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Optimal Flood Control of Nysa Kłodzka Reservoir System

Tomasz DYSARZ [1] and Jarosław J. NAPIÓRKOWSKI [2]

[1] Gdańsk University of Technology
ul. Narutowicza 11/12, 80-952 Gdańsk, Poland
e-mail: todys@pg.gda.pl

[2] Institute of Geophysics, Polish Academy of Sciences
ul. Księcia Janusza 64, 01-452 Warszawa, Poland
e-mail: jnn@igf.edu.pl

Abstract

The flood control problem is discussed in respect to the Nysa Kłodzka Reservoir System. The main goal of the paper is to present the performance of control mechanisms developed by the authors to minimize the maximum peak of the superposition of waves on Nysa Kłodzka and Odra Rivers. The control structure consists of the simple tank models to describe the reservoirs performance, the St. Venant equations to represent transformation of flow in open channels and the sequential optimisation algorithm. This algorithm makes use of characteristic features of the system and global optimisation methods. Relations resulting from the system dynamic equations enable to separate calculations for any particular reservoir in the cascade and to propagate the results to other system components so that computational costs grow linearly with the number of reservoirs. The encouraging results of test for both synthetic and historical data are presented.

Key words: flood control, flood routing models, sequential optimisation, global optimisation method.

1. Introduction

The catchment of the Nysa River (see Fig. 1) is located in the southern part of Poland. The length of the river from the source in mountain massif of Śnieżnik to the junction with Odra River is 181,7 km, and the catchment area is about 4566 km^2, i.e., 1/3 of the Odra catchment at this particular cross-section.

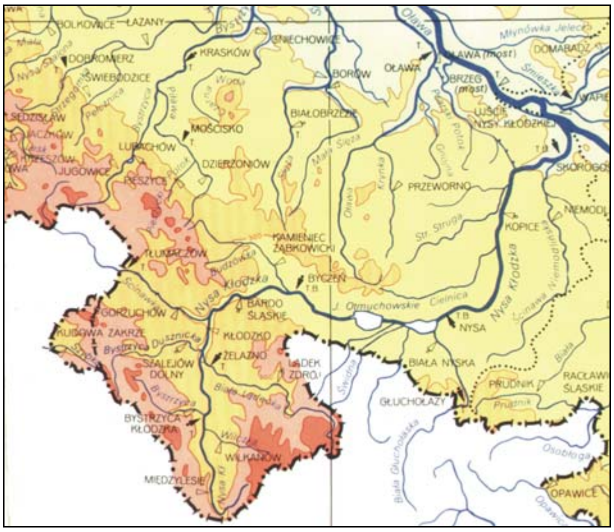

Fig. 1. Catchment of the Nysa Kłodzka River (S-W Poland).

The river reach between the source and the reservoirs is a typical mountain river reach with a bottom slope of about 10‰ and a number of highland tributaries The most important are Wilczka, Bystrzyca Kłodzka, Biała Lądecka i Bystrzyca Dusznicka rivers. On the other hand, there are only a few small tributaries feeding the lower part of Nysa Kłodzka characterized by bottom slope less than 3‰.

The hydrological features of the upper part of this catchment are characterized by massive rocky underground covered only by a small layer, and an average yearly precipitation of about 900 mm. The missing ability of storing water underground leads to dangerous floods. To achieve the ability to handle this problem, two reservoirs were built, and more are under construction.

Here we are just interested in the management of reservoirs to control flood wave in the Nysa Kłodzka River and a selected reach of the Odra River.

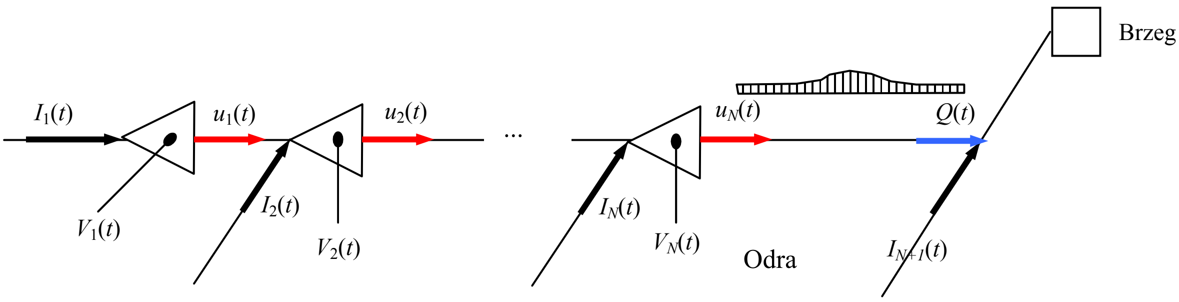

Fig. 2. Schematic representation of the system.

