\langle p_{n-1}, p_{n-1}\rangle_\beta}, \quad n=1,2,\ldots$$

and the inner product is defined as

$$\langle a, b\rangle_\beta \underset{\mathrm{df}}{=} \int_0^{T_s} a(t)\,b(t)\exp(-\beta t)\,dt. \tag{46}$$

The orthonormal form of these polynomials is obtained from the following relation

$$Q_n(t)=\frac{p_n(t)}{\langle p_n, p_n\rangle_\beta}. \tag{47}$$

The orthonormal functions used to approximate the kernel are of the form

$$\varphi_{i+1}(t)=Q_i(t)\exp\left(-\beta\frac{t}{2}\right) \tag{48}$$

and they fulfil the following conditions

$$\int_0^{T_s} \varphi_i(t)\,\varphi_j(t)\,dt=\begin{cases}0 & \text{if} \quad i\neq j,\\ 1 & \text{if} \quad i=j.\end{cases} \tag{49}$$

For the brevity of notation we use

$$q_i=\int_0^{T_s} \varphi_i(\tau)\,x(t-\tau)\,d\tau. \tag{50}$$

Now the problem of identification of the kernel function can be reduced to the minimization of the following expression

$$J(h)=\int_0^T [z(t)-\sum_{i=1}^{N} a_i q_i(t)]^2\,dt. \tag{51}$$

The necessary condition for a minimum of equation (51) is the following set of requirements for the expansion coefficients a_i (normal equations):

$$\left.\begin{array}{l} a_1\langle q_1, q_1\rangle+a_2\langle q_1, q_2\rangle+\ldots+a_n\langle q_1, q_n\rangle=\langle q_1, z\rangle,\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ a_1\langle q_2, q_1\rangle+a_2\langle q_2, q_2\rangle+\ldots+a_n\langle q_2, q_n\rangle=\langle q_2, z\rangle,\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ a_1\langle q_n, q_1\rangle+a_2\langle q_n, q_2\rangle+\ldots+a_n\langle q_n, q_n\rangle=\langle q_n, z\rangle, \end{array}\right\} \tag{52}$$

where

$$\langle q_i, q_j\rangle=\int_0^T q_i(t)\,q_j(t)\,dt. \tag{53}$$

4.1.2. *Stability of the solution.* The basic requirement that any identification method must meet is the stability of the solution. This applies also to the case when measurement values are contamined by significant errors. Using the orthonormal expansion method one can reduce the infinitely dimensional problem to the algebraic set of n equations (52) which can be written in the matrix form

$$X\mathbf{a}=\mathbf{b}. \tag{54}$$

The matrix of this set depends directly on the input $x(t)$ and the stability of the solution of these equations depends not only on the accuracy of measurements but also on the dynamics of input variability.

If the errors of the matrix X and vector $\mathbf{b}$ are δX and δb, respectively, then the following estimate of the relative solution error can be obtained (Napiórkowski, 1981):

$$\frac{\|\delta a\|}{\|a\|}\leqslant\frac{\gamma(X)}{1-\gamma(X)\dfrac{\|\delta X\|}{\|X\|}}\left(\frac{\|\delta b\|}{\|b\|}+\frac{\|\delta X\|}{\|X\|}\right), \tag{55}$$

where $\gamma(X)$ is the condition number of the matrix, e.g. the ratio of the largest to the smallest absolute values of eigenvalues. The above estimate is usually a pessimistic one when the right hand side of equation (55) is not exactly known.

There is, however, no precise check of the problem conditioning. From relation (55) it follows that the relative solution error increases with the increase of the condition number $\gamma(X)$. Therefore, if one has several input-output records in hand and one wants to identify the kernel function, one should choose the pair of records for which the set of linear equations is well conditioned. The criterion which should be applied here is the minimum of the ratio of absolute values of the largest to the smallest eigenvalues of the matrix of the set of linear equations.

Obviously, rapidly varying input signals that approximate the Dirac delta-function (e.g. rectangular pulses) will render the set of normal equations well conditioned, because these signals give large diagonal elements and small non-diagonal elements of the system matrix. For the idealized case of Dirac delta-input one gets

$$q_i(t)=\begin{cases}\varphi_i(t) & \text{for} \quad t\leqslant T_s,\\ 0 & \text{for} \quad t>T_s\end{cases} \tag{56}$$

and thus

$$\langle q_i, q_j\rangle=\begin{cases}1 & \text{if} \quad i=j,\\ 0 & \text{if} \quad i\neq j\end{cases} \tag{57}$$

and the condition number is equal to one.

