Using posterior entropy for improving accuracy of location error estimation

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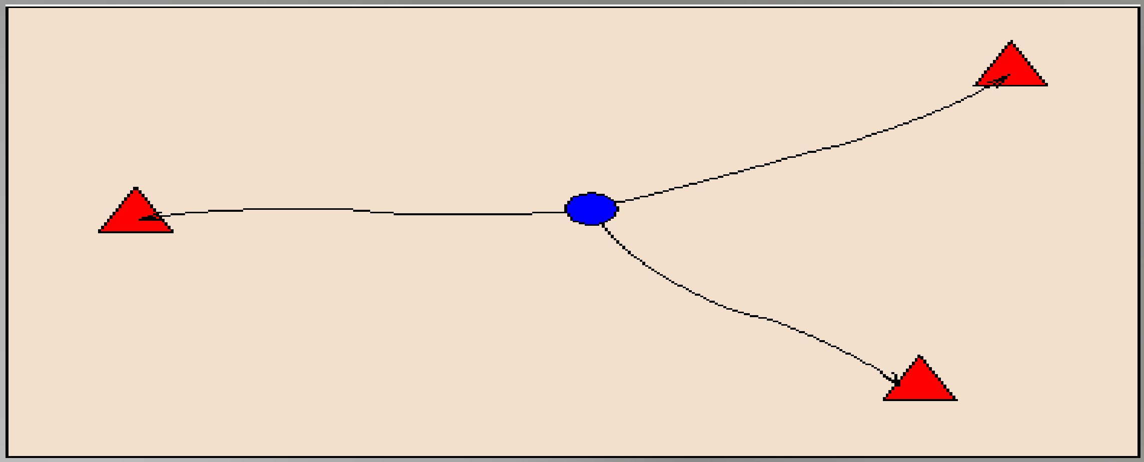
IUGG-2015, Prague

Hypocenter location



Goal:

$$(t_1,\, t_2,\, \ldots, t_N) \Longrightarrow \vec{r}_H \pm \vec{\Delta}$$

Primary: $\vec{r}_H$ (locate hypocenter)

Secondary: $\vec{\Delta}$ (estimate location errors)