2. Formulation of the problem

The considered system, schematically shown in Fig. 2, consists of N reservoirs in series and an open channel reach with lateral inflow q.

At this stage, we assume that inflows $I(t)$ to the system represent one of many possible scenarios taken into account by a decision maker. The scenarios considered could be based on rainfall-runoff prediction models, or recorded historical data. Retention in each reservoir $V_j(t)$ is described by the dynamics of a simple tank, with one forecasted inflow $I_j(t)$ and one controlled output $u_j(t)$, $j = \overline{1,N}$. According to the introduced notation, the state equations for the reservoir system are:

$$\frac{dV(t)}{dt} = B \times I(t) - C \times u(t), \tag{1}$$

where B and C are appropriate matrices. The following constraints on the reservoir storage and releases are taken into account:

$$V_{\min} \le V(t) \le V_{\max}, \qquad U_{\min} \le u(t) \le U_{\max}, \tag{2}$$

for any $t \in [0, T_H]$, where $V_{\min}$ denotes dead storage, $V_{\max}$ denotes total storage, and T_H is the optimisation time horizon, and the initial condition

$$V(0) = V_0. \tag{3}$$

To simplify the optimisation problem, the dynamics of flow in the reach between the reservoirs is omitted. The flood routing in Nysa Kłodzka River below the last reservoir is described by means of the Saint Venant equations or their simplified versions described in details in the next section, so the flow at Nysa Kłodzka outlet, $Q(t)$, can be represented as:

$$Q(t) = \varphi[u_n, q](t). \tag{4}$$

The main goal of this system is the protection of the user located below the cascade of reservoirs against flooding by minimizing the peak of the superposition of waves $Q(t)+I_{N+1}(t)$ on Nysa and Odra Rivers, respectively. This can be achieved by desynchronization of the flow peaks via accelerating or retarding flood wave on Nysa River. The second objective is storing water for future needs after flood.

Hence, the objective function of the optimisation problem under consideration can be written in the form of a penalty function:

$$\min_{u_j,\, j=\overline{1,N}} \left\{ \beta_1 \max_{t\in[0,T_H]} \left[Q(t) + I_{N+1}(t) \right] + \beta_2 \sum_{k=1}^{N} \left[V_k(T_H) - V_{\max k} \right]^2 \right\}, \tag{5}$$

where symbols β_1 and β_2 denote appropriate weighting coefficients and T_H is the optimisation time horizon.

3. Flood routing models

In the discussed optimisation problem, flood routing model for the river reach of Nysa Kłodzka between the lower reservoirs and Odra River is required. For quasi regular cross-sections and the bottom slope of about 3‰ one can expect supercritical flow conditions for this particular river reach. However, some local depressions with much higher bottom slopes cause disturbances, e.g. transition from supercritical flow to subcritical flow, backwater, etc. Moreover, irregular shape of the cross-section (see Fig.3) and the problems involved in the determination of roughness coefficient, further complicate the calculations of flow transformation in Nysa Kłodzka River.

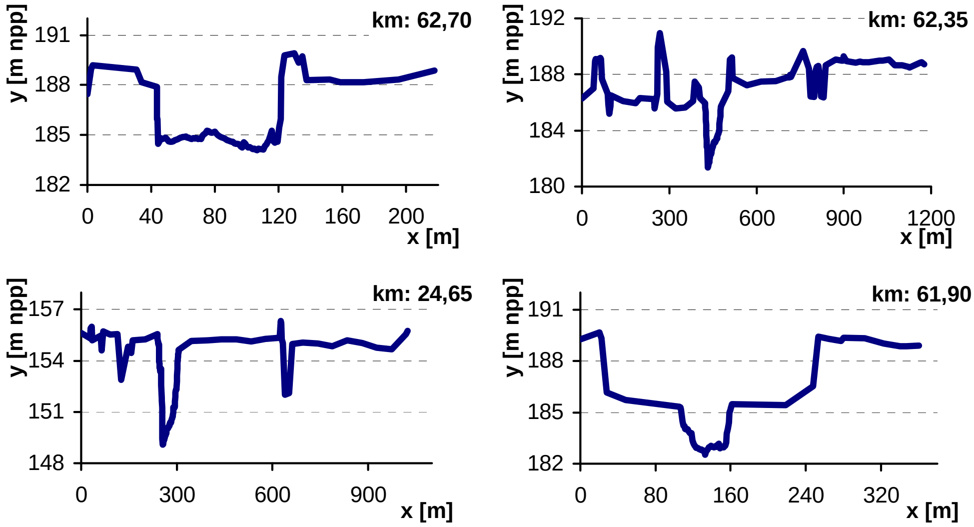

Fig. 3. Cross-sections of the Nysa Kłodzka River.

