Global Optimisation Method for Determination of Reservoir Decision Rules During Flood

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The flood control problem is discussed in respect to the Nysa Kłodzka Reservoir System. The main goal of the paper is to present performance of the particular control structures and control mechanisms developed by authors. The new control algorithm is proposed that make use of characteristic features of the system and global optimisation methods. Relations resulting from the system dynamic equations enable to separate calculations for any particular reservoir in the cascade and to propagate the results to other system components. Then the solution obtained at the previous iteration step is modified. This decomposition reduces the dimensionality of the sub-problems that leads to the reduction of the overall computational costs.

Introduction

The catchment of Nysa River is located in the southern part of Poland. The hydrological features of the upper part of this catchment are characterized by massive rocky underground covered only by small layer, and an average yearly precipitation of about 900 mm. The missing ability of storing water underground leads to dangerous floods. To achieve the ability to handle this problem two reservoirs were built, and two are under construction. Here we are just interested in the management of two existing reservoirs to control flood wave in the Nysa Kłodzka River and a selected reach of the Odra River.

Description of the Nysa Reservoir System

The considered system that consists of two reservoirs in series and open channel reach with lateral inflow q is schematically shown in Fig.1. Management and control of flooding generally require the use of forecasting techniques. At this stage we assume that inflows to both reservoirs $I_1(t)$, $I_2(t)$, lateral inflow and flow in Odra River $I_3(t)$ represent one of many possible scenarios taken into account by a decision maker. The scenarios considered are based on rainfall-runoff prediction models, or recorded historical data.

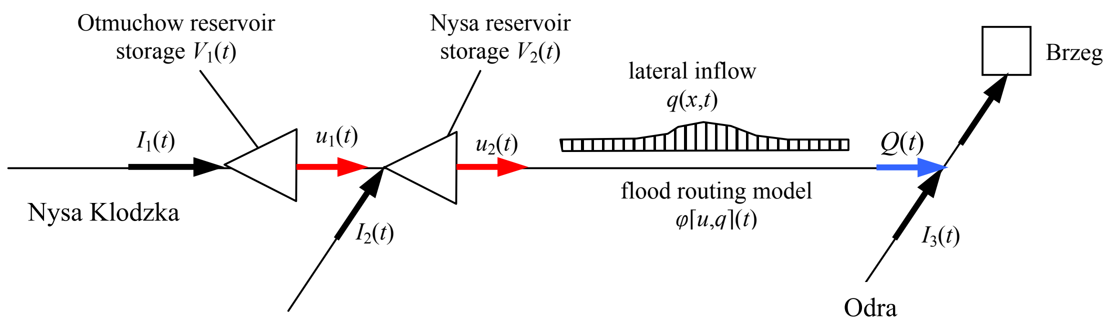

Fig. 1 Schematic representation of the system

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Retention in each reservoir $V_j(t)$ is described by the dynamics of a simple tank, with one forecasted inflow $I_j(t)$ and one controlled output $u_j(t)$, j=1,2

$$\frac{dV_1}{dt} = I_1(t) - u_1(t) \qquad 1$$

$$\frac{dV_2}{dt} = u_1(t) + I_2(t) - u_2(t) \qquad 2$$

The following constraints on the reservoir storage and releases (given in Table 1) are taken into account:

$$V_1(0) = V_{10} \qquad V_2(0) = V_{20} \ \text{(initial condition)}$$

$$V_{\min j} \le V_j(t) \le V_{\max j} \qquad U_{\min j} \le u_j(t) \le U_{\max j} \qquad 3$$

for j=1,2 and for any $t \in [0, T_H]$, where $V_{\min}$ denotes dead storage, $V_{\max}$ denotes total storage, and T_H is optimisation time horizon.

Table 1. System parameters

Reservoir	$V_{\min}$ [mln m^3]	$V_{\max}$ [mln m^3]	$U_{\min}$ [m^3/s]	$U_{\max}$ [m^3/s]
Upper (no 1)	19,38	124,66	0,0	1363,0
Lower (no 2)	20,29	113,60	0,0	1960,0

Flood routing models

To describe the flow transformation between the lower reservoir and Odra River two types of flood routing models were used. The first model was based on the de Saint – Venant equations with simplified trapezoidal geometry of channel cross – sections. This model guarantees more accurate description of the transformation process but requires more computational time. Therefore to speed up numerical computations two versions of kinematic wave models were tested, namely linear and nonlinear ones.