A posteriori pdf

$$\sigma(\vec{r}) \;=\; f(\vec{r})L(\vec{r};\vec{t}^{\,o})$$

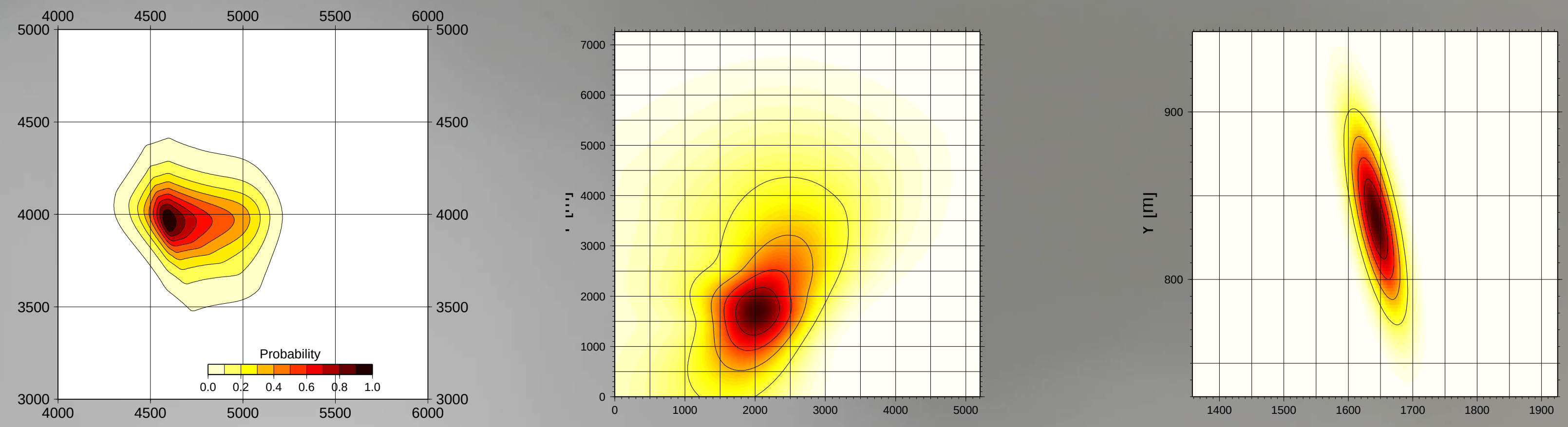
where

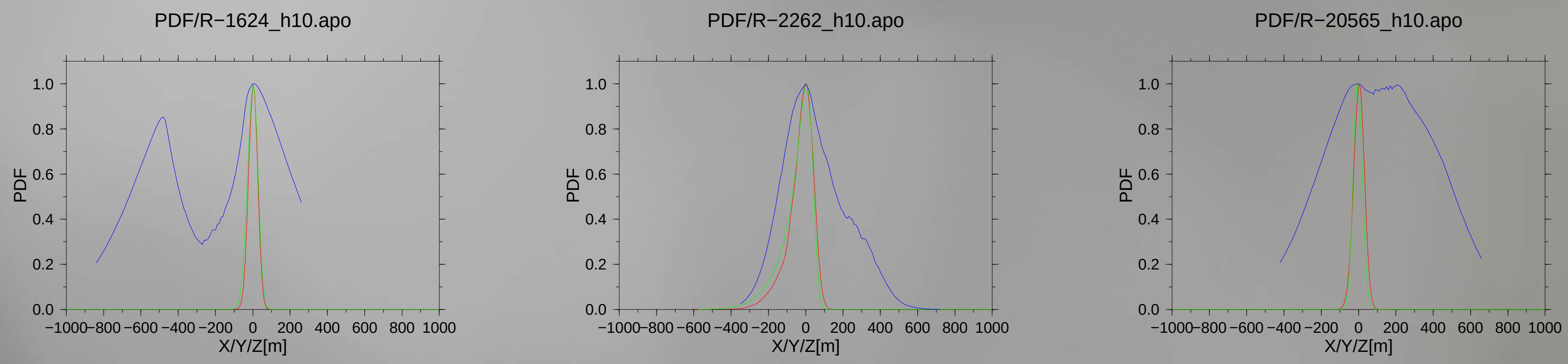
- $f(\vec{r})$ *a priori* pdf (arbitrary)
- $L(\vec{r};\vec{t}^{\,o}) = \exp\left(-||\vec{t}^{\,th}(\vec{r}) - \vec{t}^{\,o}||\right)$

for example

$$L(\vec{r};\vec{t}^{\,o}) = \exp\left[\frac{-1}{C_p^2}\sum_i \left(\vec{t}_i^{\,th}(\vec{r}) - \vec{t}_i^{\,o}\right)^2\right]$$

Bayesian inversion - solution





Bayesian inversion - subjectivity

To construct $L(\vec{r};\vec{t}^{\,o})$ we have to know errors statistics:

$$L(\vec{r};\vec{t}_{obs}) = \int \rho^o(\vec{t} - \vec{t}^{obs})\rho^m(\vec{t}^{\,th}(\vec{r}) - \vec{t})\mathrm{d}\vec{t}$$

if ρ^o or ρ^m are unknown we use *ad hock* assumed proxy statistics of sum of modelling and observational errors

$$L(\vec{r};\vec{t}^{\,o}) = \exp\left[\frac{-1}{C_p^2}\sum_i \left(\vec{t}_i^{\,th}(\vec{r}) - \vec{t}_i^{\,o}\right)^2\right]$$

but we introduce additional *a priori* parameters: $\mathbf{C_p}$

Consequences:

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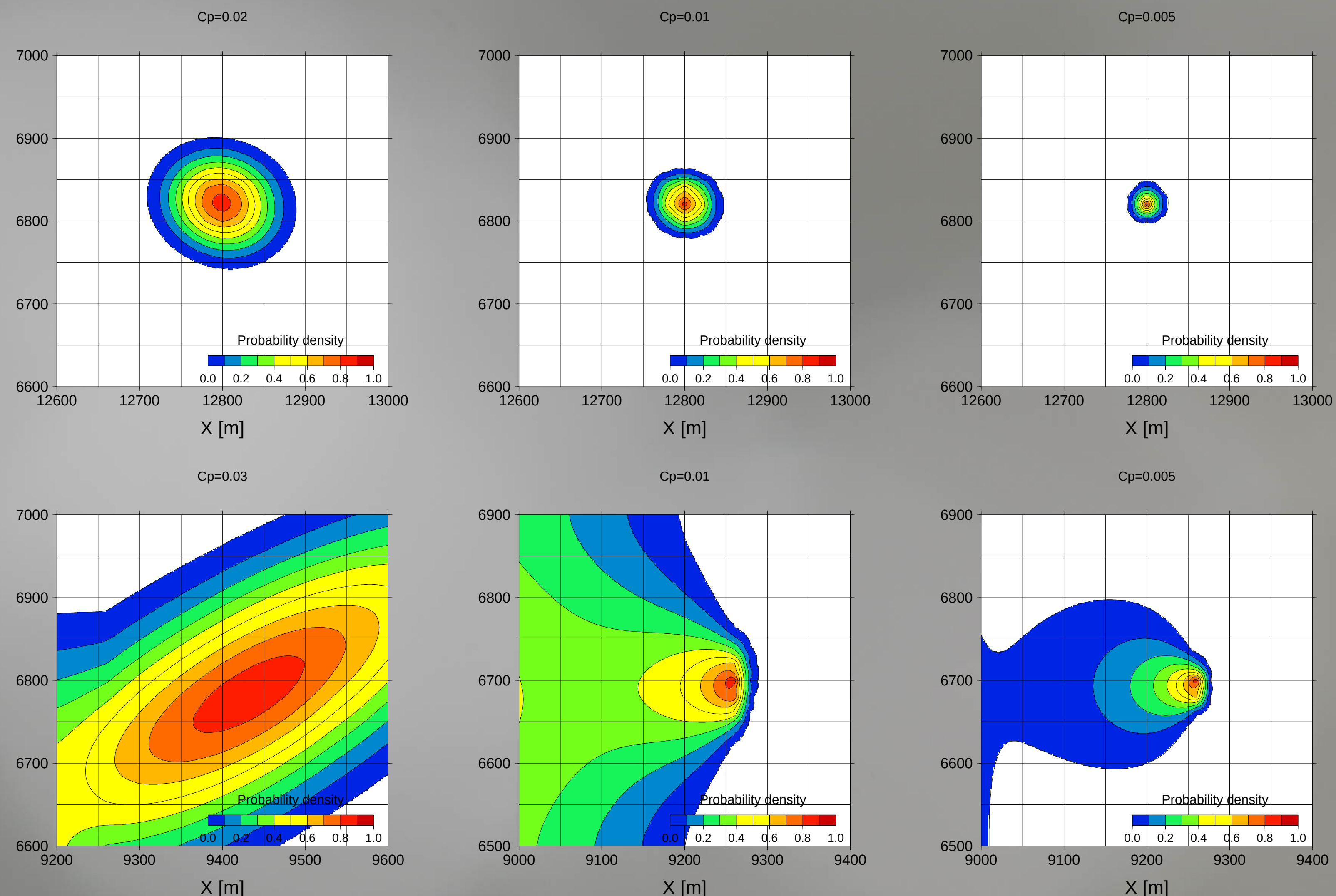
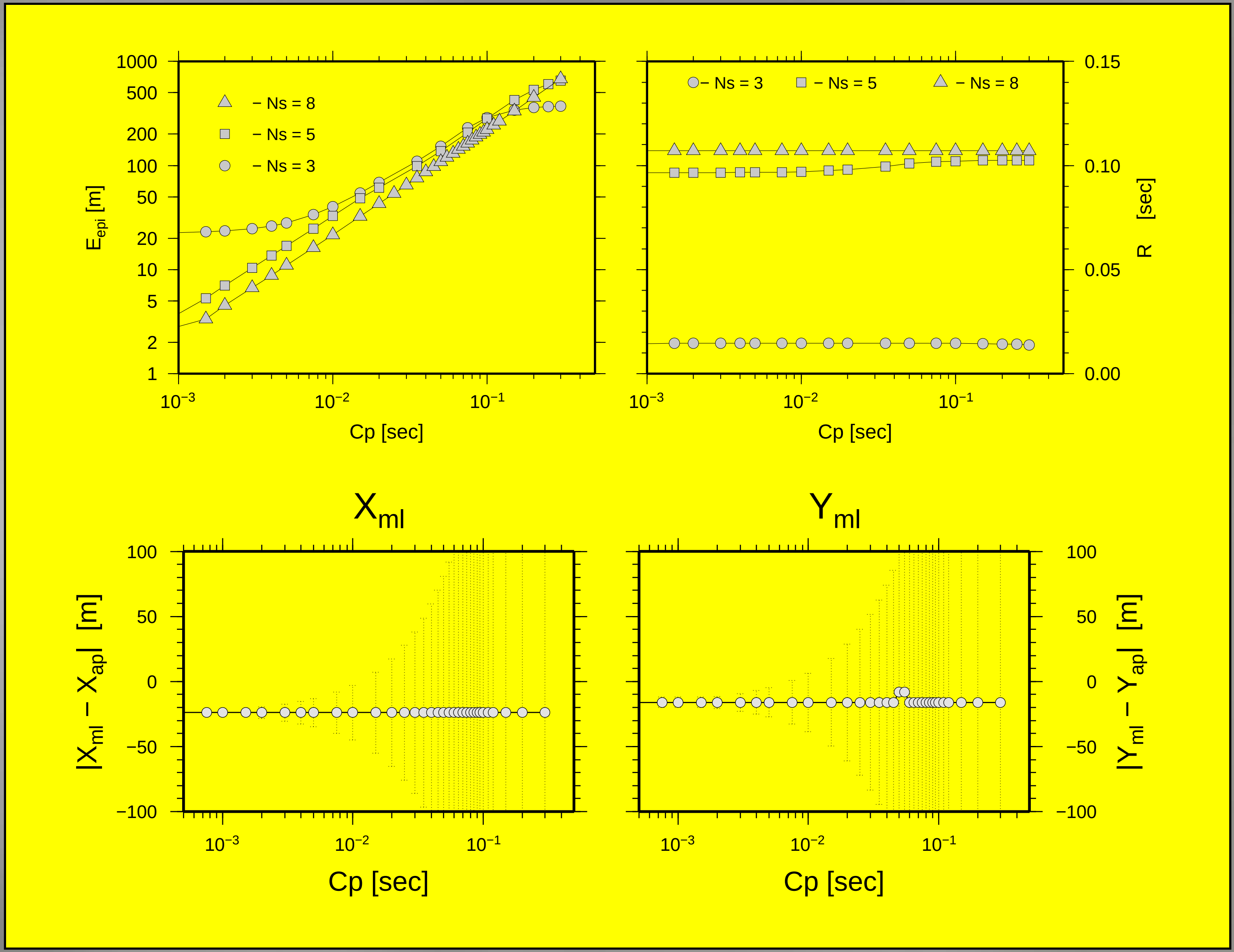
Errors uncertainties - subjectivity