On the other hand, slowly varying input signals may lead to the ill-posed set of equations. In a particular case of a constant input function all columns of the matrix X are linearly dependent and the solution of the problem is not unique.

Napiórkowski (1978) compared the results of identification of the kernel of linear integral operators for different input signals. The input signals considered ranked from weakly varying (corresponding to the p.d.f. of the Pearson III type), through moderately varying (sum of two Pearsonian input signals with different parameters) to strongly varying (rectangular pulse). The identified impulse response of the system was closest to the real one in the case of strongly varying input signal. For weakly varying input signal the result of identification was rather remote from the real impulse response; negative ordinates occurred. It is interesting that all three identified kernel functions assured good consistency of simulated and real output signals. The more varying the input signal and the longer the record of observed outflows, the better was the estimation of the impulse response.

4.2. Statistical properties of input signals and their impact on the accuracy of estimation of difference models. Let us consider an autoregressive model with one parameter a

$$y(t)-ay(t-1)=u(t-1)+\varepsilon(t), \tag{58}$$

where $\varepsilon(t)\sim N(0, \lambda^2)$ is a normally distributed white noise. This notation will be used throughout this chapter. The maximum likelihood estimator is

$$\hat{a}=\frac{\sum_{t=1}^{N} y(t)y(t-1)-\sum_{t=1}^{N} u(t-1)y(t-1)}{\sum_{t=1}^{N} y(t-1)y(t-1)}. \tag{59}$$

The variance of this estimator under the assumption of stationarity of the processes $\{u\}$ and $\{\varepsilon\}$ was determined by Budzianowski (1980) as

$$\operatorname{var}\hat{a}=\frac{\lambda^2}{NR_y(0)}, \tag{60}$$

where λ^2 is the variance of the process $\{\varepsilon(t)\}$, $R_y(0)$ is the variance of the output process $\{y(t)\}$ and N is the length of the sample.

The estimation of the parameter a is accurate if the variance of the noise ε is small enough and the variance of the output $\{y(t)\}$ is large enough (the accuracy of estimation calls for large variance and strong autocorrelation of the process $u(t)$).

Let us consider now the moving average model:

$$y(t)=b_0u(t)+b_1u(t-1)+\varepsilon(t). \tag{61}$$

The maximum likelihood estimator of the parameters b_0, b_1 and its variance can be expressed as follows (Budzianowski, 1980):

$$\begin{bmatrix}\hat{b}_0\\ \hat{b}_1\end{bmatrix}=\begin{bmatrix}R_u(0) & R_u(1)\\ R_u(1) & R_u(0)\end{bmatrix}^{-1}\begin{bmatrix}R_{yu}(0)\\ R_{yu}(1)\end{bmatrix}, \tag{62}$$

$$\operatorname{var}(\hat{b}_0)=\frac{\lambda^2}{N}\,\frac{1}{R_u(0)\left[1-\dfrac{R_u^2(1)}{R_u^2(0)}\right]}, \tag{63}$$

$$\operatorname{var}(\hat{b}_1)=\frac{\lambda^2}{N}\,\frac{1}{R_u(1)\left[1-\dfrac{R_u^2(1)}{R_u^2(0)}\right]}, \tag{64}$$

where R_u is the input autocorrelation function and R_{yu} is the cross-correlation of input and output signals.

The corollary that can be drawn from this analysis of equations (62) - (64) is as follows. The conditions for the high accuracy of estimation are:

a) large value of the signal-to-noise ratio (expressed by $R_u(0)/\lambda^2$) and

b) small autocorrelation of the input signal. The best input is the white noise which fulfils the condition

$$\frac{R_u^2(1)}{R_u^2(0)}=0. \tag{65}$$

The former condition is obvious and common to all kinds of models. The latter condition, however, is contrary to the condition developed for the autoregression model.

