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DETERMINATION OF RESERVOIR DECISION RULES DURING FLOOD

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A b s t r a c t

An application of the controlled random search method for real time operation of the Nysa Kłodzka Reservoir System in Poland is presented. To improve efficiency and accuracy of the used optimization technique a number of control structures were tested and a particular modification of the standard controlled random search method was suggested. It is shown that the introduced concept improves the performance of the control structure considerably, which can be expanded to include hydrodynamic models for flow routing in Nysa Kłodzka River.

Key words: flood control, optimization techniques, reservoir system.

1. Introduction

In the middle of the last year, the Polish Committee for Scientific Research accepted the research project ”Operational control of flood wave”, registered as 6 PO4D 032 19. The objective of the project is to design Decision Support System for flood control for Nysa Kłodzka River and a selected reach of Odra River. In order to achieve this main goal the following particular problems are to be solved: numerical forecast of precipitation for a catchment, rainfall-runoff transformation model, unsteady flow models for Nysa Kłodzka and Odra Rivers, and operational control structures. The main goal of the paper is to present some results related to the last problem, namely a control structure and control mechanisms for the cascade of Otmuchów and Nysa reservoirs.

The catchment of Nysa River is located in the southern part of Poland. The hydrological features of the upper part of this catchment are characterized by massive rocky underground covered only by small layer, and an average yearly precipitation of about 900 mm. The missing ability of storing water underground leads to dangerous floods. To achieve the ability to handle this problem two reservoirs were built, and two are under construction. Here we are just interested in the management of two existing reservoirs governing the discharges in the city of Nysa, which lies just below the reservoir No. 2. Figure 1 shows the simplified Nysa Kłodzka Reservoir System.

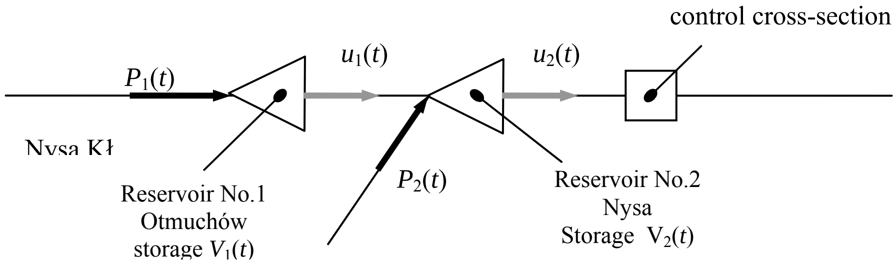

Fig. 1. Basic structure of Nysa Kłodzka Reservoir System.

Management and control of flooding generally require the use of forecasting techniques. At this stage we assume that both inflows P_1 and P_2 to the system represent one of many possible scenarios taken into account by a decision maker. The scenarios considered could be based on rainfall-runoff prediction models, or recorded historical data.

2. Formulation of the problem

The basic goal of the optimal flood control problem, i.e., minimization of the peak flow measured at a downstream cross-section and storing water for future needs after flood can be solved in two stages: the optimization of the so-called aggregated model of the system, and then disaggregation of the obtained solution (Karbowski, 1993).

To describe the original system by means of aggregated model, the system is transformed to a single reservoir with aggregated storage $V(t) = V_1(t)+V_2(t)$, aggregated inflow $P(t) = P_1(t) + P_2(t)$, and one outflow $u(t) = u_2(t)$, as shown in Fig. 2.

Thus the objective function for the optimization problem under consideration can be written in the form:

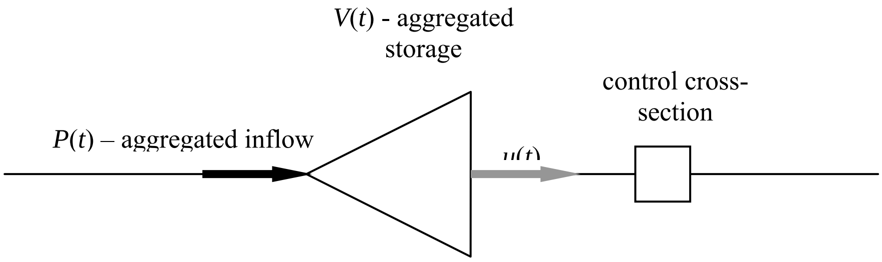

Fig. 2. Aggregated Nysa Kłodzka Reservoir System.