De Saint – Venant equations

De Saint – Venant equations constitute the mathematical description of the mass and momentum balance. The following form of this equations is adapted:

$$\frac{\partial H}{\partial t} + \frac{1}{B}\frac{\partial Q}{\partial x} = \frac{q}{B} \qquad 4$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial H}{\partial t} - gA\left(S_0 - S_f\right) = 0 \qquad 5$$

where independent variables are x – distance and t – time are independent variables, dependent variables are $Q(x,t)$ - discharge, $H(x,t)$ – water depth, appropriate parameters are A - cross-section area, B – width of the water surface, q – lateral inflow, g – acceleration of gravity, S_0 – bottom slope, S_f – hydraulic slope according to Manning equation. The initial and boundary conditions complement the formulation of model 4 and 5. The steady conditions described by Bernoulli equation are assumed as the initial condition. The upstream boundary condition is the outflow from the lower reservoir and the simplified momentum equation is taken as the downstream boundary condition. This simplification consists in neglecting the inertia and pressure elements in the original equation.

Kinematic wave models

The kinematic wave method is a straightforward simplification of the previous model. Its main idea bases on negligible influence of the inertia and pressure terms. It seems that this approach is reasonably accurate in the case of Nysa Klodzka reach. Two versions of models, namely the linear one described by equation 6 and the nonlinear one described by equation 7 were tested.

- linear version

$$\frac{\partial Q}{\partial t} + \frac{c}{m}\frac{\partial Q}{\partial x} = \frac{cq}{m} \qquad 6$$

- nonlinear version

$$\frac{\partial Q}{\partial t} + \frac{1}{\alpha m Q^{m-1}}\frac{\partial Q}{\partial x} = \frac{q}{\alpha m Q^{m-1}} \qquad 7$$

Note that equation 6 results from equation 7 when $c = 1/\left(\alpha Q^{m-1}\right) = const$. The constant model parameters *a*, *m* are to be identified.

Optimisation problem

The main goal of this system is the protection of the user located at the Brzeg cross-section against flooding by minimizing the peak of the superposition of waves $Q(t)+I_3(t)$ on the Nysa and Odra rivers, respectively. This can be achieved by desynchronization of the flow peaks via accelerating or retarding flood wave on Nysa River. The second objective is storing water for future needs after flood.

Hence the objective function of the optimisation problem under consideration can be written in the form of a penalty function:

$$\min_{u_1,u_2}\left\{\beta_1 \max_{t\in[0,T_H]}\left(Q(t)+I_3(t)\right)+\beta_2\sum_{j=1}^{2}\left[V_j(T_H)-V_{\max j}\right]^2\right\} \qquad 8$$

where symbols β_1 and β_2 denote appropriate weighting coefficients and T_H is the optimisation time horizon.

Note that minimisation of objective function 8 is subject to the constraints described by equations 1-3 and

$$Q(t) = \varphi[u_2, q](t) \qquad 9$$

where φ represents one of the transformation methods described in the previous section.

Sequential optimisation approach

In this section we describe the application of the particular optimisation procedure for two reservoirs in series. Let us assume that for k-th iteration step the control $u_1 = \hat{u}_1^{(k-1)}$ and the retention $V_1 = \hat{V}_1^{(k-1)}$ of the of upper reservoir are specified for any $t \in [0, T_H]$. Then one has to determine the control value $u_2 = u_2^{(k)}$ and retention value $V_2 = V_2^{(k)}$ of the lower reservoir. The optimisation problem for lower reservoir takes form:

$$\min_{u_2}\left\{\beta_1 \max_{t\in[0,T_H]}\left(Q^{(k)}+I_3\right)+\beta_2\left[V_2^{(k)}(T_H)-V_{\max 2}\right]^2\right\} \qquad 10$$

under constraints

$$\frac{dV_2^{(k)}}{dt} = \hat{u}_1^{(k-1)} + I_2 - u_2^{(k)} \tag{11}$$

$$Q^{(k)}(t) = \varphi\left[u_2^{(k)}, q\right](t) \tag{12}$$

$$V_{\min 2} \le V_2^{(k)}(t) \le V_{\max 2} \qquad U_{\min 2} \le u_2^{(k)}(t) \le U_{\max 2} \tag{13}$$