To describe the flow transformation between reservoirs, two types of flood routing models are used. The first model is based on the de Saint-Venant equations with simplified trapezoidal geometry of channel cross-sections (Dysarz and Napiórkowski, 2001). This model guarantees more accurate description of the transformation process but requires more computational time. Therefore, to speed up numerical computations, two versions of kinematic wave models were tested, namely linear and nonlinear ones.

3.1. De Saint-Venant equations

De Saint-Venant equations constitute the mathematical description of the mass and momentum balance. The following form of these equations is adopted:

$$\frac{\partial H}{\partial t}+\frac{1}{B}\frac{\partial Q}{\partial x}=\frac{q}{B}, \tag{6}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial H}{\partial t}-gA\left(S_0-S_f\right)=0\,, \tag{7}$$

where: independent variables are distance x and time t, dependent variables are discharge $Q(x, t)$ and water depth $H(x, t)$, appropriate parameters are cross-section area A, width of the water surface B, lateral inflow q, acceleration of gravity g, bottom slope S_0 and hydraulic slope according to Manning's equation S_f. The initial and boundary conditions complement the formulation of model (6) and (7). The steady conditions described by the Bernoulli equation are assumed as the initial condition. The upstream boundary condition is the outflow from the lower reservoir, and the simplified momentum equation is taken as the downstream boundary condition. This simplification consists in neglecting the inertia and pressure elements in the original equation.

3.2. Kinematic wave models

The kinematic wave method is a straightforward simplification of the previous model. Its main idea is based on negligible influence of the inertia and pressure terms. It seems that this approach is reasonably accurate in the case of Nysa Kłodzka reach. Two versions of models, namely the linear one described by equation (8) and the nonlinear one described by equation (9), were tested.

– linear version

$$\frac{\partial Q}{\partial t}+\frac{c}{m}\frac{\partial Q}{\partial x}=\frac{cq}{m}\,, \tag{8}$$

– nonlinear version

$$\frac{\partial Q}{\partial t}+\frac{1}{\alpha m Q^{m-1}}\frac{\partial Q}{\partial x}=\frac{q}{\alpha m Q^{m-1}}\,. \tag{9}$$

Note that equation (8) results from equation (9) when $c = 1/(\alpha Q^{m-1})$ = const. The constant model parameters α, m are to be identified.

4. Sequential optimisation

In this section we describe the application of the particular iterative optimisation procedure for N reservoirs in series. At any iteration step, the water content of only one j-th reservoir is modified. Additionally, we modify outflow from <u>this</u> particular reservoir and all outflows from reservoirs below this reservoir $u_k(t)$, $k=\overline{j,N}$.

The applied sequential optimisation procedure contains the following steps (Dysarz and Napiórkowski, 2002a; 2002d; Napiórkowski and Dysarz, 2002):

(1) assuming zero outflows from all reservoirs, i.e.

$$u_k(t):=0\,, \qquad \forall k=\overline{1,N} \quad \text{and} \quad \forall t\in[0,T_H] \tag{10}$$

(2) assuming index of the „improved” reservoir as $j:=N$

(3) solving the problem

$$\min_{u_N}\left\{\beta_1 \max_{t\in[0,T_H]}\left[Q(t)+I_{N+1}(t)\right]+\beta_2\left[V_N(T_H)-V_{\max N}\right]^2\right\}, \tag{11}$$

with $Q(t) = u_N(t - T_0)$ and following constraints

$$V_{\min N} \le V_N(t) \le V_{\max N}\ , \tag{12}$$

$$U_{\min N} \le u_N(t) \le U_{\max N} \tag{13}$$

and

$$\frac{dV_N}{dt} = u_{N-1} + I_N - u_N \qquad V_N(0) = V_N^{(0)} \tag{14}$$

(4) denoting the best solution as

$$B := \max_{t\in[0,T_H]}\left[Q(t)+I_{N+1}(t)\right]$$

(5) assuming index $j := j - 1$

(6) solving the problem

$$\min_{u_j}\left\{\beta_1 \max_{t\in[0,T_H]}\left[Q(t)+I_{N+1}(t)\right]+\beta_2\left[V_j(T_H)-V_{\max j}\right]^2\right\}, \tag{15}$$

with $Q(t) = u_N(t - T_0) + \Delta u_j(t - T_0)$, where $\Delta u_j(t)$ is the difference between trajectories of the j-th control function of the current and the previous iteration steps; the optimisation is led under the constraints