Errors uncertainties - subjectivity



Error estimation - how to do it

Due to our negligence about modelling (observational) error statistics our estimates of location errors are quite subjective

Does observational and modelling data bring any information about quality (location errors) of solution? If so, it should be possible to infere it from $\sigma(\vec{r})$

We propose to look at the meta-characteristic of $\sigma(\vec{r})$ - the entropy

Methods

Assumptions:

- *a priori* Gaussian pdf:

$$f(\vec{r}) = const.\exp(-(\vec{r}-\vec{r}_a)^2/C_m^2)$$

- **Gaussian modelling and observational errors:**

$$\sigma(\vec{r};C_p) = f(\vec{r})L(\vec{r};C_p) = f(\vec{r})\exp\left[\frac{-1}{C_p^2}\sum_i\left(\vec{t}_i^{th}-\vec{t}_i^{o}\right)^2\right]$$

- Entropy:

$$H(C_p) = -\int_r \log\left[\sigma(r)\right]\sigma(\vec{r})\mathrm{d}\vec{r}$$

Methods

If our assumption about Gaussian error statistics is correct then $H(C_p)$ should follows theoretical dependence for normal 3D distribution

$$H_G = a + 3\ln(C_p)$$

Departure of $H(C_p)$ from this reference curve (below C_p^{opt}) marks breaking underlying (Gaussian errors) assumptions. Solution

$$\sigma(\vec{r}; C_p) = f(\vec{r}) \exp\left[\frac{-1}{C_p^2}\sum_i \left(\vec{t}_i^{\,th} - \vec{t}_i^{\,o}\right)^2\right]$$