After solving optimisation problem 10, i.e. after solving for $\hat{u}_2^{(k)}$ and $\hat{V}_2^{(k)}$, the control and retention of the upper reservoir are modified so that to improve the primary objective function 8, while maintaining $\hat{V}_2^{(k)}$; all modifications of the control function u_1 are directly transferred to the control function u_2 that describes the outflow from the reservoir system. The optimisation problem for the upper reservoir takes form:

$$\min_{u_1^{(k)}} \left\{ \beta_1 \max_{t\in[0,T_H]} \left(Q^{(k_1)} + I_3 \right) + \beta_2 \left[V_1^{(k)}(T_H) - V_{\max 1} \right]^2 \right\} \tag{14}$$

under direct constraints

$$\frac{dV_1^{(k)}}{dt} = I_1 - u_1^{(k)} \tag{15}$$

$$V_{\min 1} \le V_1^{(k)}(t) \le V_{\max 1} \qquad U_{\min 1} \le u_1^{(k)}(t) \le U_{\max 1} \tag{16}$$

and indirect constraints resulting from equation 2

$$u_2^{(k_1)} = \hat{u}_2^{(k)} + u_1^{(k)} - \hat{u}_1^{(k-1)} \tag{17}$$

$$Q^{(k_1)}(t) = \varphi\left[u_2^{(k_1)}, q\right](t) \tag{18}$$

$$U_{\min 2} \le u_2^{(k_1)}(t) \le U_{\max 2} \tag{19}$$

where $u_2^{(k_1)}$ is improved outflow from the lower reservoir.

The solutions of the optimisation problem 14 – 19 are the optimal trajectories of $u_1 = \hat{u}_1^{(k)}$, $V_1 = \hat{V}_1^{(k)}$ and $u_2 = \hat{u}_2^{(k_1)}$. Therefore the solutions of the primary optimisation problem at k-th iteration step are these three functions and the $V_2 = \hat{V}_2^{(k)}$ trajectory determined at the previous stage.

After solving the problems for lower and upper reservoirs, i.e. completing k-th step of sequential optimisation, one can go to the next step, k+1, once more solving the optimisation problem 10 – 13 using the values obtained at the step k. Calculations terminate when the stopping rule is met. The chosen criterion is the difference ε between outflows from the system at first and second stage for the current iteration step k.

$$\int_0^{T_H} \left[\hat{u}_2^{(k_1)} - \hat{u}_2^{(k)} \right]^2 dt \le \varepsilon \tag{20}$$

The essential problem related to the described algorithm is the selection of initial values approximation of $\hat{u}_1^{(k=0)}$ and $\hat{V}_1^{(k=0)}$. Among different options two cases can be intuitively justified. In the first case one can assume a constant retention of the upper reservoir:

$$\frac{d\hat{V}_1^{(k=0)}}{dt} = 0 \qquad \Rightarrow \qquad \hat{u}_1^{(k=0)} = I_1 \qquad 21$$

It means that at the first stage of the next step, all inflows to the system must pass through the lower reservoir. This requirement can negatively affect the performance of the algorithm due to constraints 13. In some cases the better initial approximation is given by:

$$\hat{u}_1^{(k=0)} = 0 \qquad \Rightarrow \qquad \frac{d\hat{V}_1^{(k=0)}}{dt} = I_1 \qquad 22$$

In the majority of cases, the above formula does not guarantee that initial approximation meets the constraints 3 imposed on retention of upper reservoir. However, violation of the constraints resulting from 22 will be corrected at the second stage.

Control Random Search method

The functions $u_j(t)$, j=1,2 were represented by a train of rectangular pulses and the time horizon was divided into L unequal time intervals. The parameters to be determined were values of pulses $\hat{u}_l$ and time instances (parameters α) of switching the control function $u(t)$. This type of discretisation, denoted as TD-RP (Time Dependent Rectangular Pulses) is described in detail by Dysarz and Napiórkowski (2002).