$$J(u^*) = \min_{\substack{u(t)\in A(t)\\ t\in[0,T_H]}} \left\{ \beta_1 \max_{t\in[0,T_H]} u(t) + \beta_2 \left[V(T_H) - V_{max} \right]^2 \right\}, \tag{1}$$

where β_1 and β_2 are the weight coefficients, T_H is the estimated control time horizon, and V_{max} is the upper constraint imposed on storage. The set of admissible solutions A is determined by mass balance equation

$$\frac{dV}{dt} = P(t) - u(t) \qquad \text{with initial condition } V(t=0) = V^{(0)} \tag{2}$$

and appropriate inequality constraints imposed on storage $V(t)$ and outflow $u(t)$ for $t \in [0, T_H]$:

$$V_{min} \le V(t) \le V_{max} \qquad U_{min} \le u(t) \le U_{max}, \tag{3}$$

where V_{min} is the lower constraint of $V(t)$, and U_{min} and U_{max} are the lower and the upper constraints of $u(t)$, respectively.

To solve the optimization problem (1)–(3) we transformed it to the equivalent form by adding the penalty function to the main criterion (1):

$$Q^*(u) = \min_{\substack{u(t)\in A(t)\\ t\in[0,T_H]}} \left\{ \beta_1 \max_{t\in[0,T_H]} u(t) + \beta_2 \left(V(T_H) - V_{\max} \right)^2 + K\left[u(t), V(t) \right] \right\}, \tag{4}$$

where the penalty function depends on current storages and outflows:

$$\begin{aligned} K\left[u(t),V(t)\right] &= \kappa_{V_{\min}} f_+\left(V_{\min} - \min_{t\in[0,T_H]} V(t) \right) + \kappa_{V_{\max}} f_+\left(\max_{t\in[0,T_H]} V(t) - V_{\max} \right) \\ &+ \kappa_{U_{\min}} f_+\left(U_{\min} - \min_{t\in[0,T_H]} U(t) \right) + \kappa_{U_{\max}} f_+\left(\max_{t\in[0,T_H]} U(t) - U_{\max} \right) \end{aligned} \tag{5}$$

and where $f_+(x) = \max(0, x)$.

3. The optimization techniques

The dynamic optimization problem (4) with the equality constraints (2) can be solved as a static one after suitable parameterization of the control function $u(t)$. It should be noted, however, that because of the mini-max form of the objective function (4), the well-known gradient techniques fail and the non-gradient global optimization methods should be applied.

Recently, methods based on simulation of natural selection and evolution processes have become more popular. They have been called evolutionary algorithms (Holland, 1975). In parallel, there have been developed non-evolutionary methods of optimization. The second group contains methods based on the simplified mathematical descriptions of physical and chemical phenomena, which are naturally or artificially forced processes. This group includes the methods like generalized descent method (Griewank, 1981), and simulated annealing (Kirkpatrick *et al*., 1983; Dekkers and Aarts, 1991).

In this paper we apply the global optimization technique elaborated by Price (1983; 1987) and later developed by Ali and Storey (1994), namely the controlled random search method, and particularly the versions called CRS2 and CRS3 as described below.

Controlled Random Search – Price approach

CRS methods are global random search procedures combined with local optimization algorithms. The local optimization algorithm used can be the simplex method or in more developed applications the Nealder-Mead non-linear simplex method (Findeisen *et al*., 1980; Wit, 1986; Niewiadomska-Szynkiewicz, 1999).

The CRS2 algorithm starts from the creation of the set of points, much more than $n+1$ points in n-dimensional space, selected randomly from the domain. Let us denote it as *S*. After evaluating objective function for each of the points the best x_L (i.e., that of the minimal value of the performance index) and the worst x_H (i.e., that of the maximal value of the performance index) points are determined and a simplex in n-space is formed with the best point x_L and n points ($x_2,\ldots,x_{n+1}$) randomly chosen from *S*. Afterwards, the centroid x_G of points x_L, x_2 ,…, x_n is determined. The next trial point x_Q is calculated, $x_Q = 2x_G - x_{n+1}$. Then the last, if the point x_Q is admissible and better, i.e., $Q(x_Q) \le Q(x_H)$, replaces the worst point x_H in the set *S*. Otherwise, a new simplex is formed randomly and so on. If stop criterion, which is described further, is not satisfied, the next iteration is performed.

