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BRAZILIAN TEST - A MICROSCOPIC POINT OF VIEW ON TENSILE FRACTURE GENERATION

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Abstract. The tensile strength of solid materials is one of the most important parameter describing a behavior of the material under external mechanical loading and thus its knowledge is of great practical importance. However, the direct measurement of tensile strength especially for brittle materials is quite difficult and only limited results are available. To cope with this situation various methods of indirect measurements have been proposed among others the, so called Brazilian test is the most popular. The method relies on diametrically loading of disc-like sample of the brittle material until it splits apart due to a induced tensile stress. In this paper we report our effort of describing the fracturing process during the Brazilian test from the "microscopic"point of view. For this purpose we use an advanced implementation of the Discrete Element Method - the ESyS-Particle software. We represent rock specimen as a set of interacting spherical particles which mimic grains of real rock material. We have observed that the maximum loading force which sample can withstand almost linearly scales up with a ratio of maximum-to-minimum particles diameters.

1 INTRODUCTION

The tensile strength of solid materials is one of the most important parameter describing a behavior of the material under mechanical load and thus its knowledge is of great practical importance. However, the direct measurement of tensile strength, especially for brittle materials is quite difficult and thus only limited results are available. To cope with this situation [3] have proposed an indirect method of estimation of the tensile strength know as the *Brazilian test*. The method relies on diametrically loading of disc-like sample of the brittle material until it splits apart due to induced tensile stress as sketched in Fig. 1. Due to its simplicity and low cost of sample preparation it became very popular and has got a recommendation of the International Society for Rock Mechanics Commission as the standard method of the tensile strength estimation [9]. The justification of

the method comes from theory of elasticity which predicts that for an ideal homogeneous elastic cylinder of Radius R and length L subjected to a diametrically linear loading P the stress inside the body reads [11]:

$$\begin{aligned}
\sigma_x &= \frac{P}{\pi RL} - \frac{2P}{\pi L}\left(\frac{x^2(R-y)}{[x^2+(R-y)^2]^2} + \frac{x^2(R+y)}{[x^2+(R+y)^2]^2}\right) \\
\sigma_y &= -\frac{P}{\pi RL} - \frac{2P}{\pi L}\left(\frac{(R-y)^3}{[x^2+(R-y)^2]^2} + \frac{x(R+y)^2}{[x^2+(R+y)^2]^2}\right) \\
\tau_{xy} &= \frac{2P}{\pi L}\left(\frac{x(R-y)^2}{[x^2+(R-y)^2]^2} - \frac{x(R+y)^2}{[x^2+(R+y)^2]^2}\right)
\end{aligned} \tag{1}$$

where $x,\ y$ refer to coordinates shown in Fig. 1. As it follows from above, at the loading plane ($x = 0$) the σ_x and σ_y are normal stresses ($\tau_{xy} = 0$) perpendicular and parallel to the loading plane, the tensile stress σ_x is constant and reads

$$\sigma_T = \frac{P}{\pi RL} \tag{2}$$

while the compressional stress σ_y increases from $3\sigma_x$ at the center of the disc to infinity at the loading point.

These formulas have been extended to a more realistic laboratory situations taking into account loading over a finite, bended surface, non-homogeneity and anisotropy of the material, to name a few extensions. The usefulness and simplicity of the Brazilian test follows directly from Eq. 2 which predicts that tensile is proportional to the loading. Thus, assuming that splitting of the samples occurs when tensile stress reaches the material tensile strength, it can easily be estimated from Eq. 2 by recording the loading force when sample crushes.