Let us consider now the problem of accuracy of the estimation for the ARMA model:

$$y(t)-ay(t-1)=b_1 u(t-1)+\varepsilon(t). \tag{66}$$

Let us assume that the input signal $u(t)$ has the properties of simple Markovian process with the autocorrelation function

$$R_u(\tau)=R_0\, a^{|\tau|}. \tag{67}$$

The information matrix for the problem of the estimation of parameters of the ARMA model, defined as

$$J=-E\frac{\partial^2 \log L}{\partial\theta_i \partial\theta_j}, \tag{68}$$

where L is the likelihood function and θ_j is j-th parameter, was given by Budzianowski (1980) as

$$J=\begin{bmatrix} \frac{N}{s}\alpha+N\gamma & \frac{N}{s}\beta \\ \frac{N}{s}\beta & \frac{N}{s} \end{bmatrix}, \tag{69}$$

where

$$s=\frac{\lambda^2}{R_u(0)}, \quad \gamma=\frac{1}{1-a_1^2}, \quad \alpha=\frac{b_1^2(1+aa_1)}{(1-a_1^2)(1-aa_1)}, \quad \beta=\frac{b_1 a}{1-aa_1}.$$

The determinant of the matrix J is

$$\det J=\Delta=\frac{N^2}{s^2}(\alpha+s\gamma-\beta^2). \tag{70}$$

The area of the ellipse determined by eigenvalues of the matrix J (with the semi-axes lengths $1/\sqrt{\lambda_1}$ and $1/\sqrt{\lambda_2}$) is

$$P=\frac{\pi}{\sqrt{\lambda_1\lambda_2}}=\frac{\pi}{\sqrt{\det J}}, \tag{71}$$

where λ_1 and λ_2 are the matrix eigenvalues.

The probability that the estimates of the parameters a and b are located within the ellipse is

$$p_e=[1-\exp(-0.5)]\cong 0.39\,. \tag{72}$$

Let us consider now the problem of choice of the input signal which minimizes the area P, i.e. maximizes the determinant of the matrix J. This condition of maximum accuracy is consistent with intuition – the more information in the observation (the greater value of the determinant det J), the better estimation of parameters.

The covariance matrix of the estimator is J^{-1}. Elements of this matrix along the main diagonal are variances of particular parameters, which means that the estimate of the parameter θ_i lays within the interval $(\theta_{0i}-\sigma_{0i},\, \theta_{0i}+\sigma_{0i})$ with the probability equal to 0.682. θ_{0i} is the expected value of the estimator.

By maximizing Δ with respect to a one finds that the determinant reaches its maximum value for $\hat{a}=a_1$, no matter how large is the value of the signal-to-noise-ratio. The general corollary, valid for the order of the model equal to n, is that the frequency range of the input signal should by the same as the frequency range of the model.

The reciprocal of the information matrix is

$$J^{-1}=\frac{s}{N\Delta_R}\begin{bmatrix}1 & -\beta\\ -\beta & \alpha+s\gamma\end{bmatrix}, \tag{73}$$

where $\Delta_R=\alpha-\beta^2+s\gamma$.

Note that the area of the ellipse P or the value of the determinant Δ are the global criteria of accuracy. The individual parameters do not reach their maximum accuracies because

$$\sigma_a\geqslant\sqrt{\frac{s}{N\Delta_R}},$$

that is the limiting variance attains its minimum value for $a=\hat{a}$, but

$$\sigma_b\geqslant s\sqrt{\frac{\alpha+s\gamma}{\Delta_R N}}$$

attains its minimum for $a=0$.

The matrix J is also the matrix of derivatives of the second order of the logarithm of likelihood function. This means that the larger the value of the second derivative (measured by the value of determinant), the better the accuracy. This is, however, only theoretical accuracy. In practice, in numerical calculations, one places upon the hessian (matrix of the second derivatives) additional constraints which in multidimensional problems severely affect the accuracy of computations. The conditioning of the hessian depends on the ratio of the largest to the smallest eigenvalues. It is advantageous to keep this ratio as small as possible. This is equivalent to the demand for the shape of the ellipse of concentration to be circular-like.

In the two-dimensional case the criterion to be minimized is

$$\left|1-\frac{\lambda_1}{\lambda_2}\right|=\left|\frac{\lambda_2-\lambda_1}{\lambda_1}\right|, \qquad (74$$

where λ_1 and λ_2 are the eigenvalues of the matrix J.