The CRS3 algorithm is a combination of the CRS2 procedure with the local optimization procedure based on the Nelder-Mead simplex method. The local algorithm is switched when a newly generated point in CRS2 falls within the bottom, for example, one-tenth of the ordered array *S*. After completing the local search, the global search is continued. The CRS3 method tends to speed the convergence of the algorithm with respect to CRS2. The local optimization procedure operates only on the small part of set *S* and thus has a minimal effect on the global search performance of the CRS2 phase. The local procedure can operate at any stage of CRS3. It is triggered automatically but it can be modified to permit the user to switch the local procedure in or out according to his decision.

CRS methods have been modified many times. Most of modifications have consisted in adding certain local procedure into the main CRS2 or CRS3 algorithm. This has often improved the efficiency or accuracy in case of particular task or group of special kind of problems. Some of modifications concerned the construction of initial set.

In this paper the authors propose another kind of CRS2b and CRS3b modification. The worst point of the current simplex will be the reflected point $x_Q = 2x_G - x_H$, rather than the arbitrary chosen one .

Details of practical application

The initiation procedure and stop criterion are crucial for the performance of the Control Random Search techniques used. The initiation is a procedure which determines the choice of certain number of points from the domain of a possible solution. Following the Price results (Price, 1983), the initial set is built by sampling the domain with uniform distribution, and the suggested number of points is to be equal to $10(n + 1)$.

CRS methods are characterized by fast expansion of the best solution and slower convergence of the other points of the set *S*. This means that the value of the objective function becomes gradually constant in the set and the exploration or expansion abilities are slowly lost. Hence, in the first description of CRS2 method (Price, 1983), the stop criterion was based on evaluation of the difference between the best objective function and the worst. When this difference was equal to or lower than 10^{-6} the stop criterion was fulfilled.

It was noticed that the methods reach the optimum much faster before the above stop criterion is satisfied so it is expected the stop criterion could be weakened without loosing accuracy of the obtained solution. It was assumed in the tests that the proper stop criterion could be the difference between the mean value of objective function in converted set and the best value. To avoid unnecessary iteration, this difference should be equal to or lower than certain accuracy ε. In the first tests the accuracy chosen was 10^{-3}. Such a condition can guarantee high quality of the obtained solutions and, at the same time, reduces an effect which could be called “idle run”, i.e., performing iterations which do not improve the best solution.

CRS methods may not be convergent in certain cases. Hence, the above stop criterion is extended to include the second condition, namely maximum allowable computational costs. As to the computational cost, the number of evaluation of objective function is adopted. Maximum allowable computational cost depends on actual problem solved and is in the range of 10^5 for two-dimensional problem and 10^6 for ten-dimensional problems, respectively.

4. Accuracy of the CRS methods

In this section the comparison of the results obtained by means of CRS methods with those obtained by partially analytical approach for particular case of the control structure is presented. The analytical approach adopted here was described in detail by Karbowski (1991; 1993) and Karbowski and Malinowski (1995). However, it should be noted that the above-mentioned analytical approach cannot be applied to real world problems because it requires oversimplification of the hydrodynamic models for flow routing.

According to the analysis proposed by Karbowski (1993), the optimal outflow takes the form

$$\hat{u}(t) = \begin{cases} q & t \in [t_0, t_2] \\ P(t) & t \notin [t_0, t_2] \end{cases}, \tag{6}$$

where q is a certain constant outflow from reservoir for time instances $t \in [t_0, t_2]$. The above formula is well-known in literature as the "basic rule" and is shown in Fig. 3. The value of the discharge q and the time instances t_1 and t_2 are calculated according to the above-mentioned papers.