The Brazilian test method has gain a lot of popularity not only because of simplicity of its application but also due to more fundamental questions concerning mechanisms of creation and development of the tensile fractures under a simple initial and boundary conditions. However, it has received also a lot of criticism due to lack of robustness and proximity. This last issue arises from the fact that the Brazilian test results show a systematic overestimation of the tensile toughness with respect to values obtained in direct measurements. Thus, there is a lot of ongoing discussion on reasons of this discrepancies. There is also still an open question about dynamics of nucleation and development of the main tensile and secondary cracks resulting in a final breaking apart of the specimen. The classical theory predicts creation of such crack in a center of the sample where compressing stress is the smallest. After nucleation the crack is expected to propagate along the loading plane outwards. However, in many experiments the crack was observed to nucleate not in center but close to sample surface [12]. Wings-type and secondary cracks have also been observed [5]. Another open question is which criterion should be used for estimation of the tensile strength. The most popular approach is based on the stress criterion according to which the material breaking occurs when *tensile stress* reaches the critical value and

Eq. 2 is then directly used. However, this is fully arbitrary choice. Another, physically justified the *strain criterion* was proposed by [17]. Both criteria are equivalent (at least from mathematical point of view) for perfectly elastic media but not if large deformation effects are taken into account.

Theoretical analysis of the Brazilian test method are based mainly on the classical continues mechanics [6, 8, 14] and are limited to the simplest cases. Analysis of more realistic cases can be done only numerically. Authors of such studies use most often different variants of the Finite and/or Boundary Element Methods [15, 4, 18]. Only recently, a simple but very interesting analysis based on the Fiber Bundle Method has been presented [13]. In this paper we analyze some of the above mentioned issues from a "molecular" point of view using an advanced implementation of the Discrete Element Method (DEM) [2]. The method represents the medium as a set of interacting elements "molecules" and is particularly convenient for describing process of medium fragmentation. Moreover, it has been argued [16] that such representation is more natural for describing rock materials than methods based on the continuous mechanics. The used EsyS-Particle software allows a full 3D analysis of stress and displacement evolution in the sample under external loading, takes into account possible large deformations like the classical Finite Element approach and supports description of the material fragmentation and thus cracks nucleation and its temporal evolution [1].

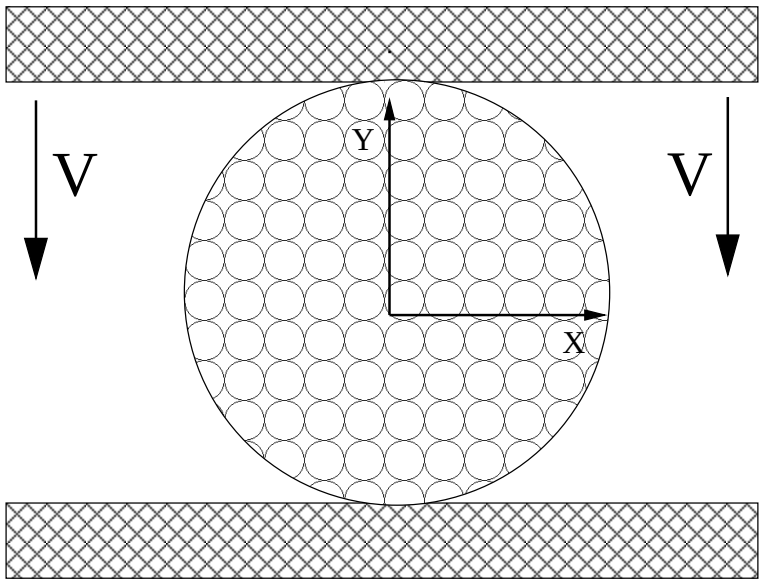

Figure 1: The sketch of the simulation setup used in numerical simulations. The horizontal loading plates are assumed to be perfectly rigid. The lower plate is fixed while the upper one moves downwards with constant velocity V providing the diameter loading of the disc.