$$V_{\min j} \le V_j(t) \le V_{\max j}\ , \tag{16}$$

$$U_{\min j} \le u_j(t) \le U_{\max j} \tag{17}$$

and

$$\frac{dV_j}{dt} = u_{j-1} + I_j - u_j\ , \qquad V_j(0) = V_j^{(0)} \tag{18}$$

or

$$\frac{dV_j}{dt} = I_j - u_j\ , \qquad V_j(0) = V_j^{(0)} \qquad \text{if} \qquad j = 1 \tag{19}$$

with additional constraints:

$$U_{\min k} \le \Delta u_j(t) + u_k(t) \le U_{\max k} \qquad \text{for} \quad k = \overline{j+1, N} \tag{20}$$

(7) assuming

$$u_k(t) = \Delta u_j(t) + u_k(t) \qquad \text{for} \quad k = \overline{j+1, N}$$

(8) solving the problem

$$A : \max_{t\in[0,T_H]}\left[Q(t)+I_{N+1}(t)\right]$$

(9) returning to (2) if $A > B$,

(10) returning to (5) if $j > 1$.

The above algorithm describes the consecutive steps of computation which was conducted from the reservoir N to 1. Of course, the starting point for the next iteration would be step (2). If the results obtained for $j = 1$ are not worse than those obtained in step $j = 2$, the procedure stops. The experiments showed that the computation time for one iteration could be very long (eight hours for $N = 4$ on PC Pentium III).

5. Control Random Search method

The functions $u_j(t)$, $j = \overline{1, N}$ were represented by a train of rectangular pulses, and the time horizon was divided into L unequal time intervals. The parameters to be determined were values of pulses $\hat{u}_l$ and time instances of switching the control function $u(t)$. This type discretisation, denoted as TD-RP (Time Dependent Rectangular Pulses) was described in detail by Dysarz and Napiórkowski (2002a; 2002c).

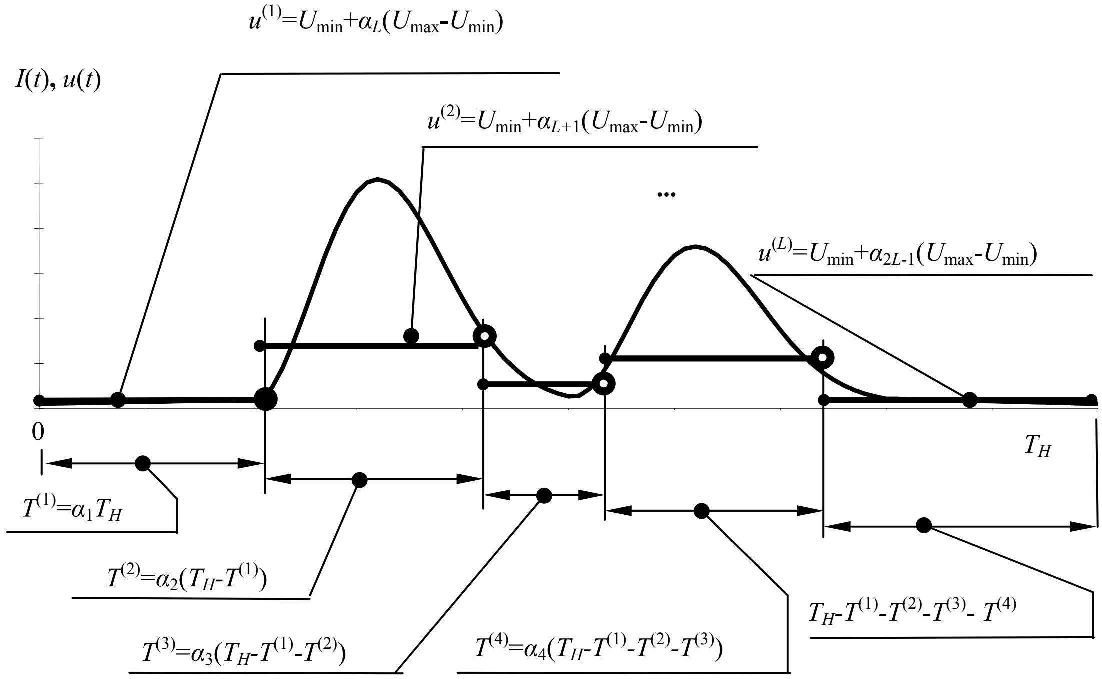

Fig. 4. Control functions as a train of rectangular pulses.