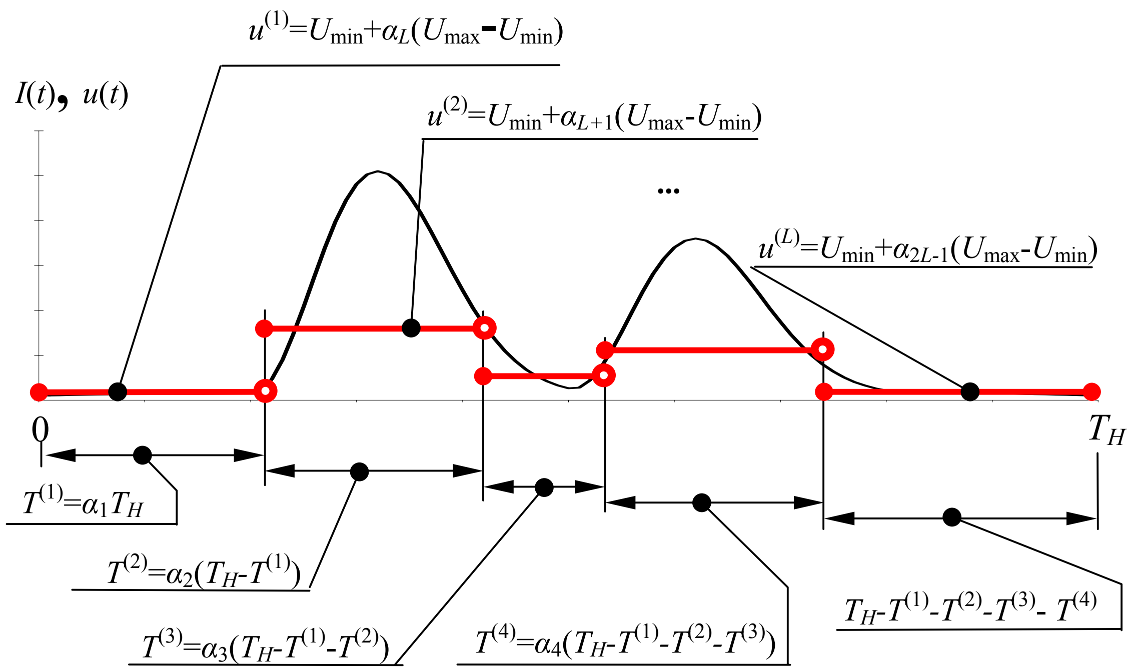

Fig. 2 Control functions as a train of rectangular pulses

The optimisation problems for the lower reservoir 10 and the upper reservoir 14 were solved by means of the global random search procedure (Ali and Sorey, 1994; Price 1987), namely the following version of Controlled Random Search (CRS2) described in details in Dysarz and Napiórkowski (2002).

The CRS2 algorithm starts from the creation of the set of points, many more than n+1 points in n-dimensional space, selected randomly from the domain. Let us denote it as S. After evaluating the objective function for each of the points, the best x_L (i.e. that of the minimal value of the performance index) and the worst x_H (i.e., that of the maximal value of the performance index) points are determined and a simplex in n-space is formed with the best point x_L and n points ($x_2,\ldots,x_{n+1}$) randomly chosen from S. Afterwards, the centroid x_G of points $x_L,x_2,\ldots,x_n$ is determined. The next trial point x_Q is calculated, $x_Q=2x_G-x_{n+1}$. Then, if the last derived point x_Q is admissible and better (i.e., $Q(x_Q)\leq Q(x_H)$), it replaces the worst point x_H in the set S. Otherwise, a new simplex is formed randomly and so on. If the stop criterion is not satisfied, the next iteration is performed. In the CRS2 version applied in the tests, the worst point of the current simplex will be the reflected point x_Q, rather than the arbitrary chosen one (Dysarz and Napiórkowski, 2002).

Results of Tests for Historical Data

The described sequential optimisation was tested and verified against a number of historical and synthetic flood events and for two types of flood routing models described above. The results for three of them, namely for the historical floods in Nysa catchment in 1965, 1977 and 1997 and for the nonlinear kinematic wave model, are presented in Fig.3, Fig.4 and Fig.5, respectively. We observed that kinematic wave model more accurately described flow transformation with lateral inflow.