It can be shown that

$$\left|\frac{\lambda_2-\lambda_1}{\lambda_1}\right|=\frac{2\sqrt{1-w}}{1+\sqrt{1-w}}, \tag{75}$$

where

$$w=\frac{4\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^2},$$

and that $w \leqslant 1$ (equality holds for $\lambda_1=\lambda_2$). One can infer from the analysis of function (75) that its minimization is tantamount to maximization of w, expressed as (Budzianowski, 1980)

$$w=\frac{4(1-a_1^2)\,[b_1^2(1-a^2)+s(1-aa_1)^2]}{[b_1^2(1+aa_1)+s(1-aa_1)+(1-a_1^2)(1-aa_1)]^2}. \tag{76}$$

The above function attains its maximum value for

$$a_{\text{opt.}}=\frac{a_1(b_1^2-1+a_1^2+s)}{-b_1^2-1+a_1^2-s(1-2a_1^2)}. \tag{77}$$

The example solution for $a_1=0.5$, $b_1=0.5$ and $s=0.2$ is

$$a=0, \quad K=1.66,$$

$$a_{\text{opt.}}=0.136, \quad K=1.6,$$

$$a=0.5, \quad K=2.22,$$

where $K=\lambda_2/\lambda_1$.

The best conditioning of the hessian can be obtained for $a_{\text{opt.}}=0.136$, that is for the input signal closer to the white noise ($a=0$) than the input signal minimizing the area of ellipse of concentration ($a=0.5$).

Table 1

a	K	$\Delta/10^6$	a_1	b_1
0.0	1.55	0.238	0.486	0.491
0.136	1.31	0.258	0.497	0.502
0.25	1.38	0.276	0.495	0.504
0.5	2.0	0.303	0.490	0.522
0.75	3.5	0.283	0.462	0.550
0.95	9.0	0.209	0.420	0.576

The optimization of the model parameters was performed with the help of generated data. The input signal was simple Markovian process (of the first order), normally distributed $N(0, 1)$. The calculations were performed for several values of the parameter a. The results are given in the Table 1, where $\lambda^2=0.25$, $R_u(0)=1$, that is $s=0.25$ and $N=200$.

The results of the numerical experiment support the theoretical analyses presented earlier. However, the results of calculations for real data were worse than the ones reported in Table 1 (cf. Budzianowski, 1980).

5. CLASSIFICATION OF INPUT SIGNALS IN HYDROLOGIC MODELS

The general form of the vector of input signals in hydrologic models can be given by the following spatial-temporal vector field

$$\mathbf{f}(x, y, z, t),$$

which, after being submitted to the input terminals of the model, gives the output signal (or signals) from the model.

There are several special cases of interest to hydrologists. Let us assume that there is only one significant input signal to the system, which means that the vector $\mathbf{f}$ reduces to the scalar f.

If the model is of distributed type with one spatial variable, then the general input signal is

$$f(x, t).$$

If the signal is lumped (as, for instance, inputs to lumped conceptual models), then it is simplified to the form

$$f(t),$$

whereas if the signal is static, then it takes the form

$$f(x).$$

When modelling particular elements of a hydrologic cycle, one deals with some or all of the special cases of input signals defined above.

The examples of specification of input signals for certain subprocesses of the hydrologic cycle are as follows:

5.1. Open channel flow. Lumped input signals – boundary conditions:

$Q(0, t)$ – inflow to the reach,

$y(0, t)$ – depth (stage) at the upstream terminating cross-section,

$Q(L, t)$ – outflow from the reach,

$y(L, t)$ – depth (stage) at the downstream terminating cross-section (there are several alternate choices of dependent variables describing the process of open channel flow: depth/stage and velocity, area and flow rate, area and velocity and so on),

$q(l, t)$ – lumped lateral inflow (or outflow) located in point l (the inflow can represent tributary or sink of wastewater, the outflow can represent water intake).

Distributed input signals:

$q(x, t), x \in (0, L)$ – distributed lateral inflow accounting losses (infiltration, evapotranspiration) and augmentation of flow (groundwater inflow, rainfall falling directly into the channel, overflow).

Statical distributed signals:

$y(x, 0)$, $Q(x, 0)$, $0 \leqslant x \leqslant L$ – initial conditions.