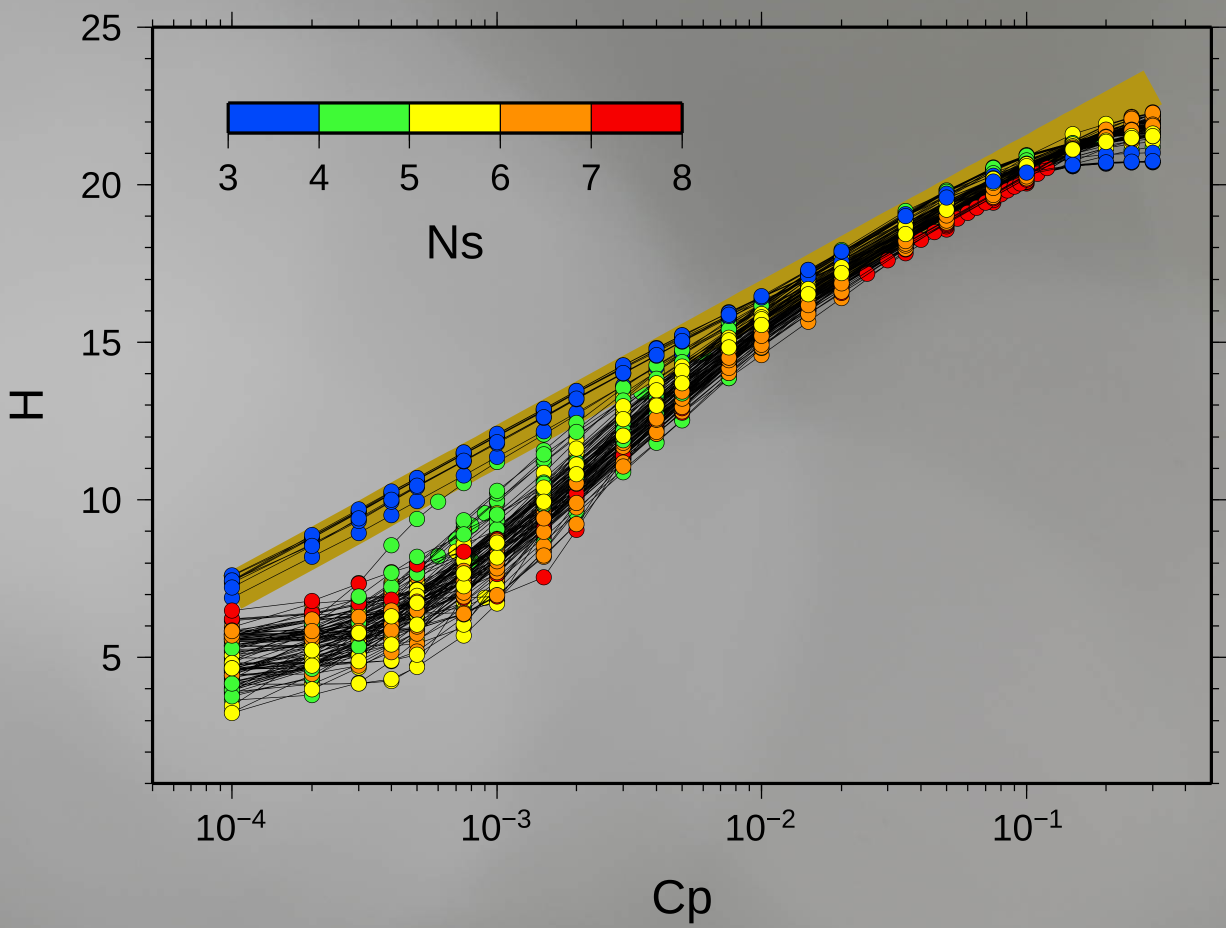
is thus valid for $C_p > C_p^{opt}$ and C_p^{opt} determines minimum consistent errors.