The floods in 1997 were caused by the most disastrous recent abundance of water in the region. During the first stage of the disaster, a rapid increase in runoff was noted after intense and long lasting rains in the 4-10 July period in the highland tributaries. Yet, a few days later, from 15 to 23 July, another series of intensive rains occurred. The highest precipitation in the Klodzko valley reached 100-200 mm. The flood virtually ruined the town of Klodzko (Kundzewicz et al., 1999), and the historic stage record was exceeded by 70 cm. Several all-time maximum stages recorded were largely exceeded by the 1997 flood.

Fig.3a-5a show the performance of the Otmuchow (upper) Reservoir, Fig.4b-5b show the performance of Nysa (lower) Reservoir, and Fig.3c-5c show the flow at the cross-section below the junction of Nysa and Odra Rivers.

As one can see, by an appropriate choice of the control functions the peaks of the waves on Nysa Klodzka and Odra rivers were desynchronised and the culminations did not overlap.

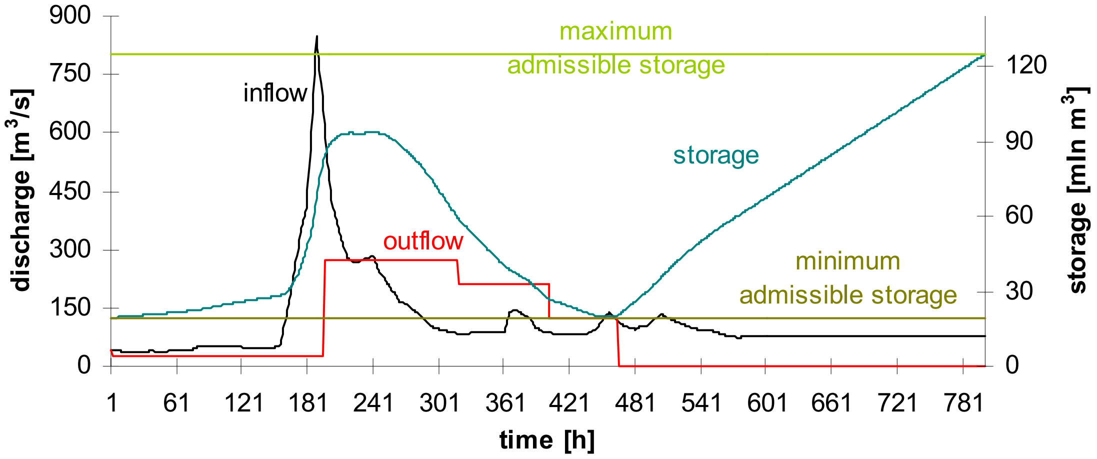

Fig. 3a Performance of Otmuchów (upper) reservoir – 1965 data

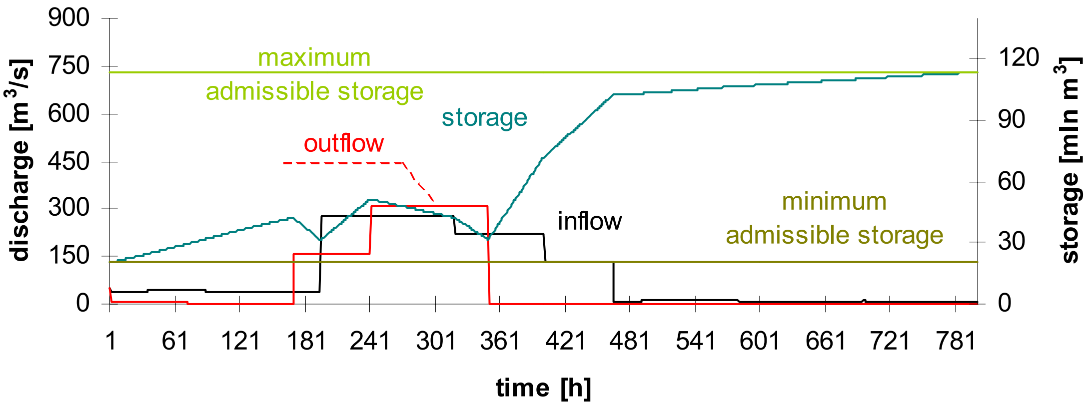

Fig. 3b Performance of the Nysa (lower) reservoir –1965 data

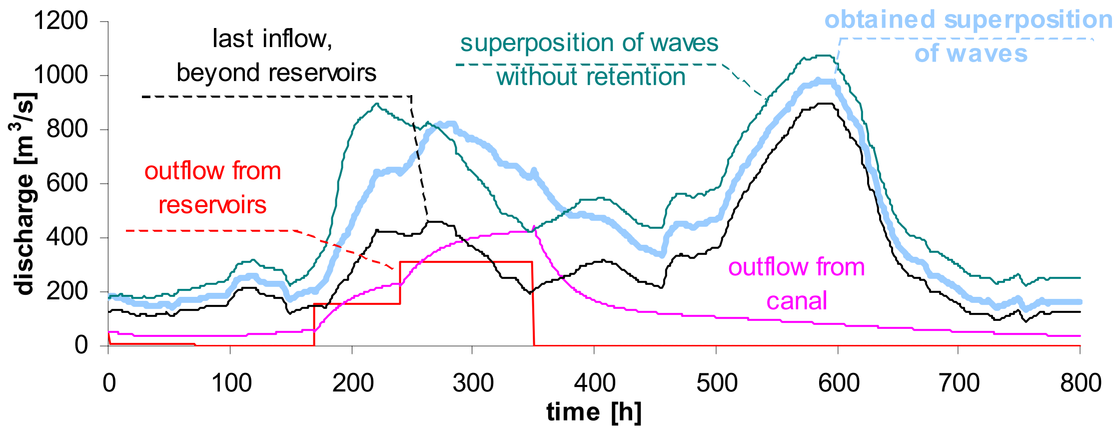

Fig. 3c Flow below the junction of Nysa and Odra Rivers –1965 data

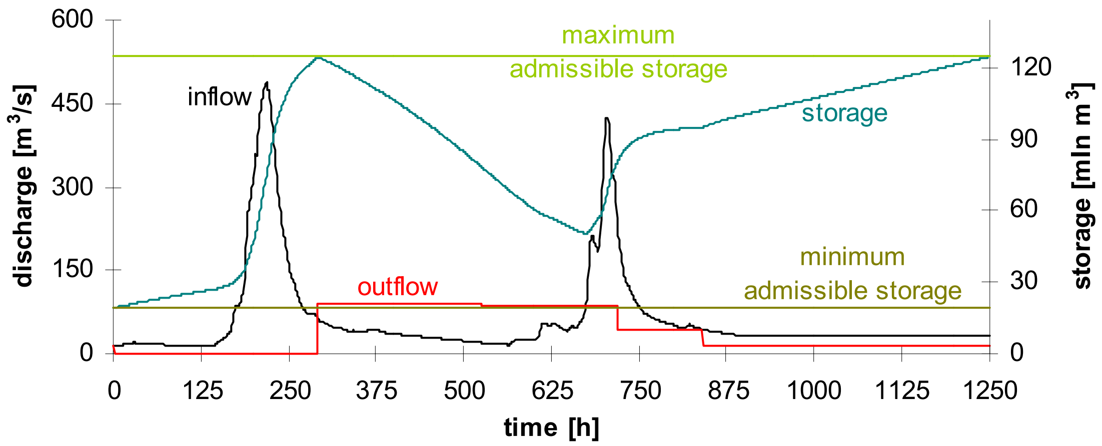

Fig. 5a Performance of Otmuchów (upper) reservoir – 1977data

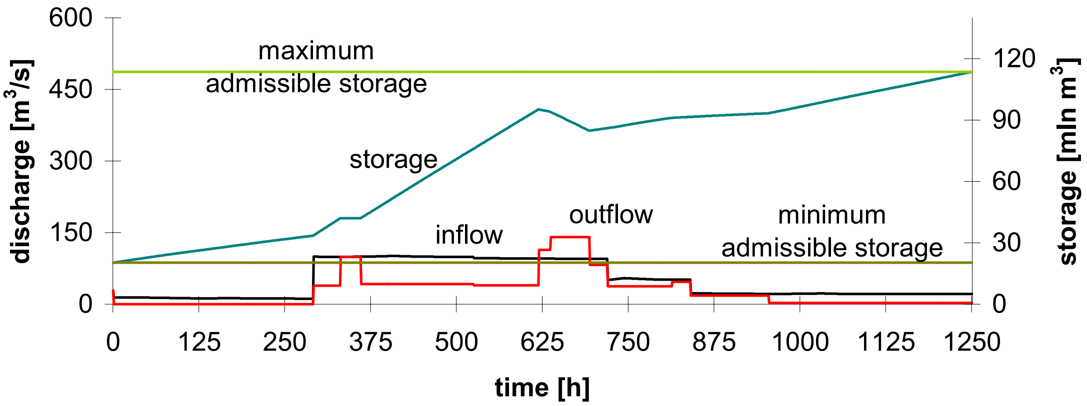

Fig. 5b Performance of the Nysa (lower) reservoir –1977 data

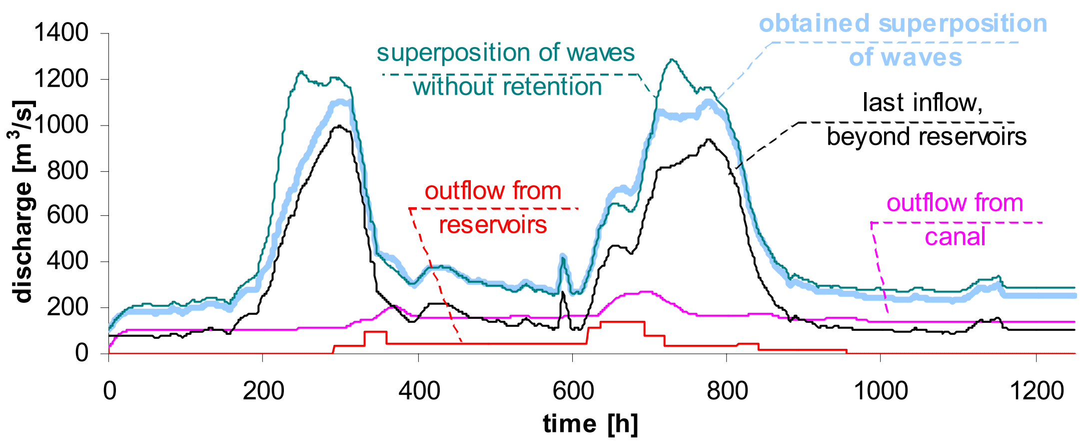

Fig. 4c Flow below the junction of Nysa and Odra Rivers –1977 data

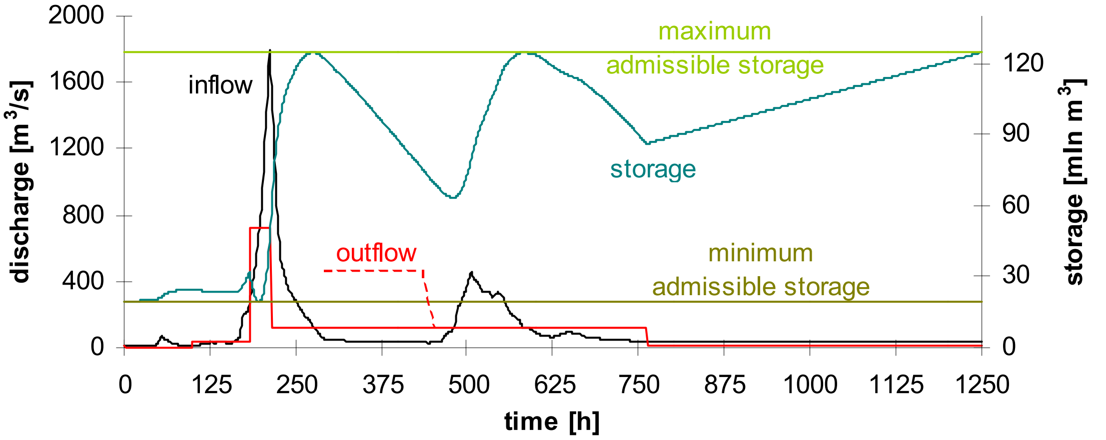

Fig. 5a Performance of Otmuchów (upper) reservoir – 1997data

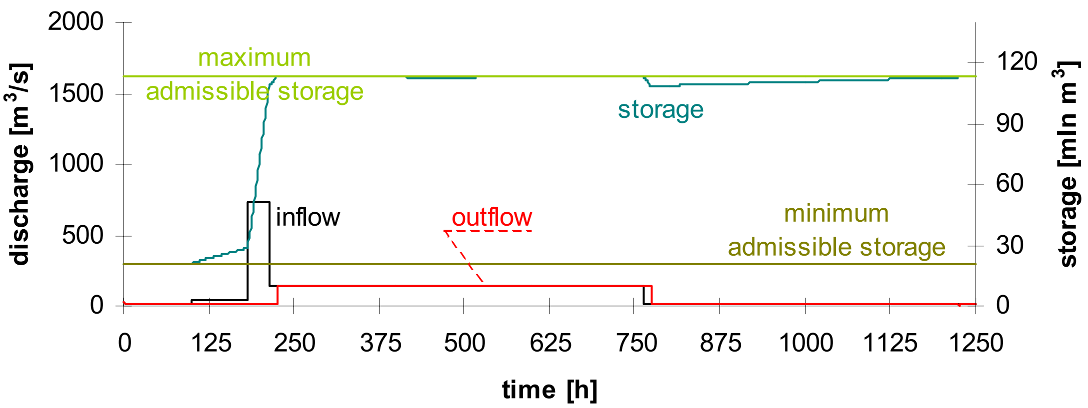

Fig. 5b Performance of the Nysa (lower) reservoir –1997 data