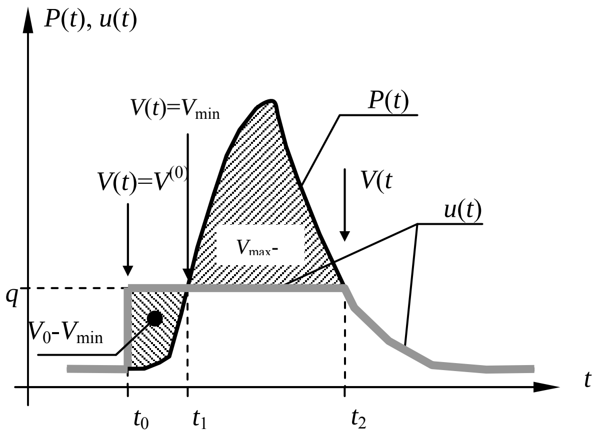

Fig. 3. Outflow from reservoir described by the "basic rule".

In the numerical examples discussed below, the flood wave is represented by the synthetic inflow and is described by

$$P(t) = P_0 + P_m \left(\frac{t - t_P}{T_m} \right)^2 \exp\left[1 - \left(\frac{t - t_P}{T_m} \right)^2 \right], \tag{7}$$

where P_0, P_m, t_P and T_m are the parameters that represent base flow, maximum amplitude of flood wave, time to pick and time scale, respectively. Computations are carried out for three sets of input data and different initial conditions. In each case reservoir parameters were taken as aggregated storage of Nysa Kłodzka system, i.e., $V_{min} = 36.67 \times 10^6$ m^3, and $V_{max} = 238.26 \times 10^6$ m^3. The appropriate time horizon is $T_H = 600$ h. Both the data and the obtained analytical results are shown in Table 1.

Table 1

Values of parameters used in tests

Test	Inflow				$V^{(0)}$ [10^6 m^3]	Results			
	P_0 [m^3/s]	P_m [m^3/s]	T_m [h]	t_P [h]		q [m^3/s]	t_0 [h]	t_1 [h]	t_2 [h]
Set 1	10.0	700.0	120.0	0.0	36.67	201.22	40.24	40.24	226.74
Set 2	10.0	700.0	120.0	0.0	100.00	258.94	0.00	46.84	214.52
Set 3	10.0	700.0	120.0	60.0	100.00	205.54	0.00	100.75	285.75

The CRS methods described in previous section were used to solve the same optimization problem and the same synthetic (7) inflow with constrains on q and t_0:

$$U_{\min} \le q \le U_{\max} \qquad 0 \le t_0 \le T_H \,. \tag{8}$$

Each version of the proposed methods were run 20 times for each set of data. Averaged results are shown in Table 2. They were divided into four sections, and each section contains two columns. The first column represents the mean cost (number of objective function evaluations) that is considered as a measure of efficiency. The second column represents accuracy and reliability of the solution and is based on mean distance between the values q, t_0 obtained by means of the CRS methods and those obtained analytically ($\hat{q}$ and $\hat{t}_0$ shown in Table 1) given by

$$K\,r_D = \sqrt{\left(\frac{q - \hat{q}}{U_{\max} - U_{\min}} \right)^2 + \left(\frac{t_0 - \hat{t}_0}{T_H} \right)^2} \times 100\% \,. \tag{9}$$

Note, that the value in the second column in each section of Table 2 is the number of runs of the particular CRS method for which the value of criterion (9) is less than 5%.

One can see from Table 2 that the high efficiency and accuracy can be expected for each algorithm. Original CRS2 and CRS3 methods are more accurate but the modified techniques seem to be faster. The main conclusion is that the CRS methods can be applied for real reservoir systems when hydrodynamic equations are used to describe flood routing in open channels.

Table 2

Efficiency and reliability of the CRS methods, 20 runs

Test	CRS2		CRS3		CRS2b		CRS3b	
	Mean cost	Runs with $Kr_D < 5\%$	Mean cost	Runs with $Kr_D < 5\%$	Mean cost	Runs with $Kr_D < 5\%$	Mean cost	Runs with $Kr_D < 5\%$
Set 1	608	18/20	645	19/20	487	16/20	576	17/20
Set 2	365	17/20	409	15/20	322	18/20	426	15/20
Set 3	398	18/20	401	18/20	238	14/20	384	16/20

5. Proposed representations of the control function

To solve the optimization problem (1) for real systems, four types of approaches that lead to the determination of function $u(t)$ depending on finite number of parameters, are considered. They are discussed in detail below.