Data

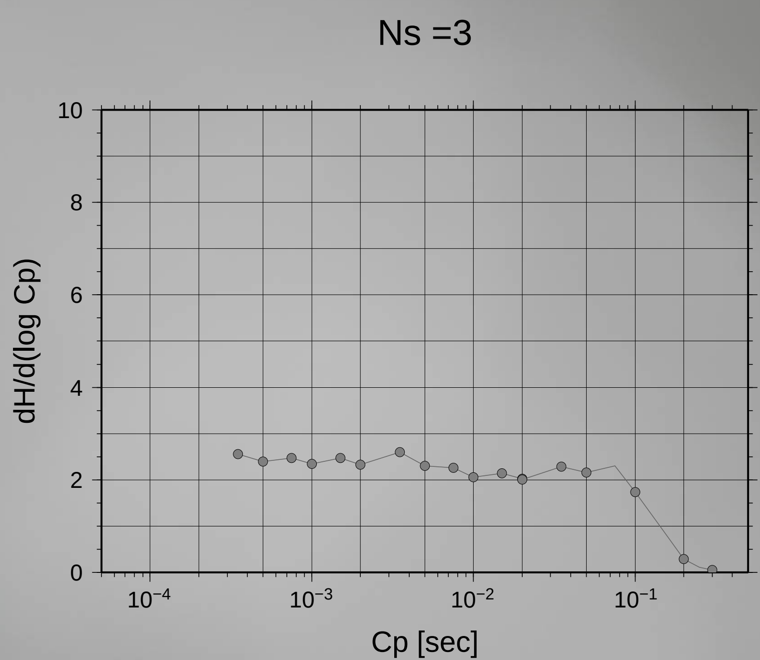
- 101 events from Rudna copper mine ($M_W \sim 2.1 - 3.0$)
- *a priori* solution provide by mine with (exagerated) accuracy $C_M \approx 500m$
- location based on 3-9 underground recording stations operated by mine
- relocation by very fast, fully Bayesian TRMLOC software (Acta Geophys, *submitted*)

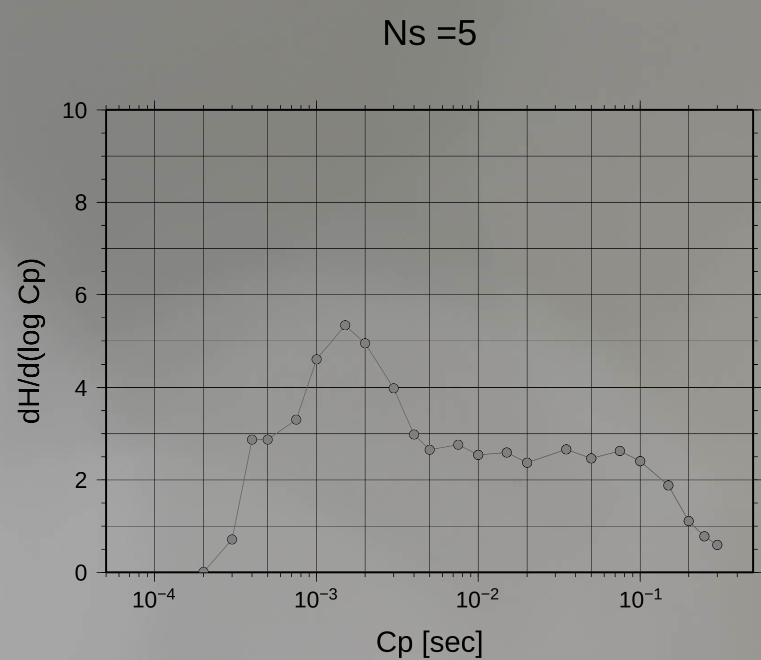
Results

H=H(Cp)