5.2. Propagation of pollutants in rivers. The inflow signals listed for an open channel flow are also valid in the present case. Lumped input signals – boundary conditions:

$\mathbf{c}(0, t)$, $\mathbf{c}(L, t)$ – vector of concentration of polluting elements, biochemical oxygen demand and dissolved oxygen in inflow to the reach and outflow from the reach,

$q_c(l, t)$ – point lateral source of pollution (e.g. municipal or industrial waste inflow) or of oxygen (aerators).

Distributed input signal:

$q_c(x, t)$, $0 \leqslant x \leqslant L$ – distributed lateral waste inflow, e.g. supply of agricultural and farming pollutants (nitrates, fertilizers, pesticides, herbicides).

Statical distributed signal:

$\mathbf{c}(x, 0)$, $0 \leqslant x \leqslant L$ – initial conditions.

5.3. Movement of soil moisture. Lumped input signals:

$Q_i(t)$ – drawdown from a well.

Distributed input signals:

$i(0, t)$ – rainfall rate at the soil surface,

$h(0, t)$ – depth of surface storage (ponding if rainfall rate exceeds the infiltration capacity),

$r(0, t)$ – net water exchange between the soil surface and soil (infiltration, evapotranspiration),

$p(X, t)$ – net water exchange between saturated and unsaturated zones (deep percolation, capillary rise).

5.4. Transformation of effective rainfall into runoff. Distributed input signals:

$r(x, y, z, t)$ – rainfall rate at the point (x, y, z),

$i(x, y, t)$ – losses (infiltration, evaporation) of rainfall at $z=0$,

$r_e(x, y, t)$ – distributed effective rainfall at $z=0$, equal to rainfall rate minus losses.

Lumped input signals:

$R_e(t)$ – value of effective rainfall, representative of the whole catchment or subcatchments. Determination of this lumped value of effective rainfall is probably the weakest point of the rainfall-runoff modelling. The rainfall data usually available pertain to one or a few gauges only.

From the list presented it is obvious that in lumped models all input signals are lumped, whereas in distributed models some of input signals can be distributed and other – like boundary conditions – are lumped.

6. SOME PRACTICAL REMARKS

In the hydrologic rainfall-runoff modelling one deals with effective rainfall and surface runoff as the system input and system output, respectively. The effective rainfall is, by definition (e.g. Linsley et al., 1975), that part of the total precipitation that contributes directly to the surface runoff. Thus, the total runoff in a river channel, represented by the hydrograph, is conceptually distributed into direct runoff and baseflow.

A river channel transforms spatially distributed precipitation into a runoff, which is a spatially lumped process at a given cross-section. Effective rainfall and direct runoff, that is the input and output processes in the rainfall-runoff modelling, are considered as spatially lumped.

Therefore a practical problem occurs of (a) finding the representative lumped value of the precipitation over the watershed, and (b) determination of the lumped effective rainfall from the representative lumped total precipitation. Though there are some well established techniques of these evaluations (e.g. Chow, 1964; Linsley et al., 1975), they are not equivalent and their results are different (Singh, 1977).

In all these procedures the effective rainfall is detached as that part of rainfall which produces the direct runoff. Thus, the effective rainfall is defined from its result, i.e. the direct runoff. In these idealized systems the causality is not clear since the input precipitation is inferred from the direct runoff. Then the idealized effective rainfall is created without relevance to the "direction" of the hydrologic cycle (Mitosek, 1984a).

The common practice in engineering design of urban sewer facilities, road culverts, and many small water-control structures is a runoff estimation as a fixed percentage of rainfall. The method used in the design of storm drains is based on the criterion that for so-called reliable storms of uniform intensity, distributed over the basin, the peak discharge occurs when the entire basin area participates in a runoff formation and the peak is equal to a percentage of the storm. This forms the foundations of the rational formula being used by hydrologists, hydraulic engineers and environmental planners for many years. The authorship of the approach is attributed to Mulvaney and dates back to 1851 (cf. Dooge, 1973):

$$Q = CIA, \tag{78}$$

where C is the runoff coefficient, $0 \leqslant C \leqslant 1$, I is the storm intensity, and A is the basin area.