2 DEM SIMULATIONS

2.1 Simulation setup

We have performed a number of numerical simulations of Brazilian test experiment considering the specimen in a form of disc (cylinder) of diameter 10 mm and thickness 5 mm, diametrically loaded as it is shown in Fig. 1. The sample was build of spherical particles of varying size within predefined ranges. The maximum radius of used particles was always kept fixed and equal to $R_{max} = 0.2$ mm. The minimum radii were varying among simulations in a range from 0.027 mm up to 0.1 mm. The external load was supplied by two perfectly rigid plates. The lower plate was fixed while the upper one moved downward with constant velocity V=4mm/sec. Simulations ended when the vertical displacement of the upper plate reached the predefined level of 0.2 mm, selected as a large enough to include the sample breaking and a beginning of the post failure stage. In Fig. 2 examples of the facial views of one of the sample prior and after creating a main tensile crack (but still before complete breaking of the sample) are shown. During loading we have recorded at each time step the vertical position of the upper plate and total vertical force acting on it, so a stress - strain relation was continuously monitored. Besides that, we have also recorded total kinetic energy of particles and total potential energy of inter-particles interactions.

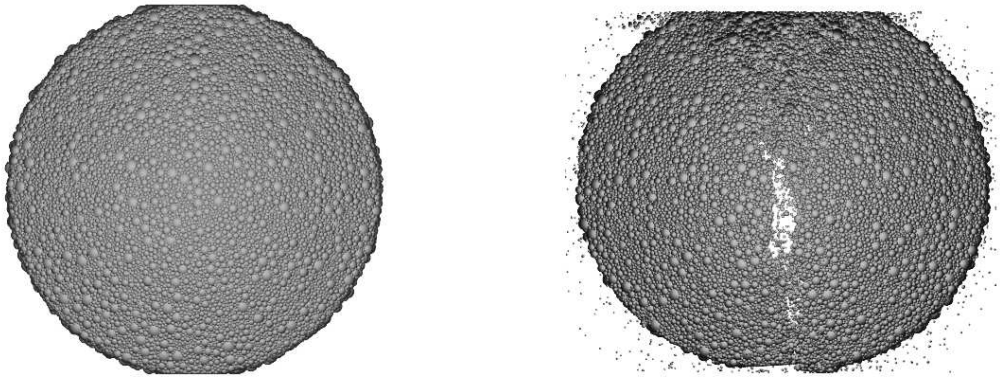

Figure 2: An example view of the sample face prior to loading (left) and after creating a tensile crack (right) in the middle of the sample.

The most nontrivial element of any DEM simulation is defining the particle interactions and breaking conditions. We have used the model of "elastic-brittle interactions" supported by ESyS-Particle and illustrated in Fig. 3.
In this model the near-neighborhood particles interact with each other with repulsive/cohesive radial forces and non-radial "shearing" ones. If interacting particles separate by fixed distance (in terms of percentage of their radii) the interacting bond is broken and interaction is reset to zero. Breaking of shearing forces is based on the Coulomb-Mohr criterion [2]. Under these assumptions our specimen represents a medium which for small external load-

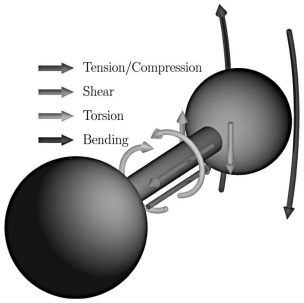

Figure 3: A sketch of forces and moments between particles interacting according to used the advanced interaction model implemented in ESyS-Particle (after [2]).

ing behaves as an ideal elastic body. For larger loads, when some inter-particles bonds break and particles can significantly move away from their initial location the material exhibits some plasticity. Finally, in a large stress concentration regions the particles can separates "en bloc" due to a significant stress redistribution when interaction bonds break (a similar effect is observed in soft-clamp Fiber Bundle Model [10] which finally leads to an initiation and development of cracks. This way, with this simple setup we can simulate behavior of a variety of solid materials.