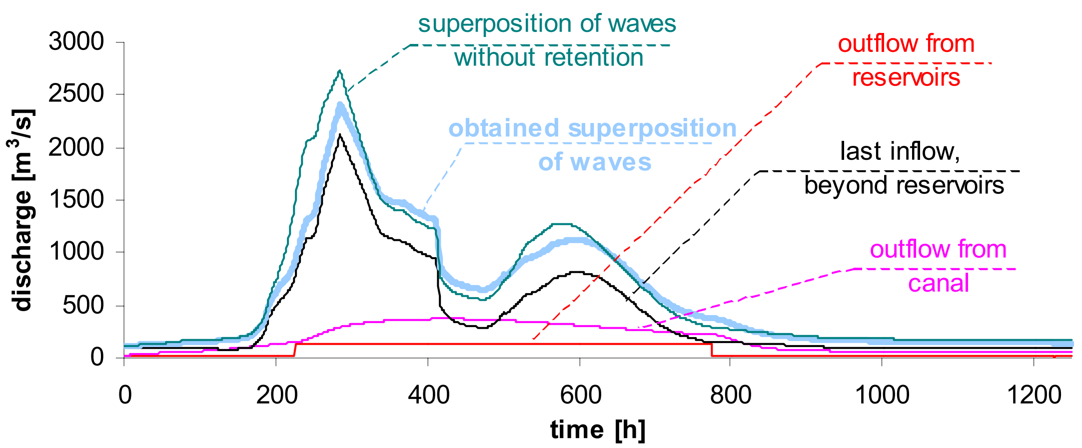

Fig. 5c Flow below the junction of Nysa and Odra Rivers –1997 data

CONCLUSIONS

It is necessary to take into account the uncertainty of the inflows forecast in operation control of reservoirs system during flood. Hence the optimisation problem has to be solved repetitively for many scenarios using actual measurements and updated forecasts. Therefore, from the decision making point of view, the access to a quick and reliable, especially designed for the particular system optimisation module, is very important.

The approach presented in the paper makes a decomposition of the general problem possible, so that computational costs grow linearly with the number of reservoirs. Hence, more complex representation, than that described by Niewiadomska-Szynkiewicz et al. (1996) and Niewiadomska-Szynkiewicz and Napiórkowski (1998), of the control functions $u_j(t)$ can be adopted.

Because of nondifferentiability of global and two local performance indices, the global optimisation technique CRS is used. The authors have not proved the convergence of the proposed method yet, however convergence was observed in all carried out tests.

The results from applications of the sequential optimisation by means of control random search methods to determine the reservoir decision rules during flooding are encouraging. Accuracy of the proposed method is satisfactory. The initiation procedure and the stop criterion were cautiously investigated, so high efficiency does not cause losses in accuracy. As a result, the described control structure of Nysa Kłodzka reservoirs system includes transformation by means of hydrodynamic flood routing model, because the proposed technique guarantees that the solution of the optimisation problem can be obtained in reasonable time.

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