There are a number of variations of rational formula (78). The modified equation used up-to-date in Polish practice (cf. Błaszczyk et al., 1974) is

$$Q = \varphi CIA, \tag{79}$$

where φ is a reduction coefficient, $0 \leqslant \varphi \leqslant 1$.

The reliable storm intensity is in general assumed to be a function of a storm duration and given exceedence probability only. In hydrologic manuals there are many formulas expressing the storm intensity. The reduction coefficient φ takes into account a basin and drains retention, whereas the runoff coefficient C defines that part of rainfall which enters the drains. It is worth noting that the product CIA in expression (78) is similar to the effective rainfall considered above. Błaszczyk et al. (1974) estimated the average value of the runoff coefficient C to be equal to 0.5 for newly urbanized areas in Poland.

Yen and Chow (1980) analysed an alternate approach to storm drains design accounting the temporal distribution of rainfall. The results obtained are consistent with the findings of chapter 3 of this paper. For large catchment the knowledge of the average rainfall rate is sufficient for engineering design purposes. For small catchment, on the other hand, the form of temporal distribution of rainfall influences the runoff properties. According to the terminology introduced in chapter 3 one can call the former case the rapid input/slow system and the latter case the rapid input/rapid system. The former case of different dynamics of the input signal and system can be simplified, while the latter remains complex.

Since one has to be aware of large errors in determination of the input signal in the rainfall-runoff model, it is important to evaluate the impact of these errors on the accuracy of simulation. This problem was approached by Singh and Woolhiser (1976), and Singh (1977), who compared models of different complexities. The essence of their findings is that even a perfectly identified nonlinear model cannot be proved uniformly better than a perfectly identified linear model. In other words, the input sensitivity (i.e. the sensitivity of the model with respect to errors in the input signal) can grow with the degree of complexity of the model.

7. CONCLUSIONS

Compact and elegant linear system theory, that is frequently applied to hydrology, deals with standard idealized input signals. In hydrologic reality, however, the input signals (e.g. effective rainfall) can result from processing scarce and inaccurate data that aim to represent the spatial-temporal field of recorded rainfall. The accuracy of this process is a severe and, frequently, underestimated source of deficiences of the modelling process.

If dynamics of the system is much different from the dynamics of the input signal, simplified analysis can be performed. In such a situation the slower element of the two (i.e. input or system) plays the decisive role.

It has been shown possible to formulate guidelines for the correct choice of input signals easing the process of identification of model parameters. Deterministic identification calls for strong variability of the input signal. Slowly varying, monotonous, input signals may cause bad posedness of the identification problem. The basic requirement of identification of stochastic model parameters is — large value of the ratio-signal-to--noise. Moreover, for AR-models large variance and strong autocorrelation of input signals is required, whereas small autocorrelation of input signal is advantageous for MA-models identification. Conditions of optimal choice of input signal for identification of ARMA-models have been shown to be — maximizing the area of ellipse of concentration, i.e. maximizing the determinant of the information matrix.

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HYDROLOGICZNE SYGNAŁY WEJŚCIOWE – IDEALIZACJA I RZECZYWISTOŚĆ

Streszczenie

Przedstawiono ogólną analizę hydrologicznych sygnałów wejściowych w postaci skupionej i rozłożonej. Wykazano znaczną lukę między idealizacją sygnałów wejściowych a rzeczywistością. Dokonano przeglądu podstawowych standardowych sygnałów wejściowych w postaci schematyzowanej. Podano związki między odpowiedziami systemu na wymuszenia standardowe. Rozważono szczególne przypadki, w których pojawiają się znaczne różnice między dynamiką sygnału wejściowego i systemu (tzn. przypadki powolnego wymuszenia i szybkiego systemu oraz szybkiego wymuszenia i powolnego systemu). W przypadkach tych można przeprowadzić uproszczoną analizę. Podano wskazówki dotyczące wyboru sygnałów wejściowych dla identyfikacji parametrów pewnych modeli hydrologicznych. Problem ten pojawia się w co najmniej dwóch praktycznych sytuacjach: jeżeli możliwe jest kształtowanie sygnału wejściowego lub jeżeli dostępne są dane dotyczące szeregu zjawisk. Dokonano analizy wad znanych procedur określania opadu skutecznego jako sygnału wejściowego do modelu opad – odpływ i konstrukcji opadu projektowego.