In the current simulations we have concentrated on a question how size of particles building the sample influences simulation results. For this reason we have kept particle interactions fixed and also the loading rate was kept constant. The build disc samples consisted from about $5 \cdot 10^4$ up to almost $2 \cdot 10^6$ particles and typical $5 \cdot 10^4$ time steps were required to break the samples apart. The time step we have used for the temporal integration (evolution) have been chosen as the compromise between numerical stability and computational time and reads $dt = 5 \cdot 10^{-6}$. With this simulation setup computational time on 10 cores CPU workstation ranged between 6 and 90 hours.

2.2 Simulation results

Let us begin the discussion of obtained results from the analysis of a relation between applied load and vertical displacements (strain) of the sample. The scaled together (for presentation purpose) strain-stress relations for five simulations are shown in Fig. 4 and few particular features are clearly visible in this figure.

Firstly, the maximum attained values of load P_{max}, at which samples break apart strongly depend on the minimum size R_{min} of particles building the sample. On the contrary, the critical strain d_c at which $fmax$ is reached feeble depends on R_{min} ranging from about 0.1 mm up to 0.12 mm. In the consequence the slope of the initial part of the strain-stress curves also significantly varies with R_{min}. Secondly, for small strains the response of the sample to the load is essentially elastic, manifesting itself in an almost linear strain-stress dependences. The visible undulation of the curve is partially due to an acoustic wave generated at the beginning of loading when the upper plate "hits" the

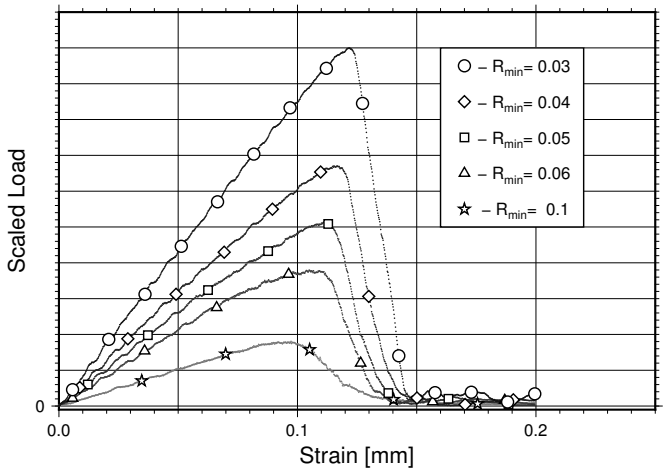

Figure 4: The load against the vertical displacement (strain) for five different samples composed of particles with different minimum radii R_{min}. The maximum radius was fixed for all samples $R_{max} = 0.2$ mm. The weak undulation in first part of curve is mainly due to acoustic waves generated by abrupt beginning of loading of the samples.

sample with the constant speed and partially due to a numerical noise. Only for very small strains (less than 0.01mm) this linearity is broken and a "flattening" of the stress-strain curves is observed. This is a purely numerical effect connected to a non-optimum initial packing of particles in the samples. Thirdly, a departure from the elastic behavior appears at larger strains and manifest itself as a flattening of the curves. This plastic regime starts earlier for samples built of larger particles. In case of samples with $R_{min} = 0.03$ mm it appears just before reaching the maximum withstand load, while for samples built of particles with $R_{min} \sim 0.1 \sim 0.06$ mm it starts almost in half of the strain value at given sample breaks. Finally, in the post failure stage the stress drop rate is smaller for samples built of larger particles. It is interesting to note, that the post failure stress reaches minimum (complete breakage of the sample) for similar values of strain in all cases. To summarize, we have observed a significant mechanical strengthening for samples composed of smaller particles with respect to softer samples build of larger particles. However, the vertical deformation at which samples break apart only weakly depends on R_{min}.