The local optimisation problems for all reservoirs were solved by means of the Global Random Search procedure, namely the following version of Controlled Random Search (CRS2) described in details in Dysarz and Napiórkowski (2002d).

The CRS2 algorithm starts from the creation of the set of points, many more than $n + 1$ points in n-dimensional space, selected randomly from the domain. Let us denote it as S. After evaluating the objective function for each of the points, the best x_L (i.e., that of the minimal value of the performance index) and the worst x_H (i.e., that of the maximal value of the performance index) points are determined and a simplex in n-space is formed with the best point x_L and n points ($x_2, \ldots, x_{n+1}$) randomly chosen from S. Afterwards, the centroid x_G of points $x_L, x_2, \ldots, x_n$ is determined. The next trial point, x_Q, is calculated, $x_Q = 2x_G - x_{n+1}$. Then, if the last derived point, x_Q, is admissible and better (i.e., $Q(x_Q) \le Q(x_H)$), it replaces the worst point x_H in the set S. Otherwise, a new simplex is formed randomly and so on. If the stop criterion is not satisfied, the next iteration is performed. In the CRS2 version applied in the tests, the worst point of the current simplex will be the reflected point $x_Q = 2x_G - x_H$, rather than the arbitrarily chosen one (Dysarz and Napiórkowski, 2002b).

6. Results of tests for synthetic data

The sequential optimisation technique described above was successfully tested for the system of four reservoirs in series depicted in Fig. 1 (without flood routing model) and for the objective function given by Eq. (5). All reservoirs are identical with the parameters $V(0) = 20$ mln m^3, $V_{\min} = 20$ mln m^3, $V_{\max} = 120$ mln m^3, $u_{\min} = 0$ m^3, $u_{\max} = 800$ m^3. The synthetic inflows to the system for $k = 1{,}5$ are described by the following equation

$$P_k(t) = P_{0k} + P_{mk}\left(\frac{t - t_{0k}}{T_{mk}}\right)^2 \exp\left(1 - \left(\frac{t - t_{0k}}{T_{mk}}\right)^2\right). \tag{21}$$

The parameters of particular waves are given in Table 1 and respective hydrographs are shown in Fig. 5.

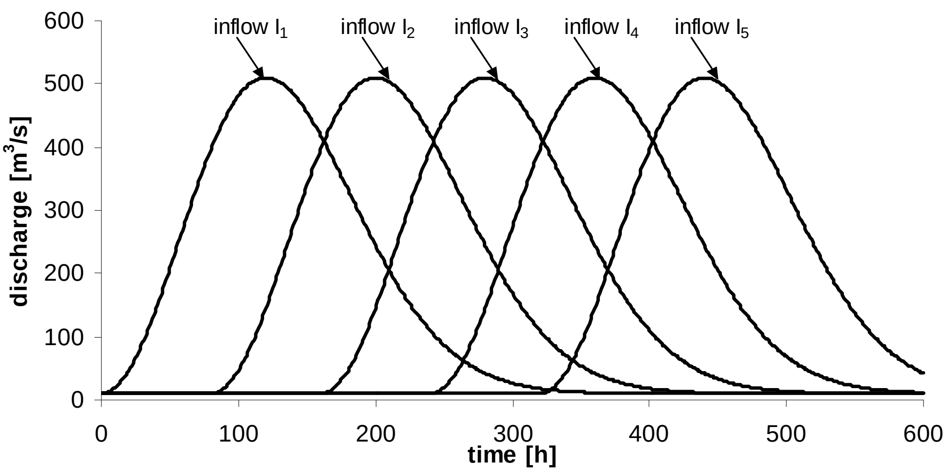

Fig. 5. Inflow hydrographs to the system of four reservoirs in series.

Table 1

Parameters of the inflows to the system

k	P_{0k} [m^3/s]	P_{mk} [m^3/s]	T_{mk} [h]	t_{0k} [h]
1	10.0	500.0	120.0	0.0
2				80.0
3				160.0
4				240.0
5	10.0	500.0	120.0	320.0

The performances of reservoirs are shown in Figs. 6a–6d and total reduction of the flood wave from 964 m^3/s to 742 m^3/s at the cross-section below the last junction is depicted in Fig. 6e.

