Conceptual catchment water balance model

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ABSTRACT: A water balance model aimed at quantifying catchment water storage, runoff and actual evapotranspiration is considered in the paper. A concept of two connected reservoirs is adopted and the corresponding equations for upper and lower reservoirs are successively solved. The computer system based on the model consists of two main independent parts. The first one CLIRUN_4s simulates water balance components on a basis of historical values of catchment precipitation and potential evapotranspiration. In the second part CLIRUN_4g the input climatic data are obtained by means of stochastic generation procedures. In the paper exemplary results for a selected catchment are presented to reveal a better model performance in comparison with its former simpler versions.

1 INTRODUCTION

Time series of catchment water storage, river runoff and actual evapotranspiration may be simulated by means of a conceptual hydrological model CLIRUN. The earlier version of the model (Kaczmarek 1994) was extensively tested at a number of rivers in Poland and in other countries, and was recommended as a climate impact assessment tool for participants of the U.S. Country Studies Programme. The purpose of this paper is to describe a framework of PC software for CLIRUN_4 rainfall-runoff model, recently developed at the Institute of Geophysics of the Polish Academy of Sciences. It belongs to an assembly of continuous water balance simulation models, with discrete in time input data formed by measured or synthetic time series of meteorological variables. The model structure is based on the similarity between conceptualisation of the traditional water balance model and the dynamics of water reservoirs described by mass conservation equations. The use of two reservoirs corresponds to the division of the soil into two zones, namely upper (rooted) soil and lower soil zone. The system CLIRUN_4 is mainly used as a tool for the analysis of the sensitivity of water balance components to changes in climate characteristics.

2 WATER BALANCE MODEL

CLIRUN_4 forms a two-reservoir model (Fig.1), in which river runoff is simulated as an aggregation of overland (quick) flow R_s, delayed subsurface flow R_g, and a base-flow R_b.

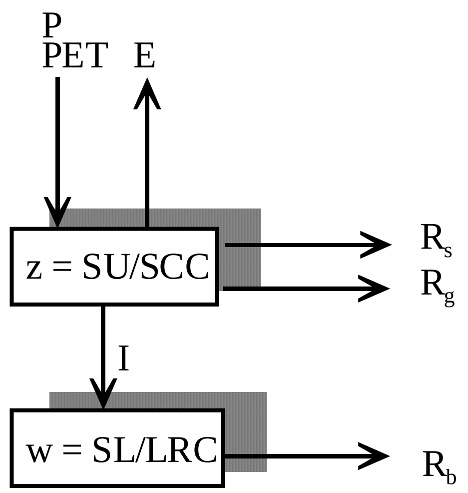

Figure 1. Water balance model structure.

The governing water balance equations are:

$$SCC\frac{dz}{dt} = P - R_s - R_g - I - E \quad \textbf{(1)}$$

$$LRC\frac{dw}{dt} = I - R_b \quad \textbf{(2)}$$

where *z=SU/SCC*, *SU* is the upper reservoir storage, *SCC* - soil (upper reservoir) catchment capacity, *w=SL/LRC*, *SL* is the lower reservoir storage, *LRC* - capacity of the lower reservoir, I is the infiltration rate from the upper reservoir to the lower one, and E - actual evapotranspiration, P is the catchment rainfall.

Conceptualisation of the water balance model consists in assuming functional relationships describing components in equations (1, 2). Various expressions were discussed in hydrological literature to represent these relationships (Delworth & Manabe 1988, Wood et al. 1991, Ripple et al. 1972). For the purpose of the CLIRUN_4 model the following relationships are used:

$$R_s = P\frac{\varepsilon}{1+\varepsilon - z^{\mu}} \quad \textbf{(3)}$$

$$E = PET[1-(1-z)^{1.67}] \quad \textbf{(4)}$$

$$R_g = \alpha z \quad \textbf{(5)}$$

$$R_b = \beta w \quad \textbf{(6)}$$

where PET is potential evapotranspiration and ∀, ∃ ,, μ, are constant values.