In the next step we have analyzed variation of number of broken inter-particle bonds with progressing load. The results are shown in Fig. 5 where separately scaled number of broken bonds for five considered samples are shown together. The behavior of plotted curves is very similar in a narrow vicinity of critical strain d_c when samples start to break for all but one cases . An abrupt increase of bonds breaking starts just around reaching critical load P_{max} and continue until the sample fully breaks and load reaches minimum value. Only for the very soft sample ($R_{min} = 0.1$) inter-particle bonds start to break massively earlier and the process goes smoother through larger range of sample

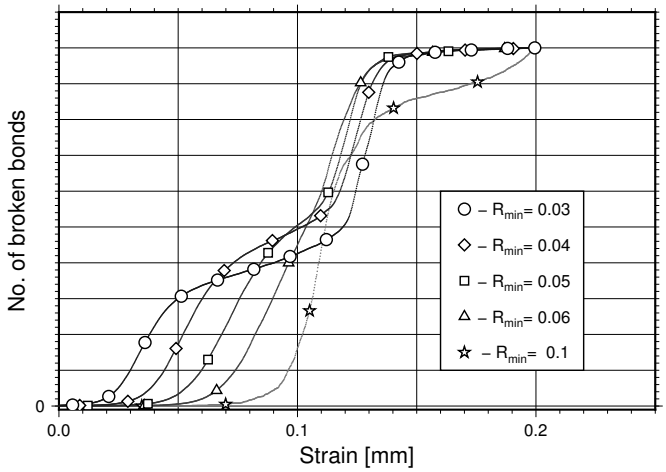

Figure 5: Number of broken inter-particle bonds as the function of the sample vertical deformation.

deformations. The particle composition of the samples influences significantly the initial part of curves corresponding to elastic behavior of samples. For deformations smaller than 0.1 mm we clearly see in Fig. 5 that smaller particles are used to build the sample, sooner inter-particle bonds starts to break. Simultaneously, smaller R_{min} is, larger hardening of a sample before the final breakage is observed. This effect manifests itself by flattening of curves when strain approaches 0.1 mm. Since during the simulations interaction bonds were not allowed to re-heal after breaking, the observed initial increase of bonds breaking can be interpreted as beginning of a visco-plastic deformation of the samples. Let us note, that this effect is quite feeble and can hardly be visible in strain-stress curves in Fig. 4. The degree of this induced viscosity significantly depends on the particles size. From a physical point of view this observation suggests, that at intermediate values of loading some dislocations are induced in the samples. Indeed, such dislocations are more probable for smaller particles and for this reason viscous behavior would start earlier for samples with smaller R_{min}. Moreover, the particle rearrangement by dislocations very soon leads to denser particle packing and in the consequence to the hardening of samples. Finally, for the very small deformation we observe no particle bonds breaking. This is a perfectly elastic regime.

Next, let us consider the change in the potential energy of particle interaction and the particles kinetic energy (without rotational energy) with progressing vertical deformation. The results are shown in Fig. 6.

In the first approximation one can assume that an external compressional load induces harmonic repulsive forces between interacting particles. In consequence, a change of total potential energy should be proportional to the squared sample deformation. Such a behavior is indeed visible in the right panel in Fig. 6 for small and intermediate deformations. For larger deformations when the rupture process nucleate potential energy starts to be