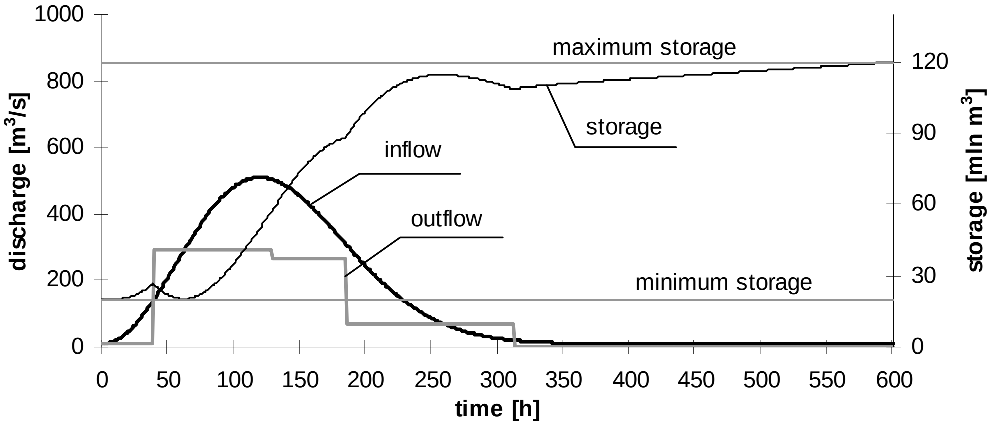

Fig. 6 (a). Trajectories of inflow, outflow and storage for the first reservoir.

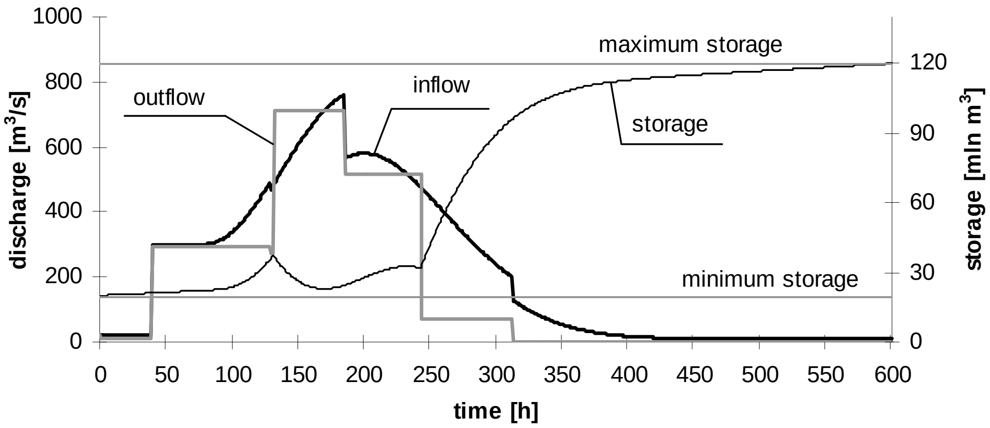

Fig. 6 (b). Trajectories of inflow, outflow and storage for the second reservoir.

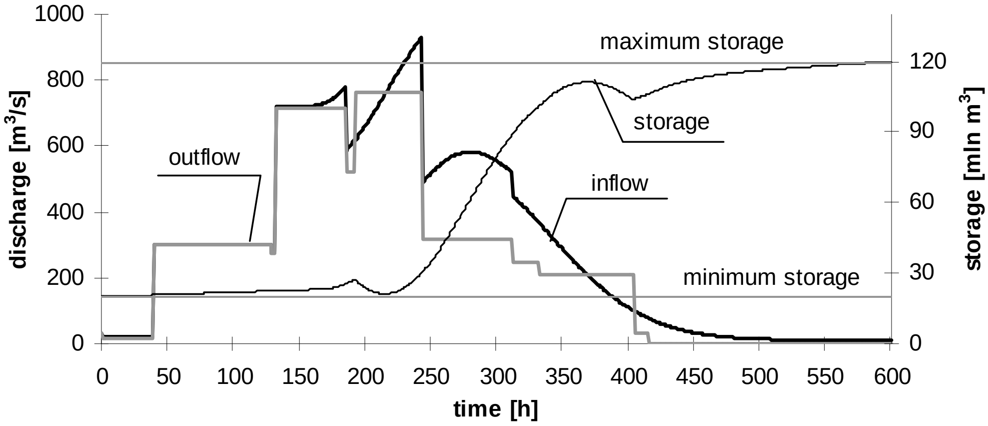

Fig. 6 (c). Trajectories of inflow, outflow and storage for the third reservoir.