In the first case, the time horizon is divided into L equal time intervals and the function $u(t)$ is represented by a train of Rectangular Pulses, so this type is denoted by RP

$$u(t) = \hat{u}_l = \text{const} \quad \forall t \in [t'_{l-1}, t'_l), \qquad \text{for every } l = 1, ..., L . \tag{10}$$

This is one of the simplest forms of the control function and it is also convenient from the practical point of view. In this case the values of pulses $\hat{u}_l$ are determined directly by the chosen CRS technique.

In the second case, the time horizon is still divided into L equal time intervals and the function $u(t)$ is represented by a train of rectangular pulses, but the values of pulses $\hat{u}_l$ are calculated indirectly. First, the required reservoir storage $\hat{V}_l$ at the end of any time interval is calculated by means CRS technique, and then outflow $\hat{u}_l$ is determined as

$$\hat{u}_l = \frac{1}{\Delta\tau}\left[\int_{t_{l-1}}^{t_l} P(t)\,\mathrm{d}t - \left(\hat{V}_l - \hat{V}_{l-1}\right)\right], \qquad l = \overline{1, L}, \tag{11}$$

where $\Delta\tau = t_l - t_{l-1}$. This type is denoted as RPV, that stands for Rectangles Pulsed controlled by water Volume. In this procedure $\hat{V}_{l-1}$ is known from the initial condition or from previous computations.

In the third case the function $u(t)$ is represented by a train of rectangular pulses, but the time horizon is divided into L unequal time intervals. The parameters to be determined are values of pulses $\hat{u}_l$ and time instances of switching the control function $u(t)$. Note that in this case the number of time intervals $\Delta\tau_l = t_l - t_{l-1}$ can be much smaller than in the case of RP and RPV, so the overall number of parameters to be determined by means of CRS method is smaller. This type is denoted by TD-RP (Time Dependent Rectangular Pulses).

The last case is similar to the third case but the values of pulses $\hat{u}_l$ are calculated indirectly as in the case of RPV with the use of eq. (11). This type is denoted by TD-RPV (Time Dependent Rectangular Pulses controlled by water Volume).

6. Tests with single reservoir

In this section the results of numerical experiments for three different scenarios are presented. The first two scenarios were generated artificially and they represent both unimodal and bimodal flood waves. The third scenario represents the real flood wave recorded at Bardo cross-section in Nysa Kłodzka River in the time period between August 01 and 25, 1980.

The tests were carried out for aggregated Nysa Kłodzka Reservoir System as described in Section 4. Each of four versions of the control function, namely RP, RPV, TD-RP and TD-RPV, was run 20 times for each of three inflow scenarios. For the first two versions, i.e., RP and RPV, the optimization time horizon was divided into 10 equal time intervals, i.e., the number of variables to be determined by CRS method is 10. For the third and fourth versions, TD-RP and TD-RPV, the optimization time horizon was divided into 5 unequal time intervals, i.e., the number of variables to be determined by CRS method is 9.

The initiation procedure and the stop criterion were used as described in Section 3.

Table 6 shows the results of the applied optimization techniques, namely the number of objective function calls, the best solutions F_{best} and the number of good runs (out of 20) for which coefficient $Kr_{[\%]}$ defined by

$$Kr_{[\%]} = \frac{F - F_{best}}{F_{best}} \times 100\% \tag{12}$$

is less than 5%, where F is objective function value obtained from current run.