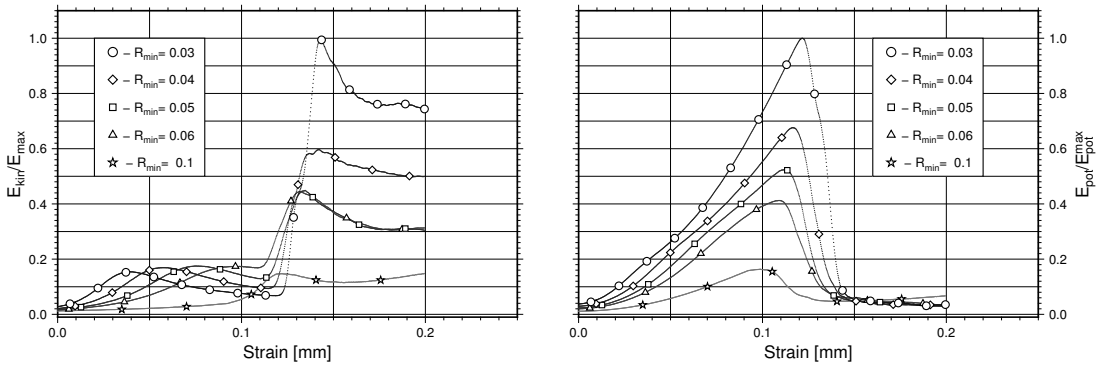

Figure 6: The absolute value of the kinetic (left) and potential energy of particles interactions (right) scaled to the maximum value obtained in all simulations.

released and diminishes. Besides these changes of the potential energy with deformation is different for samples with different R_{min}. It increases most rapidly for samples build of smaller particles and reaches larger values, as expected. The samples build of smaller particle can accumulate larger elastic deformation (potential) energy. In consequence, taking into account that the breaking (critical) strain d_c only slightly increases with decreasing R_{min} and deformation at which samples breaks apart is almost the same for all samples the potential energy released is more abrupt for samples with smaller R_{min}. Thus, we can expect that breaking process will go faster for samples build of smaller particles.

As far as the kinetic energy E_{kin} is concerned we can see in the left panel of Fig. 6 that it is quite stable in the first phase of loading. Some increase and variations of E_{kin} at the intermediate loading can be attributed to the dislocations processes discussed above. During the breaking stage it increases significantly due to a release of potential energy. The changes (mostly decrease) of E_{kin} in the post-failure stage are more complex due to a possible secondary cracking of the samples [5]. However, at this stage, the behavior of E_{kin} is also strongly influenced by a numerical dumping implemented in the used code and thus include a non-physical component. For this reason we do not analyze this stage any more.

Considering energy transformations during the loading and breaking stage of the process we have analyzed the difference between work of the external load and sum of the kinetic and potential energies. The obtained results are shown in Fig. 7

For small, elastic deformations the work of the loading force (ΔW) fully converts into the elastic ($E_T = E_{kin} + E_{pot}$) energy. In consequence $\Gamma = \Delta W - E_T$ vanish. At the intermediate loading stage we observe a monotonic increase of Γ with the sample deformation. This is a signature of non-elastic transfer of external energy. We attache it to breaking of inter-particle interactions bonds and particle dislocations. In real materials at this stage the external energy is also efficiently transformed into the heat. However our

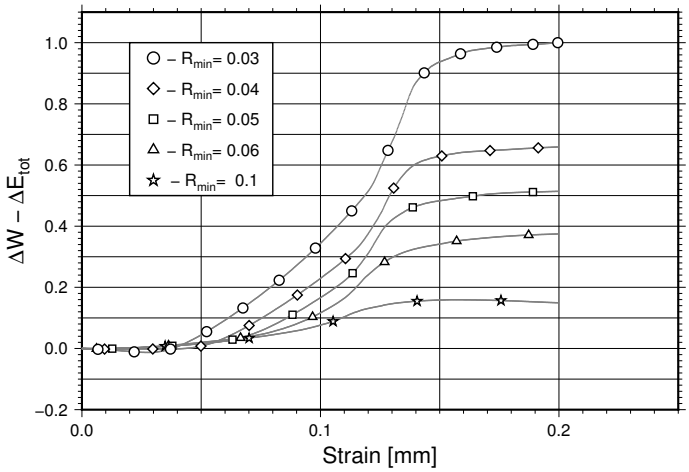

Figure 7: The difference between external load work (ΔW) and sum of kinetic and potential energy. The obtained values were scaled by the largest value obtained in all simulations.

simulations do not take into account thermal effects yet. It is interesting to observe that at this stage Γ exhibits a strong dependences on particle size (R_{min}). During the braking stage ($0.11 < dl < 0.14$) Γ further increases and finally saturates, as expected.