After integration over time period T_m one gets:

$$\int_{z_m}^{z_{m+1}} \frac{dz}{\Phi_1(z, P_m, PET_m, I)} = \frac{T_m}{SCC} \quad \textbf{(7)}$$

$$\int_{w_m}^{w_{m+1}} \frac{dw}{\Phi_2(w, I)} = \frac{T_m}{LRC} \quad \textbf{(8)}$$

Functions M_1 and M_2 represent the right sides of equations (1) and (2). Solving (3) and (4) for given z_m, P_m, PET_m, and the infiltration rate I, one gets:

$$z_{m+1} = F_1(z_m, P_m, PET_m, I, SCC, T_m) \quad \textbf{(9)}$$

$$w_{m+1} = F_2(w_m, I, LRC, T_m) \quad \textbf{(10)}$$

Average values of water balance variables for the time interval <0, T_m> may then be calculated by means of formulae:

$$R_m = \frac{SCC}{T_m}\int_{z_m}^{z_{m+1}} \frac{\varepsilon + \alpha z(1+\varepsilon - z^{\mu})dz}{(1+\varepsilon - z^{\mu})\Phi_1(z, P_m, PET_m, I)} + \frac{LRC}{T_m}\int_{w_m}^{w_{m+1}} \frac{\beta w}{I - \beta w} dw \quad \textbf{(11)}$$

$$E_m = \frac{SCC}{T_m}\int_{z_m}^{z_{m+1}} \frac{PET_m[1-(1-z)^{1.67}]dz}{\Phi_1(z, P_m, PET_m, I)} \quad \textbf{(12)}$$

3 COMPUTER IMPLEMENTATION OF THE MODEL

The flow diagram of computer implementation of the model is given in Figure 2. Assuming that all the model parameters are known, the upper and lower reservoir storage are obtained based on a chosen finite difference scheme. Water balance equations are solved step by step for all consecutive time intervals. The fourth-order Runge Kutta method was incorporated in the solution of Equations 1-2 and it turned out to produce results of sufficient high accuracy when the specified stepsize is small enough. This approach surely does not minimise computer time but this fact is of some importance only when the identification of model parameters is considered, i.e. when the computations have to be repeated many times. Average values of water balance components in selected time periods are obtained by integration of these values with the use of simple closed Newton-Cotes formulae.

Input data comprise either of time series of periodical (5-days to one month) average values of catchment precipitation and potential evapo-transpiration - or are used as statistical parameters of these variables allowing to simulate synthetic time series of P_m and PET_m. Because of the effects of wind, wetting, evaporation from gauge surface, and other factors, the amount of precipitation measured should be corrected if the data are to be used for hydrologic calculations (WMO, 1982). To assess the impact of snow accumulation and snow melting processes on catchment water balance, the P_m values for winter season must be modified, based on energy budget of the snow cover, or approximately on air temperature only. Time series of potential evapotranspiration may be calculated by means of Penman formula, or by any other credible method (Brutsaert 1982).

Simulation of the water balance must be usually preceded by the identification of model parameters which are: soil catchment capacity, lower reservoir capacity, infiltration rate, and four constant values ∀, ∃, , and μ. Runoff characteristics are needed for a calibration procedure. Any combination of the parameters can be calibrated, assuming that some of them may be known from soil characteristics or other physiographic catchment characteristics.