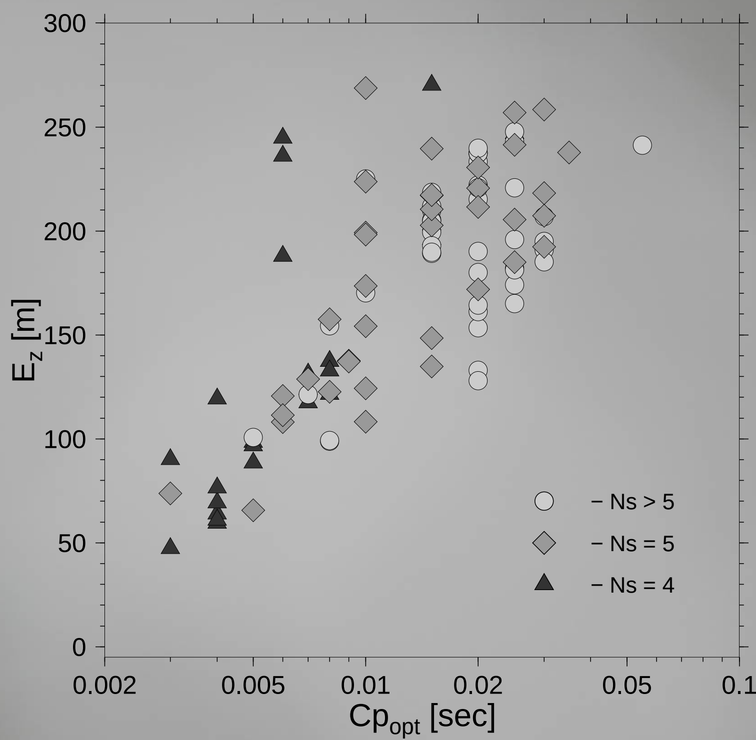
Results - optimum C_p



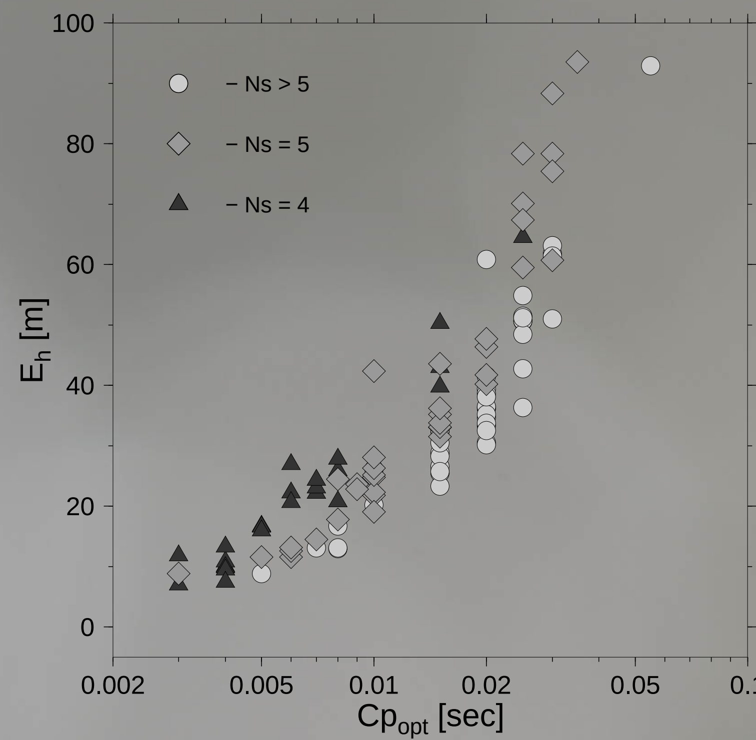


Estimated errors

Depth



Epicentral



Conclusions

Probabilistic (Bayesian) inverse theory provides tools for quantitative estimation of inversion errors. However, it requires knowledge of the statistics of modelling and observational errors. If unknown, the statistic of sum of these errors is postulated (likelihood function) but at the cost of introduction additional (*a priori*) set parameters. We have demonstrated that in the case of simple inverse task, when data are redundant it is possible to evaluate an “optimum” values of these parameters.

- W. Debski (2015), Using meta-information of a posteriori Bayesian solutionsof the hypocentre location task for improving accuracy of location error estimation, GJI,201 (3), doi:10.1093/gji/ggv083
- W. Debski, P. Klejment (2015) The new algorithm for fast probabilistic hypocenter locations Acta Geophys., submitted
- W. Debski, (2010), Probabilistic Inverse Theory, Adv. Geophys. 52, doi: 10.1016/S0065-2687(10)52001-6

Thank you

Acknowledgements:
Research supported in part by NCN under project 2011/01/B/ST10/07305