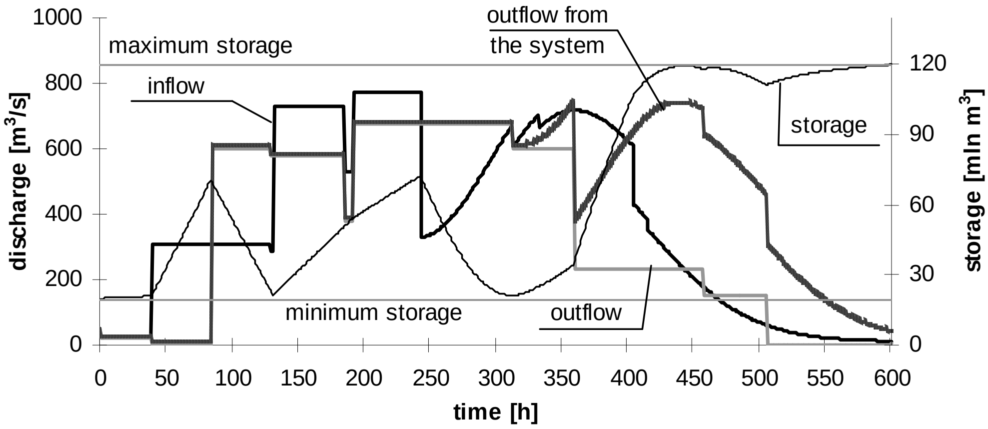

Fig. 6 (d). Trajectories of inflow, outflow and storage for the fourth reservoir.

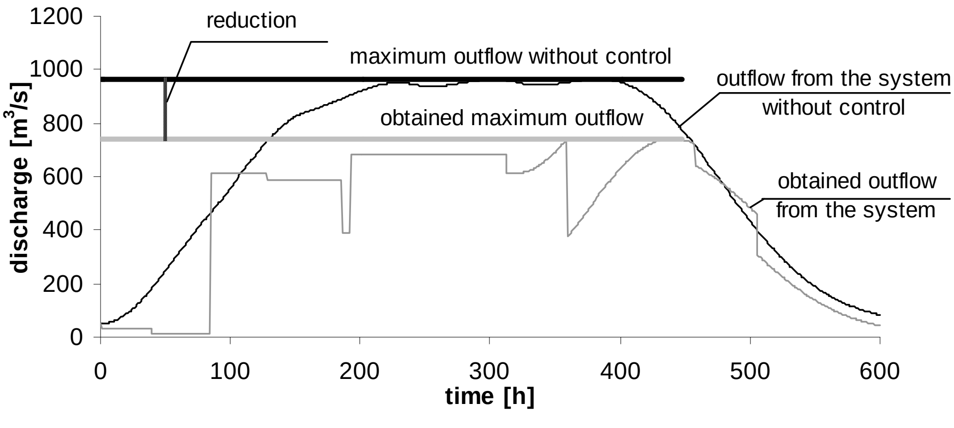

Fig. 6 (e). Comparison of the uncontrolled outflow from the reservoirs with that obtained by means of CRS method.

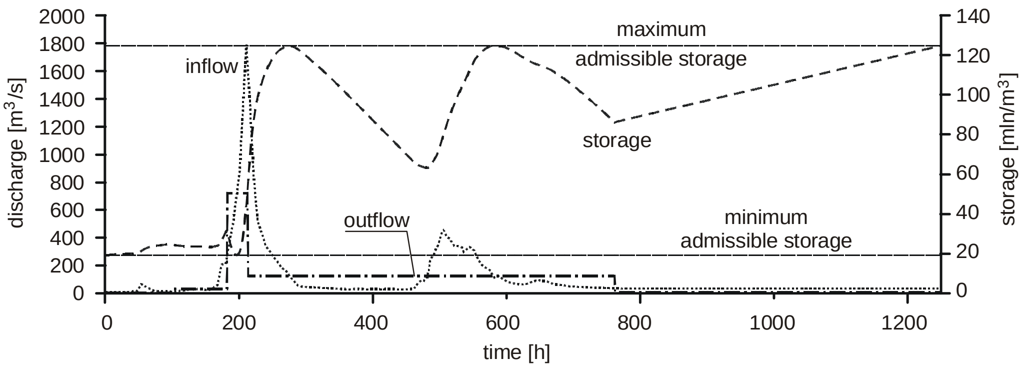

Fig.7 (a). Performance of the Otmuchów (upper) Reservoir .