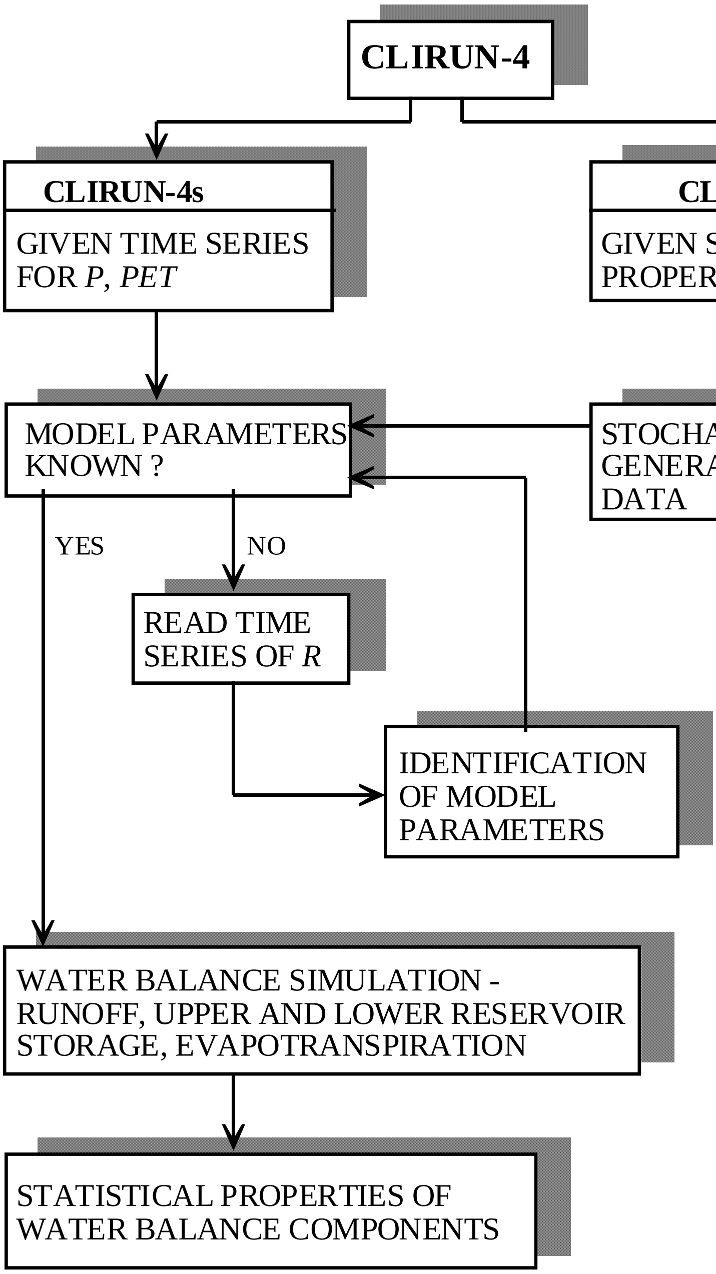

Figure 2.CLIRUN_4 flow diagram.

For example, some authors (e.g. Manabe & Wetherald 1985) employ a constant value SCC = 150 mm to describe the catchment capacity of the root zone, and the infiltration rate I between the upper and lower reservoirs may be approximated by the average value of minimum yearly runoff. If calibrated for several river catchments the model parameters may be regionalised, e.g. interpolated to a grid of required resolution.

Identification of parameters was performed by minimisation of the mean square error of catchment outflows. Historical data and simulation results are used to construct the objective function. A non-gradient multidimensional optimisation technique is used. This method requires only function evaluation, not its derivatives. Therefore, we have to give an algorithm a starting guess, that is a vector of independent variables as the first point to try. Then the algorithm makes its own way downhill through some complex topography until it encounters the searched minimum. The time of computations depends of this first guess as well as the time step in Runge-Kutta simulations. The described method works efficiently in the solution of our specific problem.