Table 3a

Classic CRS methods, stop criterion: $\varepsilon = 10^{-3}$, 20 runs

	CRS2			CRS3		
	Cost	The best value	Runs with $Kr < 5\%$	Cost	The best value	Runs with $Kr < 5\%$
	Rectangular Pulses (RP)					
Scenario 1	7862	2.2E+08	1/20	108171	1.2E+08	1/20
Scenario 2	8135	9.7E+05	1/20	9002	3.3E+06	1/20
Scenario 3	1103775	3.6E+08	2/20	1086269	2.7E+08	1/20
	Rectangular Pulses controlled by water Volume (RPV)					
Scenario 1	12760	244.4	11/20	12885	244.4	13/20
Scenario 2	14915	148.9	13/20	17923	148.9	19/20
Scenario 3	18054	65.3	18/20	19143	65.3	19/20
	Time Dependent Rectangular Pulses (TD-RP)					
Scenario 1	1802399	206.5	7/20	1702498	206.5	7/20
Scenario 2	1424697	138.5	16/20	1451865	138.5	15/20
Scenario 3	591021	62.7	20/20	582604	62.7	20/20
	Time Dependent Rectangular Pulses controlled by water Volume (TD-RPV)					
Scenario 1	1940155	206.5	20/20	2000000	206.5	20/20
Scenario 2	1153309	138.5	20/20	1261611	138.5	20/20
Scenario 3	796894	63.8	2/20	392993	62.7	1/20

One can see from Tables 3a and 3b that the performance of the modified CRS2b and CRS3b methods is much better when compared to the classic ones and that the classic methods fail completely in case of RP function.

Note that if the reservoir is filled up at the end of optimization time horizon and the constraints imposed on current storage are fulfilled, then the value of objective function represents the maximum outflow from the reservoir. Hence, very high values of the objective function, for example 2.2×10^8, indicate the violation of the constraints.

Table 3b

Modified CRS methods, stop criterion: $\varepsilon = 10^{-3}$, 20 runs

	CRS2b			CRS3b		
	Cost	The best value	Runs with $Kr < 5\%$	Cost	The best value	Runs with $Kr < 5\%$
	Rectangular Pulses (RP)					
Scenario 1	92686	244.4	20/20	99959	244.4	20/20
Scenario 2	162464	148.9	20/20	147457	148.9	20/20
Scenario 3	176186	65.3	20/20	170535	65.3	20/20
	Rectangular Pulses controlled by water Volume (RPV)					
Scenario 1	28008	244.4	20/20	28161	244.4	20/20
Scenario 2	52346	148.9	20/20	52846	148.9	20/20
Scenario 3	66739	65.3	20/20	64393	65.3	20/20
	Time Dependent Rectangular Pulses (TD-RP)					
Scenario 1	1601442	206.5	20/20	1442677	206.5	20/20
Scenario 2	1691753	138.5	20/20	1664119	138.5	20/20
Scenario 3	905483	62.7	20/20	985361	62.7	20/20
	Time Dependent Rectangular Pulses controlled by water Volume (TD-RPV)					
Scenario 1	1254978	206.5	20/20	1070384	206.5	20/20
Scenario 2	1134968	138.5	20/20	1233857	138.5	20/20
Scenario 3	326951	75.3	20/20	735728	75.0	20/20

It also results from Tables 3a, and 3b that techniques with variable time intervals, namely TD-RP and TD-RPV, give better solutions then techniques with constant time intervals length. However, lower values of the objective function entail higher computational costs.

Examples are shown in Figs. 4–6. These are simulations of reservoir performance which were obtained from runs of CRS3b method with TD-RP control function.

We observed quite often during numerical tests that, for the CRSb methods with TD-RP and TD-RPV structure of the control function, the stop criterion related to