2.3 Scaling

Discussing results shown in Fig. 4 we have noticed that the maximum load the sample can withstand depends significantly on R_{min}. To analyze this issue more in depth we have plotted in Fig. 8 the maximum loads P_{max}, d_c, and the potential energy at the d_c strain for all considered samples.

The obtained results show that the dependence of P_{max} on inverse of R_{min} is almost perfectly linear. Very similar behavior exhibits potential energy calculated for the strain when load reaches its maximum value. The critical strain d_c also almost linearly but, much weaker depends on the R_{max}/R_{min} ratio. At the moment we have no explanation of this observed P_{max} (potential energy) scaling. Actually, a preliminary analysis carried out for a larger set of samples with different R_max confirmed that with a quite good approximation P_{max} depends only on the ratio R_{max}/R_{min} and thus the reported scaling holds.

3 CONCLUSIONS

With the performed numerical simulations we have reached a few goals. From the technical point of view we have proved that ESyS-Particle software is working correctly with this type problems provided the most advanced particle interaction model is used. The obtained results are in a full agreement with similar results obtained both analytically as well as by a more traditional FEM methods. We have also demonstrated exceptional

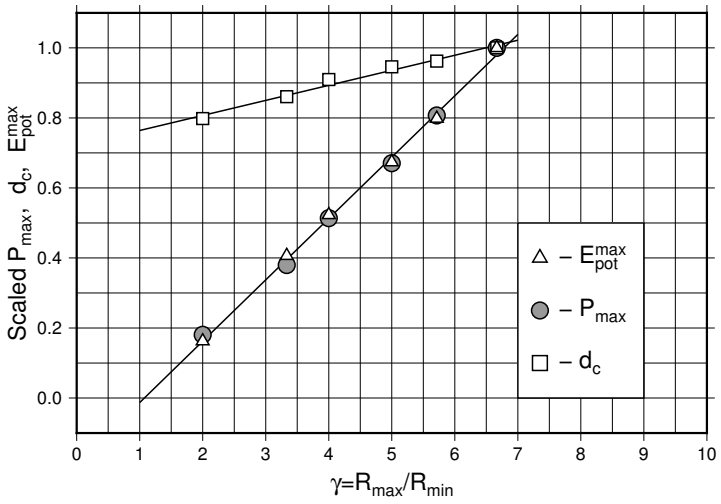

Figure 8: The maximum critical load, critical strain d_c and potential energy at d_c strain as the function of $\gamma = R_{max}/rmin$.

abilities of the DEM method with solving problems including sample fragmentation. The method has allowed a detailed monitoring of internal microscopic state of loaded sample including changes of particles kinetic and potential energies to name a few. From the physical point of, we have got an insight into creation of the tensile crack under the simulated laboratory conditions. We were able to monitor a nucleation and temporal evolution of tensile crack which finally lead to breaking apart of Brazilian test samples. Following an evolution of the total kinetic and potential energies during lading we were able to identify a few stages in a response of the samples to constant speed loading. At the beginning of loading the elastic response of the samples was clearly visible. For the intermediate loading the samples exhibited a visco-elastic properties due to inducing particles dislocations. At the end of this stage large dislocation occurring *en block* resulted in a visible plastic behavior of the samples and finally lead to crack nucleation and breaking the sample apart. The most interesting is, however, observing how size of used particles influenced each of the above stage. Although we used relatively small range of particle sizes the obtained results clearly shows that the most sensitive to the material composition is the intermediate loading stage when dislocations start to change the properties of materials and lead to a crack nucleation. On the other hand, the rather weak dependence of the critical strain and the strain when the crack fully breaks the sample into two pieces shows that this failure stage is rather insensitive to the material composition. However, in our judgement, the most important result of the performed simulations is reporting of the scaling of the critical load which the sample can withstand with inverse of the size of the smallest particles building the sample. This unexpected observation is further investigating and results will soon be published elsewhere. At this time we have no

physical explanation of the observed scaling.

4 ACKNOWLEDGEMENTS

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