Computer system based on the above-described algorithm consists of two independent parts. The first one CLIRUN_4s simulates water balance components on a basis of measured values of input data: precipitation and potential evapotranspiration. If runoff data are available then this version of PC software may also be used for identification of model parameters. In the second part CLIRUN_4g

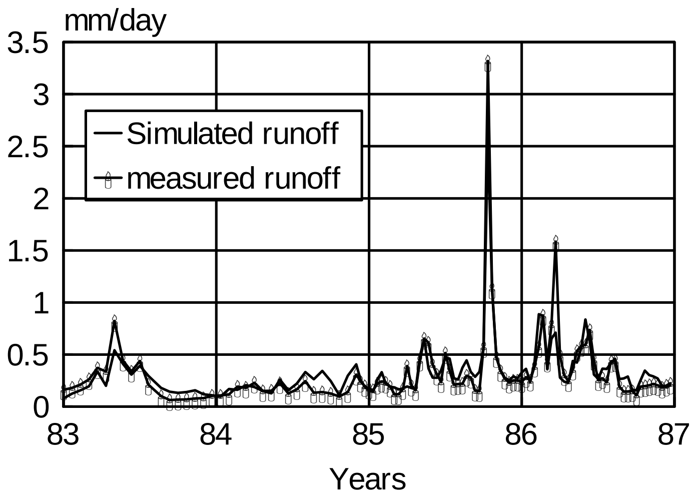

Figure 3. Comparison of measured runoff with that obtained from CLIRUN_4 for Prosna River.

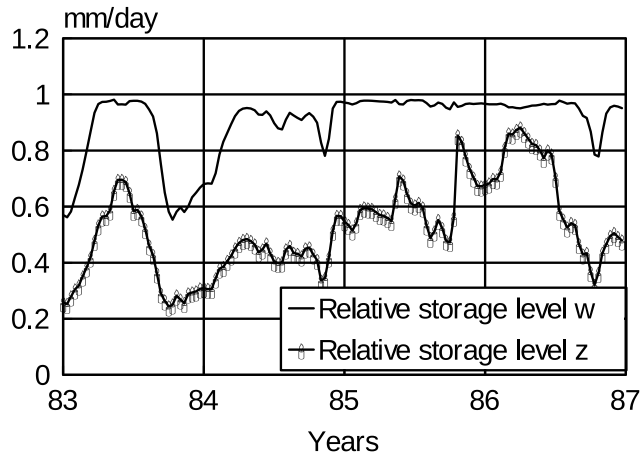

Figure 4. Relative storage levels for upper and lower reservoirs obtained from CLIRUN_4 for Prosna River.

model parameters must be known, and the input climatic data are obtained by means of stochastic generation procedures assuming that random behaviour of both *P* and *PET* variables may be described by the Weibull probability distribution (Weibull, 1951). In both cases output files include time series of monthly(or other selected period) catchment storage, runoff, and actual evapo-transpiration and their statistical properties, namely mean values, standard deviations and matrices of correlation functions.

4 SIMULATION RESULTS AND CONCLUDING REMARKS

The study of model performance implemented on the basis of data for a number of river catchments indicates that the model provides satisfactory results. Figure 3 and Figure 4 show an example of a comparison of simulated and observed runoff for Prosna River in Central Poland. That catchment of a size of 4300 km^2 is characterised by the average yearly precipitation of 628 mm, and the yearly sum of potential evapotranspiration of 605 mm. To calibrate the model, ten-day values of precipitation, potential evapotranspiration and runoff for the years 1966-1995 were used. For the purpose of illustration, only four years period is presented in Figures 3-4. Figure 3 reveals a good agreement between the simulated and the measured runoff. One can observe some differences at selected peak values (for example in 1985), however the relative error does not exceed 20%. Figure 4 shows that the relative storage level w of the lower reservoir is close to one for most of the time and it does not influence the total runoff very much. However, in some periods, this impact is meaningful and cannot be avoided - see for instance the last months of 1983, when the value of w is below the value of 0.6. This is the main reason of the modification of former CLIRUN programs with the inclusion of the second reservoir in the model, that is the base flow component. It was found that this factor is especially important for lowland catchments.

In general, an application of the stochastic storage theory with “two reservoirs” leads to an improvement of results in comparison with the former version CLIRUN_3 of the model based on one reservoir.

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