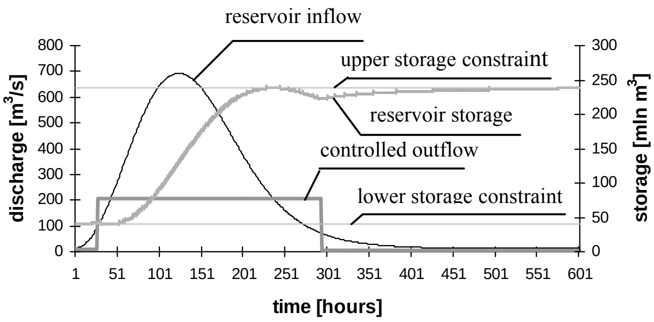

Fig. 4. CRS3b method, PS function, Scenario 1

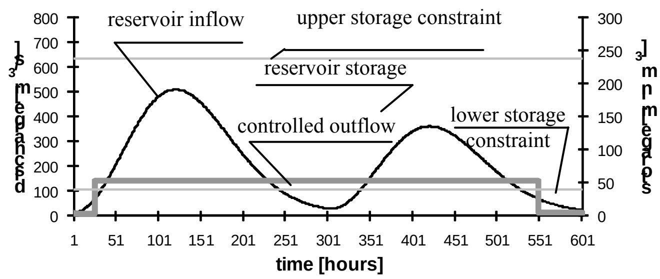

Fig. 5. CRS3b method, PS function, Scenario 2

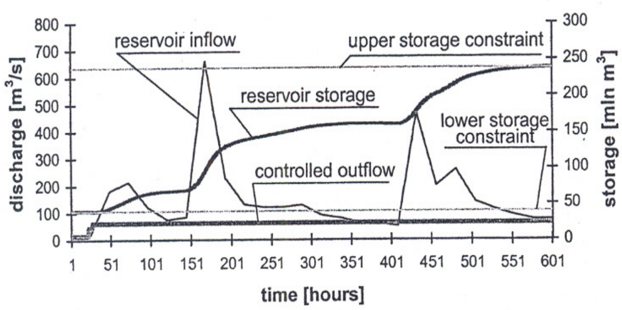

Fig. 6. CRS3b method, PS function, Scenario 3

maximum allowable calls of objective function was first met. This meant the solutions were not converged to the best ones. It can be easily explained with the help of Fig. 7, which shows the relation between the value of the objective function and computational cost (number of calls of the objective function).

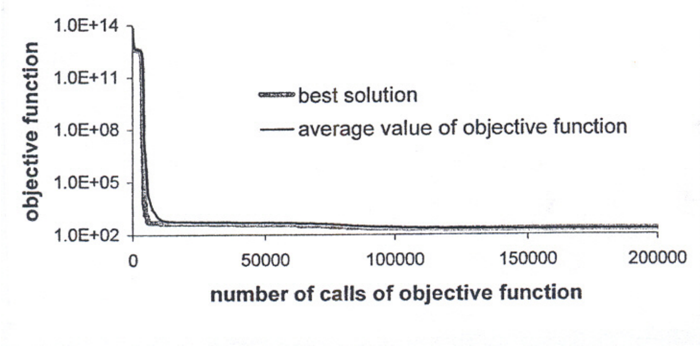

Fig. 7. Objective function *vs*. computational cost.

Objective functions are reduced very fast at the beginning of the optimization process, then one can observe a kind of plateau, where the decrease of the objective function requires considerable computational cost. Hence, the number of objective function calls meets the appropriate stop criterion before the optimal solution is reached.

Therefore, additional tests were carried out with the weaker stop criterion, namely for $\varepsilon = 0.5$ and $\varepsilon = 0.3$, in which the computational cost was significantly reduced by 50% to nearly 90%. The price for dramatic decrease in optimization time is the slight increase in the obtained value of objective function. The results obtained for $\varepsilon = 0.5$ are shown in Table 4.

Table 4

Modified CRS methods, stop criterion $\varepsilon = 0.5$

	CRS2b			CRS3b		
	Cost	The best value	Runs with $Kr < 5\%$	Cost	The best value	Runs with $Kr < 5\%$
	Time Dependent Rectangular Pulses (TD-RP)					
Scenario 1	180124	206.5	17/20	215915	206.6	19/20
Scenario 2	452923	138.5	20/20	553201	138.6	20/20
Scenario 3	401282	62.8	20/20	355563	62.8	20/20
	Time Dependent Rectangular Pulses controlled by water Volume (TD-RPV)					
Scenario 1	140348	206.9	20/20	107706	206.9	20/20
Scenario 2	192899	138.8	20/20	179927	138.7	20/20
Scenario 3	64522	76.4	20/20	60311	75.2	17/20

7. Conclusions

Results of application of the control random search methods for the determination of the reservoir decision rules during flooding are very encouraging. Accuracy of the proposed methods are more than satisfactory. For special case of control function it was shown that the numerical solution is consistent with the analytical one. Computational tests have shown that the best are CRS2b and CRS3b methods with TD-RP decision rule but similar results can be also achieved with TD-RPV representation of the control function. The initiation procedure and the stop criterion were cautiously investigated, so high efficiency does not cause losses in accuracy. As a result, the described control structure of Nysa Kłodzka Reservoir System can be easily extended to include transformation by means of hydrodynamic flood routing model, because the proposed technique guarantees the solution of the optimization problem can be obtained in reasonable time.

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