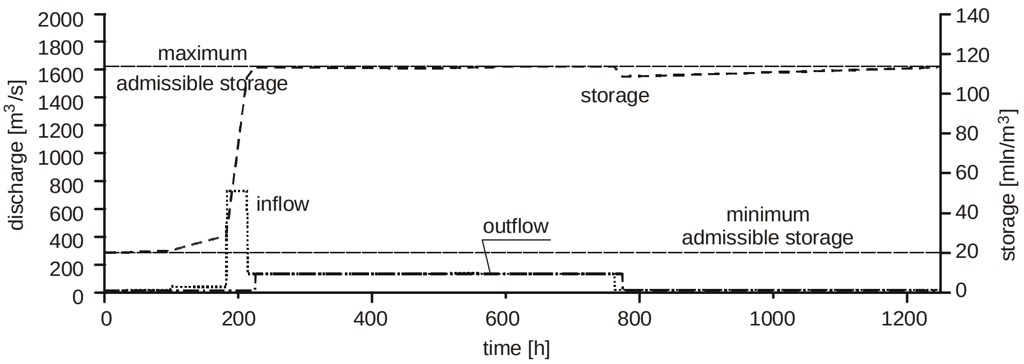

Fig.7 (b). Performance of the Nysa (lower) Reservoir.

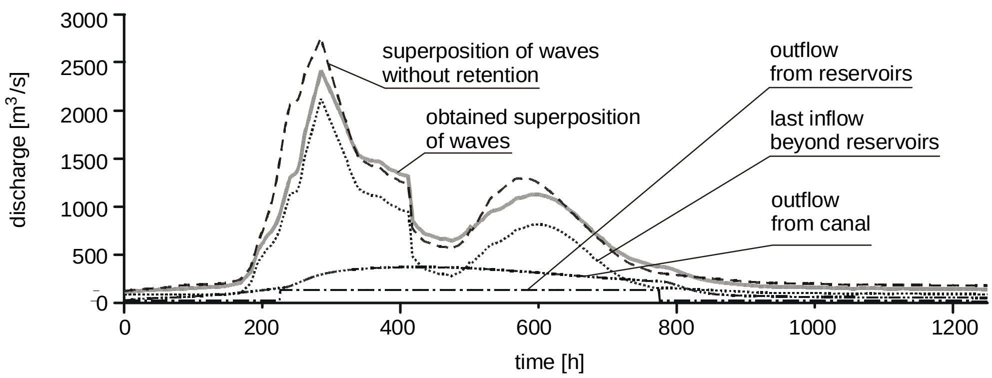

Fig.7 (c). Flow at the cross-section below the junction of Nysa and Odra Rivers.

7. Results of tests for historical data

In the next step, the sequential optimisation was verified for the system of existing two reservoirs against a number of historical flood events and for two types of flood routing models described above. The results for one of them, namely for the historical floods in Nysa catchment in 1997 and for the nonlinear kinematic wave model, are presented in Fig. 7a, Fig. 7b and Fig. 7c, respectively. We observed that kinematic wave model described flow transformation with lateral inflow more accurately.

The floods in 1997 were caused by the most disastrous recent abundance of water in the region. During the first stage of the disaster, a rapid increase in runoff was noted after intense and long lasting rains in the 4–10 July period in the highland tributaries. Yet, a few days later, from 15 to 23 July, another series of intensive rains occurred. The highest precipitation in the Kłodzko valley reached 100–200 mm. The flood virtually ruined the town of Kłodzko (Kundzewicz *et al.*, 1999), and the historic stage record was exceeded by 70 cm. Several all-time maximum stages recorded were largely exceeded by that flood.

Figure 7a shows the performance of the Otmuchow (upper) Reservoir, Fig. 7b shows the performance of Nysa (lower) Reservoir, and Fig. 7c shows the flow at the cross-section below the junction of Nysa and Odra Rivers.

As one can see, by an appropriate choice of the control functions, the peaks of the waves on Nysa Kłodzka and Odra Rivers were desynchronised and the culminations did not overlap.

8. Conclusions

It is necessary to take into account the uncertainty of the inflows forecast in operation control of reservoirs system during flood. Hence, the optimisation problem has to be solved repetitively for many scenarios using actual measurements and updated forecasts. Therefore, from the decision making point of view, the access to a quick and reliable optimisation module is very important.

The approach presented in the paper makes a decomposition of the general problem possible, so that computational costs grow linearly with the number of reservoirs. Hence, a more complex representation of the control functions than that described by Niewiadomska-Szynkiewicz and Napiórkowski (1998), can be adopted.

Because of nondifferentiability of global and two local performance indices, the global optimisation technique CRS is used.

The results of applications of the sequential optimisation to determine the reservoir decision rules during flooding are encouraging. The accuracy of the proposed method is satisfactory. The initiation procedure and the stop criterion were cautiously investigated, so high efficiency does not cause losses in accuracy. The described control structure of Nysa Kłodzka reservoirs system includes transformation by means of hydrodynamic flood routing model, because the proposed technique guarantees that the solution of the optimisation problem can be obtained in reasonable time.

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